**Quantum Technology and Quantum Phenomena in Macroscopic Systems**
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**Lecture - 31**
**Cavity Optomechanics: Basic Physics - I**

Hello, welcome to 2nd lecture of Module 3. This is Lecture number 23 of the course. In this lecture, initially, we will discuss the very important parameter called quality factor of Fabry-Perot cavity. Then, we will try to guess the Hamiltonian of a typical cavity optomechanical system based on reasoning. Then, we will start discussing the basic physics of a cavity optomechanical system assuming the radiation pressure force to be an instantaneous function of the movable mirror. So, let us begin.

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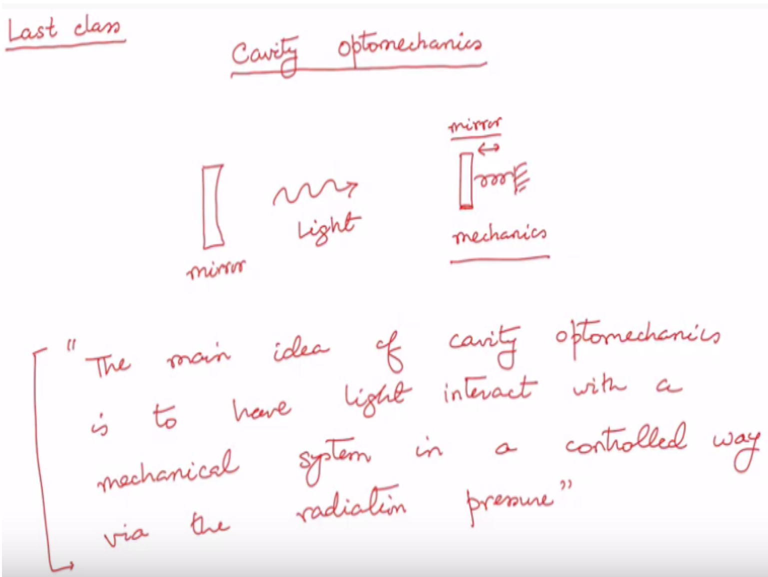


In the last class, we started discussing cavity optomechanics. A typical cavity optomechanical system consists of 2 mirrors, where one of the mirror is fixed and the other mirror is movable or we can consider it as a mechanical oscillator. The main idea in cavity optomechanics is to make light interact with the mechanical system or the mechanical oscillator in a controlled way by the higher the so called radiation pressure force.

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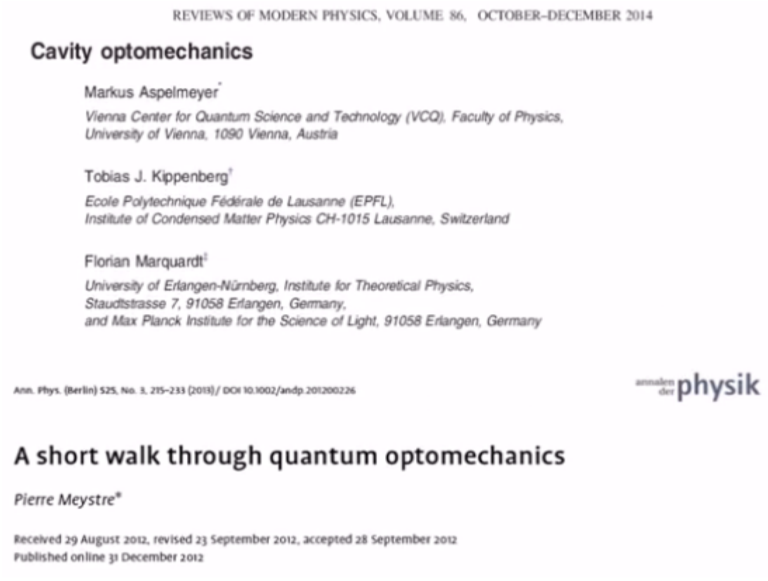
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**Cavity optomechanics**

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**A short walk through quantum optomechanics**

*Pierre Meystre**

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And, as I said, the 2 articles that we are going to rely you know our discussion is one by Markus Aspelmeyer, Tobias Kippenberg and Florian Marquardt. That is Cavity Optomechanics. And, another one, this article by Pierre Meystre called A Short Walk Through Quantum Optomechanics.

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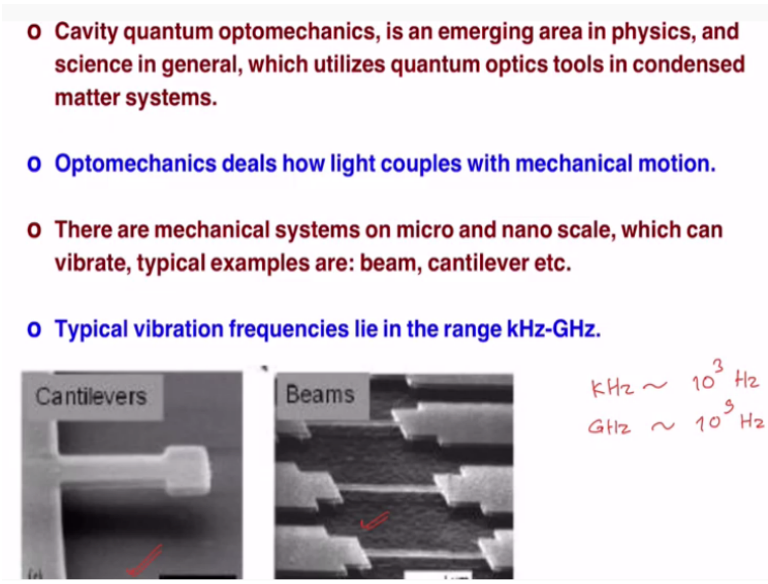


So, cavity optomechanics, which is also shortly called quantum optomechanics more popularly. Now, it is its known like that, is an emerging area in physics and which utilizes quantum optics tools in condensed matter system. And, these systems are typically cantilevers, beams and these are oscillators. These oscillators vibrate in the frequency range of kilohertz to gigahertz.

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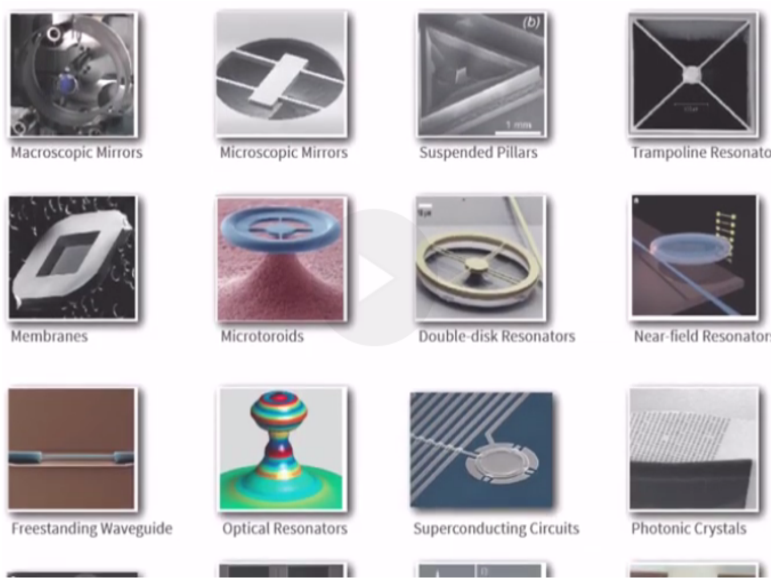


And, there are varieties of system, in fact, the plethora of such systems are available which we I pointed out in the last class.

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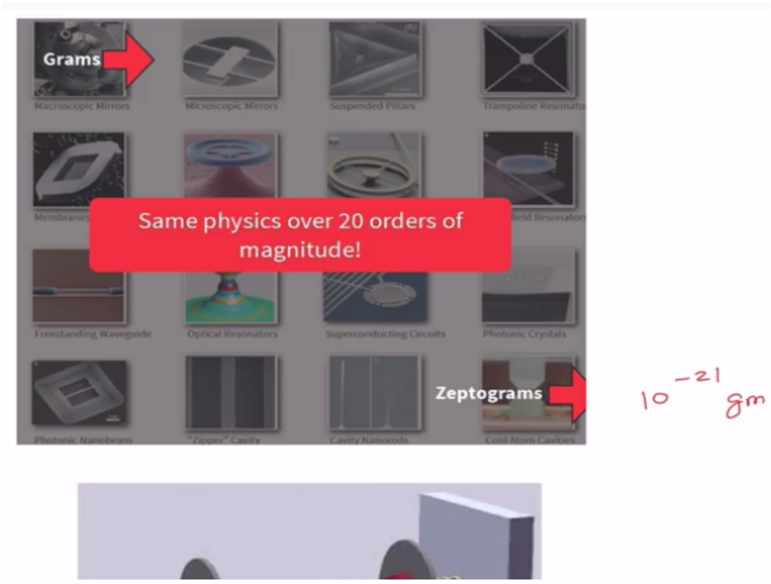

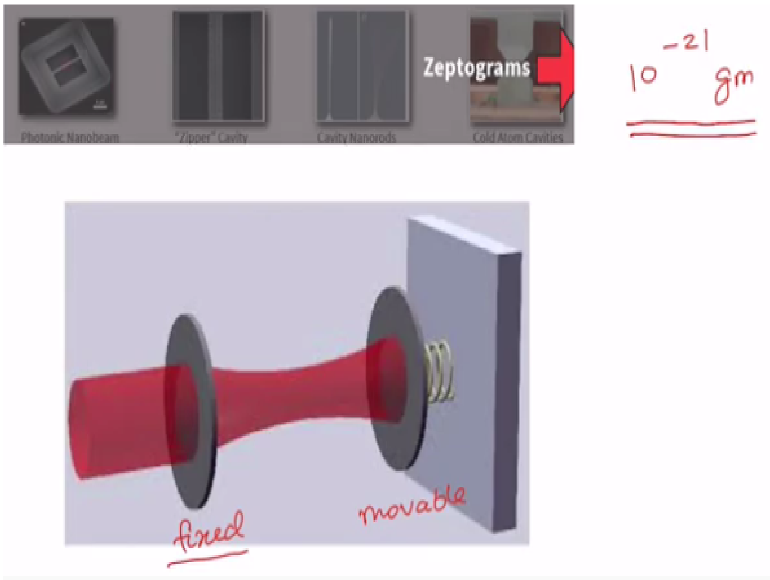


And, the sizes in terms of the masses, they can be from 1 gram to very small mass very tiny mass even in the range of zeptogram. That is say 10 to the power minus 21 gram is 1 zeptogram. Interestingly, the same physics over these 20 orders of magnitude is applicable. And, what is striking is that all these models various looking different looking or systems or optomechanical devices can be modeled by this simple model where we have one fixed mirror and another one is the movable mirror.

And we can, we will see in today's class, how we can actually model it a typical Hamiltonian. We can write for this particular cavity optomechanical system. And, this Hamiltonian would contain the all the necessary physics.

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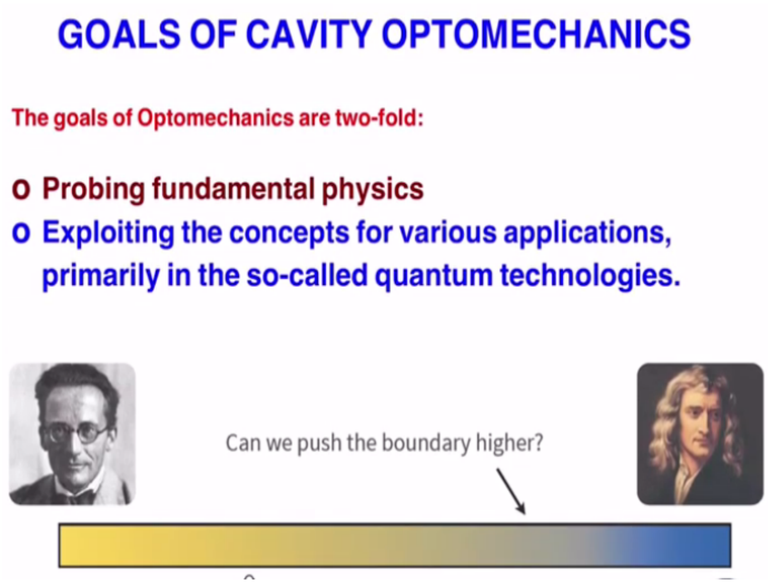

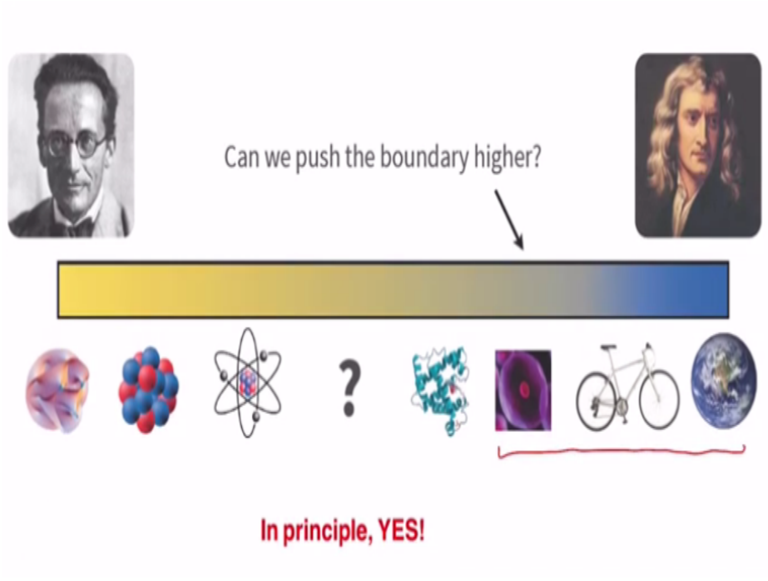


And, goals of cavity optomechanics are 2 fold. Firstly, to probe, not necessarily firstly, one of the goal is to probe fundamental physics and to exploit these concepts for various applications primarily in the so called quantum technologies. And, one of the very ambitious goal is to push the so called classical and quantum boundary. And, the idea is to whether we can push this quantum boundary towards the higher.

That means whether we can apply quantum mechanics to massive objects like this. And, that is one of the very ambitious goals. In principle, yes, we can do that.

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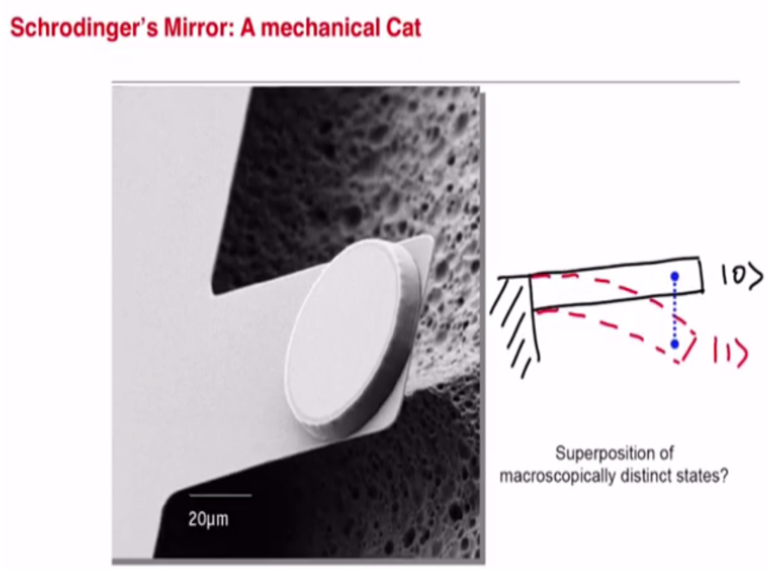


And, in this regard, the so called Schrodinger cat problem that has a mechanical analog where we have this cantilever and we can create a situation where this cantilever maybe in a superposition of 2 states. One location is denoted by ket 0 and another one is ket 1. This kind of superposition state can be created in the laboratory. And, this way, we will be able to test quantum mechanics in an entirely new domain.

And, also to, it is possible to produce non classical states of heavy mechanical object and test quantum mechanics.

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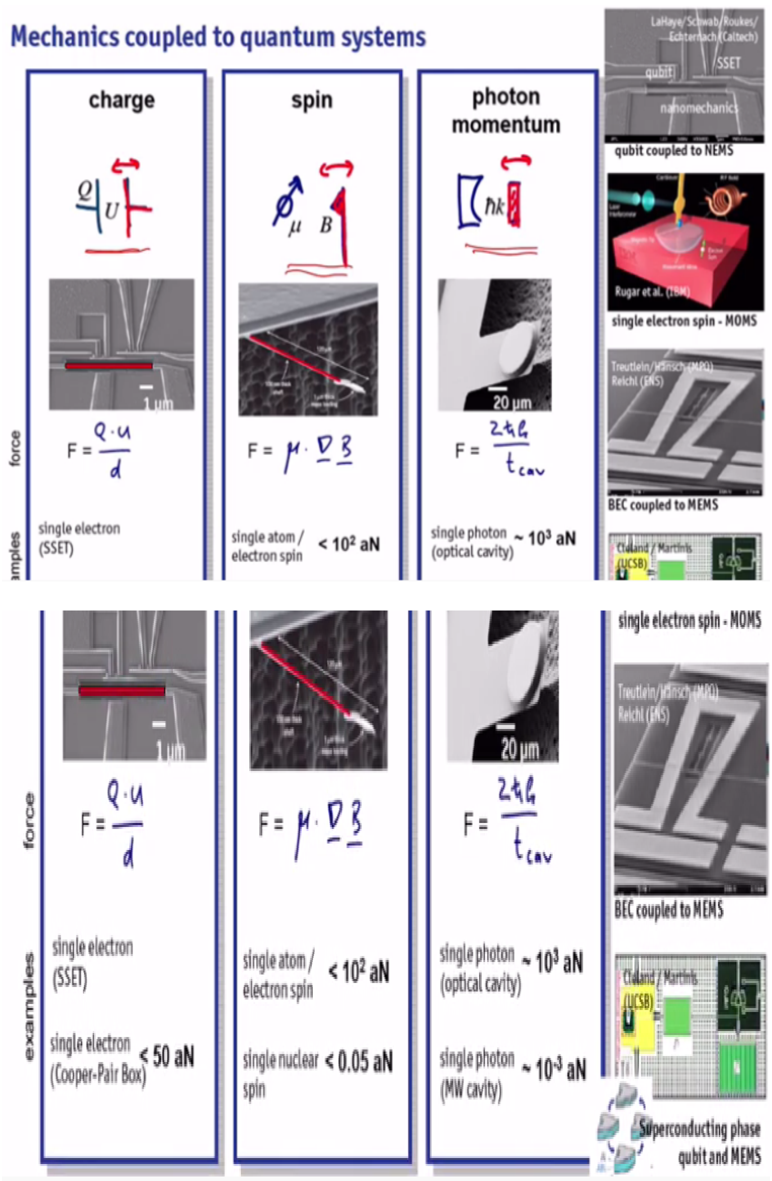


One main advantage of this mechanical system is that they can be coupled to various quantum systems and they can act as a transducer as I explained in the last class. And this is very important actually very useful. As we will see later on, to just give an example or to remind you, for example, we cannot directly transfer our information that is stored in a superconducting qubit or a circuit to a optical fiber, because we cannot couple it directly to light.

What we can however do is that we can couple the superconducting circuit or that qubit to a mechanical system. And, that mechanical system then can be coupled to optical fiber. And, that optical fiber will carry away the information.

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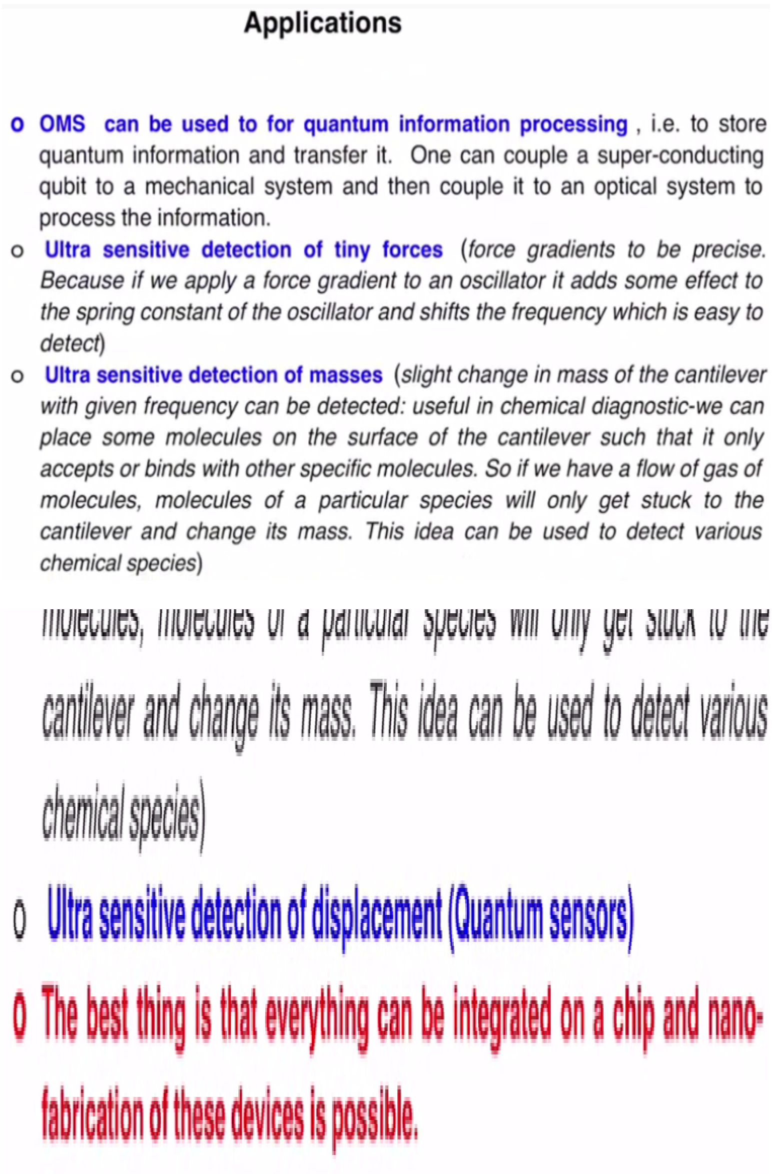


Now, there are various applications we talked about one is to so called quantum information processing, then ultra-sensitive detection of very tiny forces, ultrasensitive detection of masses and detection of very sensitive detection of displacement. And, the best thing is that everything can be integrated on a chip and nano-fabrication techniques are already available. So, solid state system in particular is mostly studied in this regard.

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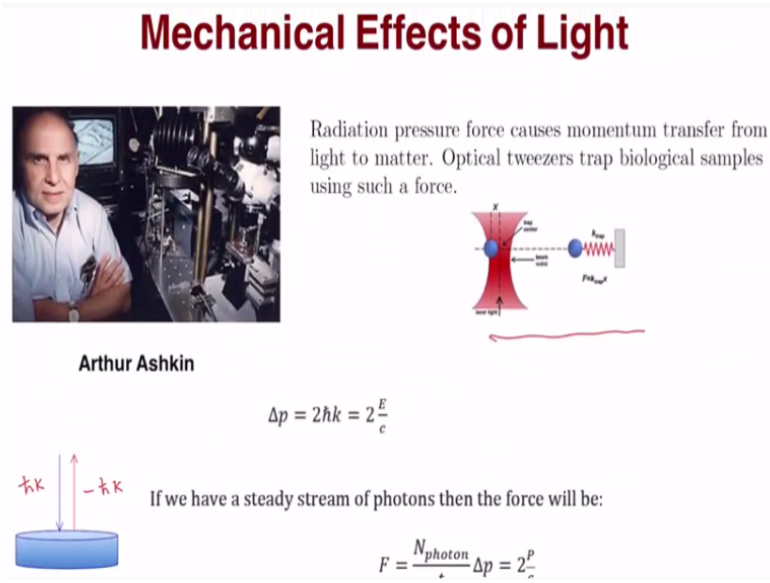


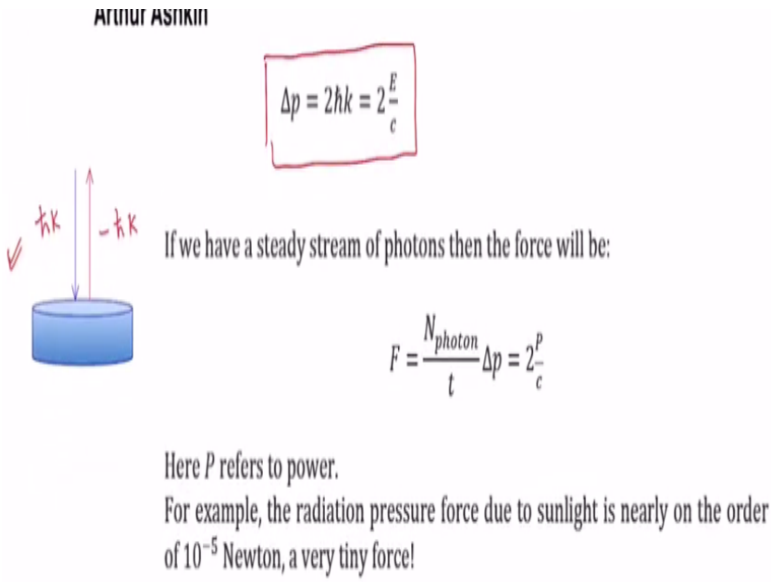


The main mechanism behind this interaction between light and mechanics is the so called radiation pressure force. And, we discussed about it. It was primarily due to Arthur Ashkin who started this area and he studied these things in great detail. It is very easy to understand. Say, a photon is getting incident and then with momentum h cross k. Then, if it is reflected from a perfectly reflecting mirror the momentum that after reflection would be minus h cross for the photon.

Therefore, change in momentum would be twice h cross k. And this way, we can find out the force if we have a stream of n number of photons. And, that relation of the force would be 2 into power divided by c.

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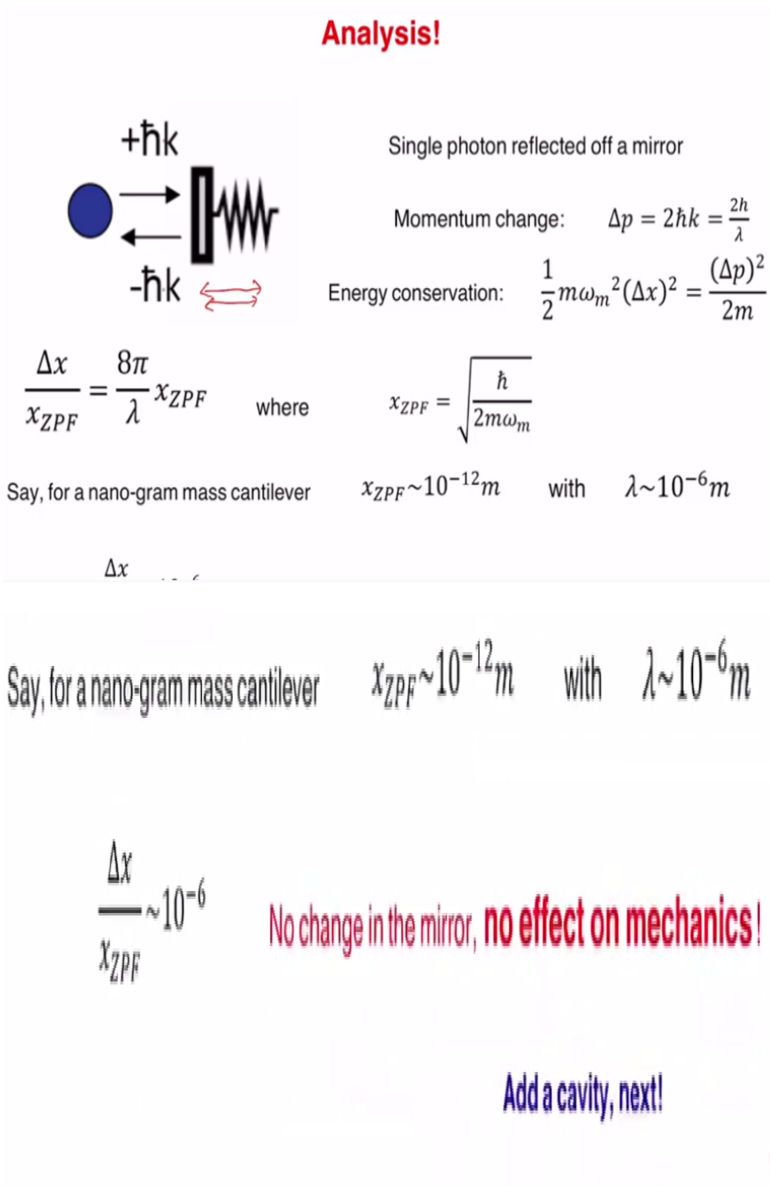


And, we analyze it little bit in details. If we just have a one mechanical oscillator or a vibrating mirror like this, then it turns out that when a photon is interacting with the displacement of the mirror is actually very small. In fact, it is not going to, it is far smaller than the so even the so called zero point fluctuations. So, therefore, it is impossible to observe quantum effects.

In order to observe quantum effects, the displacement of this mirror has to be greater than the fluctuation due to the, or displacement due to the vacuum fluctuation. And, that can be from our analysis we see that can be possible if we put another mirror.

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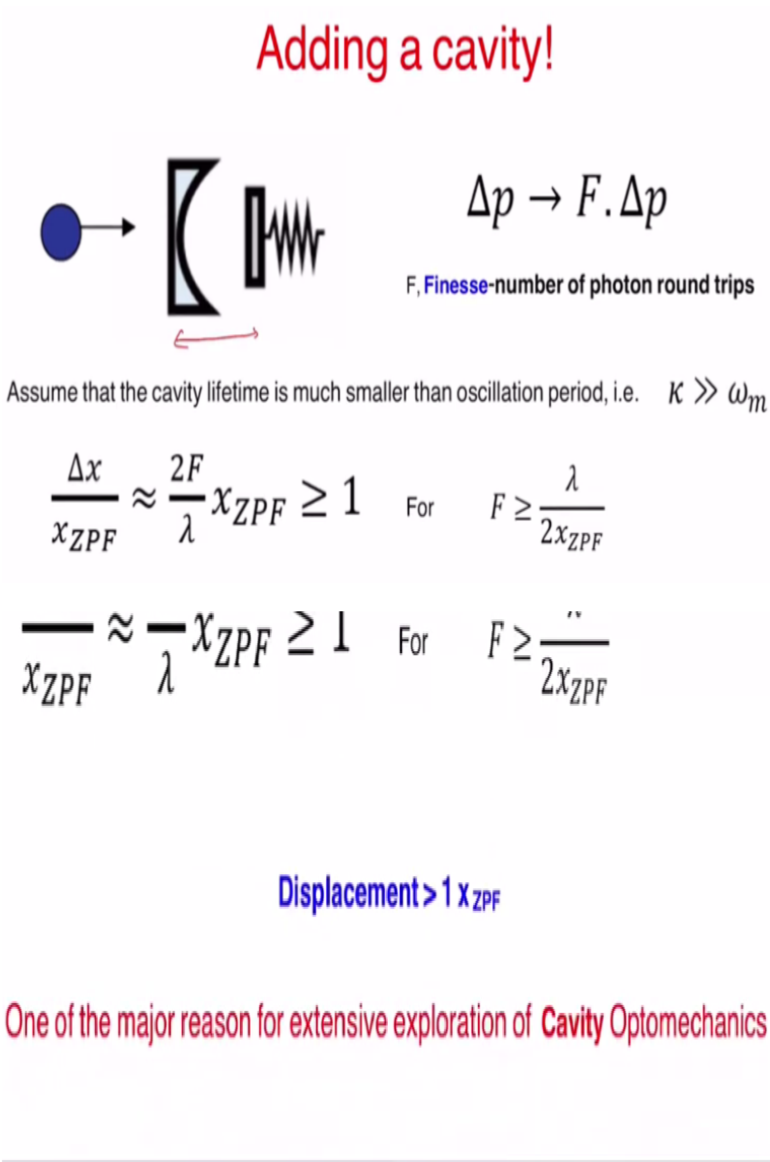


That means we make it a cavity. When this is the reason, why this cavity system is so popular and so effective?

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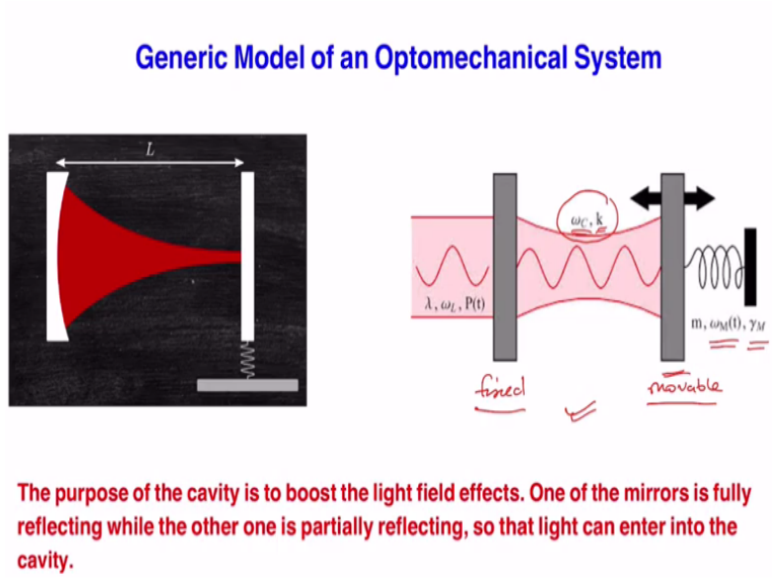


And, thus, as I say it already the generic model of a optomechanical system is represented by this diagram here as it is shown. We have a fixed mirror here. And, another mirror is movable. And, this is movable mirror because this is a vibrating oscillator. So, we can consider this vibrating oscillator has some mass m and oscillation frequency omega M and the phonon, quanta of vibration is phonon.

The phonon may decay at the rate of gamma M. On the other hand, the cavity is, this is characterized by the cavity resonance frequency omega C and the cavity decay photon decay rate that is kappa and a laser is getting incident on this mirror and that has a wavelength lambda and frequency angular frequency omega L and power P. So, the purpose of the cavity is to boost the light field interaction.

Or, and, this is already I said that one of the mirror is fully reflecting. For example, this one is fully reflecting while the other one is partially reflecting so that we can get transmitted beam from inside of the cavity and that transmitted beam if we make measurement that is going to tell us a lot of things what is going on inside the cavity.

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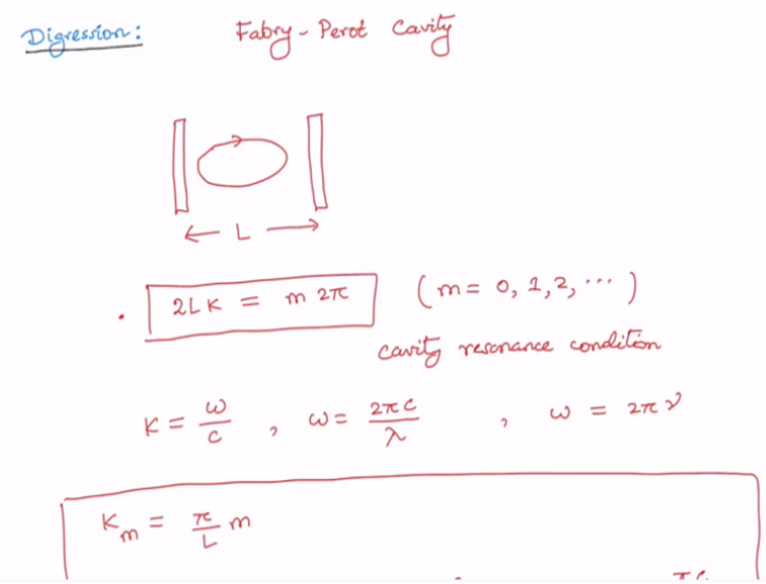


As you see that the Fabry-Perot cavity is basically the backbone of the whole system. So, therefore, we discuss Fabry-Perot cavity in some more details. And, we firstly, we talked about the cavity resonance condition. This cavity resonance condition can be written in various forms.

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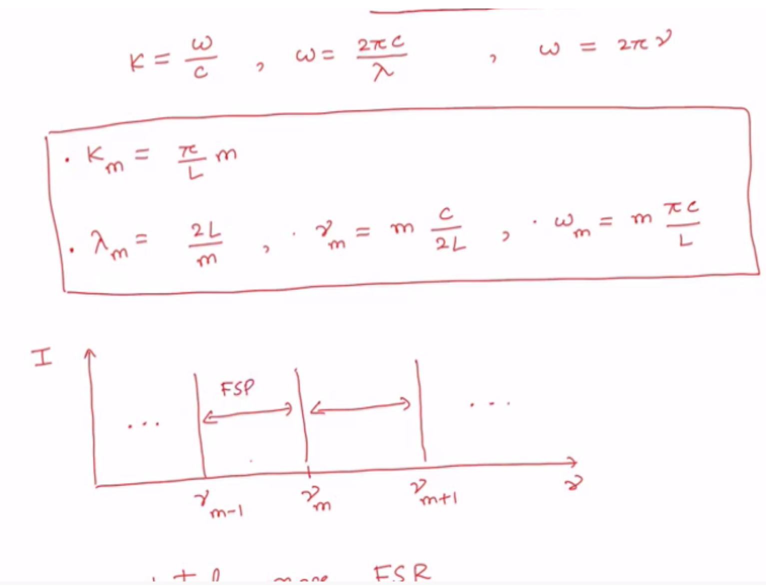


So, what does it basically means? That if light is to enter into the cavity then this resonance condition has to be satisfied. Those light which has this wave vector like this or wavelength satisfy this condition or frequency or angular frequency that light will be able to enter into the cavity and will have a various peaks. And, these peaks would be separated in frequency. And, this separation is known as free spectral range.

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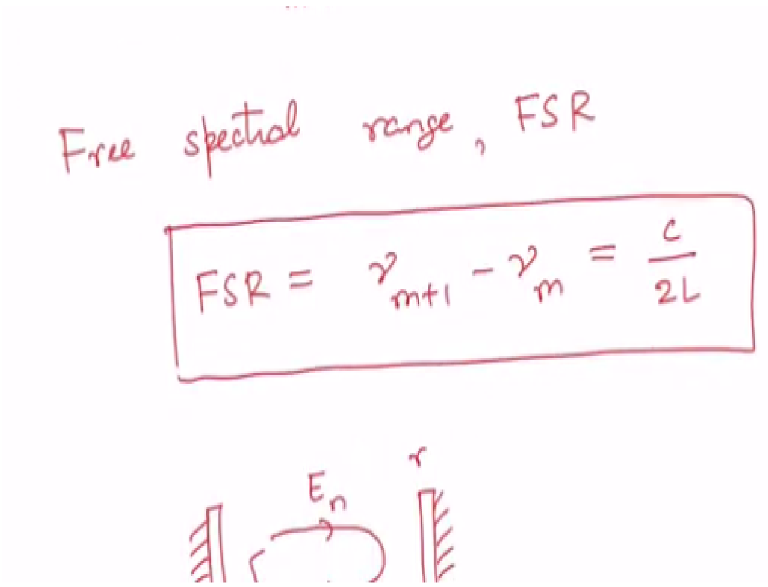


This is a very important quantity and that is given by c by 2L.

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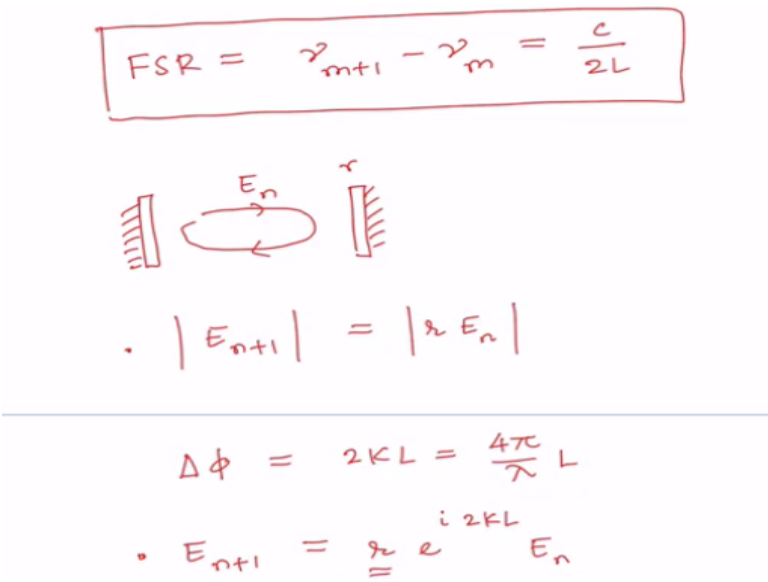


And then, we went on to work out the intensity inside the cavity.

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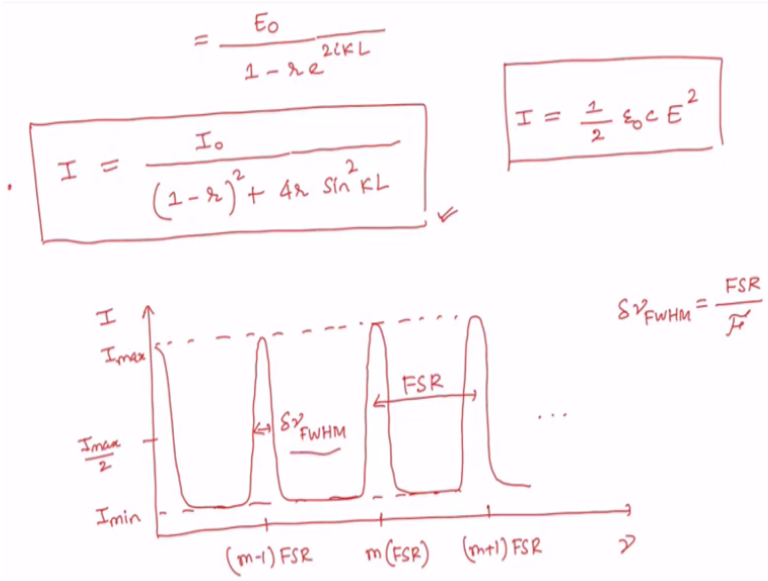


And, we found out this expression there. From here, we can also find out that these resonance peaks has some width. These are called known as the full width at half maximum. And, or, it is also called a bandwidth. And, we calculated it.

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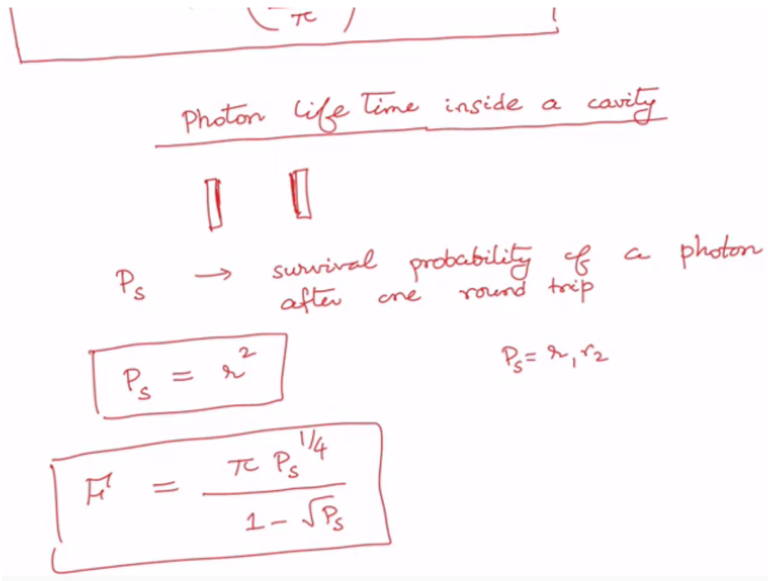


And then, we finally in the last class, we discussed about photon lifetime inside the cavity, because photon would not stay inside the cavity for eternity. After some time, it will leak away. And, there is a certain lifetime of the photon inside the cavity. And, we went on to work it out assuming that the survival probability of the photon is very high because generally these mirrors are very high quality mirrors.

So, because there will be hardly there will be very few losses or the reflectivity is around 98 to 99%. So, survival probability of the photon is pretty large. So, under that approximation, then we went on to work out the photon lifetime.

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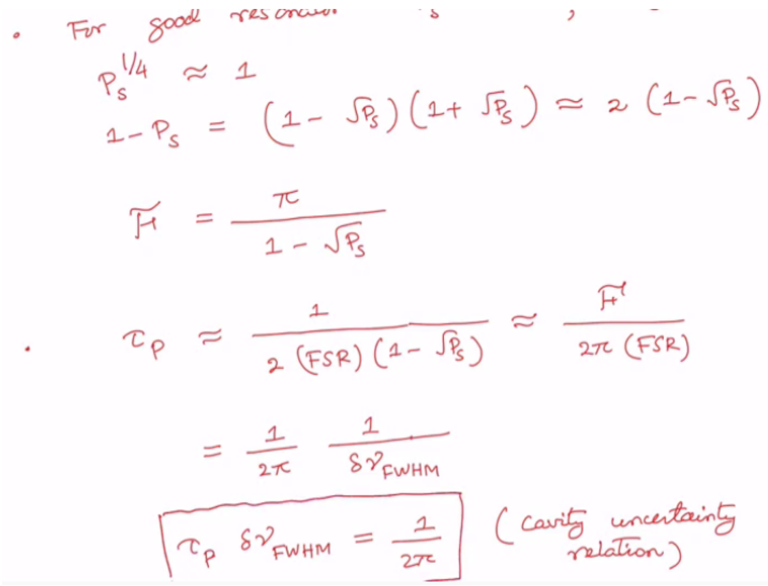


And, we worked out an interesting relation. And, this relation is called a cavity uncertainty relation, where this is basically the product of the cavity lifetime and the bandwidth is equal to 1 by 2 pi. So, this is an important relation. It tells about depending on the bandwidth. This basically means that if the bandwidth is very small, full width at the half maximum, then the photon lifetime would be larger.

So, that means, if you have a very sharp peak, then from that we can say that the photon will be able to survive inside the cavity for longer amount of time.

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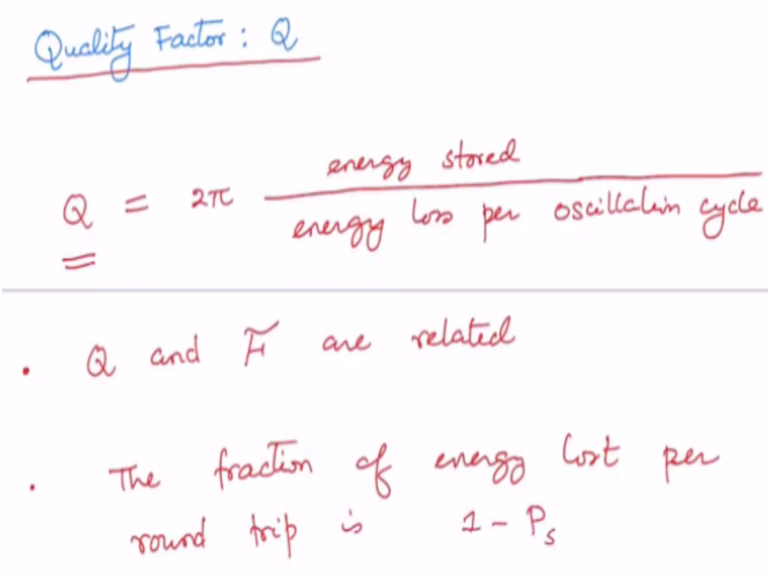


Now, we are going to discuss a very important quantity. That is the so called quality factor. I think all of you know how quality factor of an oscillator is defined. It is defined as it is the ratio between energy stored by energy loss per oscillation cycle. And, it is multiplied by a factor of 2 pi. And, in cavity optomechanics, we need a cavity with high quality factor because higher is the quality factor higher would be the number of oscillations of the photon inside the cavity.

In fact, finesse also has the same meaning. Higher the finesse, higher would be the number of oscillation of the photon before it leaked away from the cavity. And, we will see shortly that quality factor Q and finesse are related. In fact, they are actually the same thing. But, we will see the relation between them very soon. Now, you know the, if the survival probability of the photon inside the cavity is P s, then for an optical cavity, the fraction of energy lost per round trip is 1 minus P s.

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→ The rate at which energy leaves the cavity is $\frac{1}{\tau_p}$

→ $$P_s^n = (P_s)^{t/t_{rt}}$$
$$= e^{t/t_{rt} \log_e P_s}$$
$$\approx e^{-t/t_{rt}(1-P_s)}$$
$$\approx e^{-t/\tau_p}$$

The rate at which energy leaves the cavity is inverse of the photon lifetime, so, 1 by T P because T P is the photon lifetime and it is relative to the amount of energy that is stored in the cavity. We can understand it this way also because you see that in terms of survival probability the probability to survive n round trips in the cavity would be P s to the power n. For 1 round trip per survival probability is P s. For n round trip, it is P S to the power n.

This, we can write as P s to the power. This number of round trip would be time divided by 1 round trip time. This we can write in this form, e to the power t by t rt. That is the round trip time into logarithm of P s with base e. So, these I can I will explain. This I can also write it as e to the power minus t by t rt 1 minus P s. In fact, this I can write because I already know the expression for the lifetime of the photon. That is tau P.

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$$\approx e^{-t/\tau_p}$$

$\log(1+x) \approx x$ for small $x = 1 - P_s$

$$\tau_p = \frac{t_{rt}}{1-P_s}$$

→ Energy loss rate is $\frac{1}{\tau_p}$

→ Energy loss per oscillation cycle

$= $ optical period × energy loss rate

$$= \frac{1}{\nu_m}\frac{1}{\tau_p}$$

While writing it I have used this, using this relation that logarithm of 1 + x. For very small x, I can write it simply as x for small x where I am taking x is equal to say 1 minus P s. So, and photon round trip, photon lifetime is round trip time divided by 1 minus P s. So, using this I see that the energy loss is exponential in time. So, and it has a loss rate is 1by T P. So, this way also we see that energy loss rate is 1 by tau P.

So, in fact, the energy loss per oscillation cycle is just the optical period multiplied by this rate. So, I can write that energy loss per oscillation cycle is equal to optical period multiplied by energy loss rate. An optical period is in terms of frequency. I can write it as 1 by nu m. That is the resonance frequency of the cavity. And, this loss rate is 1 by tau P. So, you can with respect to the stored energy.

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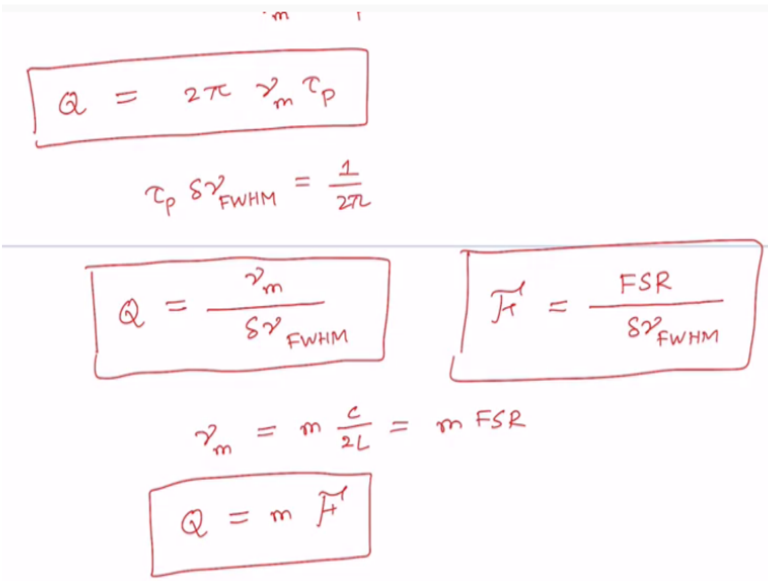


So, therefore, the quality factor as you can see it would be 2 pi into we have simply nu m tau P. So, this is the expression for the quality factor. Now, due to the, this relation that we had earlier the uncertainty relation, the tau P into delta nu full width at half maximum is equal to 1 by 2 pi. So, clearly we can write that quality factor is equal to the resonance frequency divided by the full width at half maximum. So, this is what we have.

By the way if we compare with the expression for the finesse which was defined as free spectral range divided by delta nu at full width at half maximum. This is the bandwidth and if I compare this relationship and also you note the fact that this resonance frequency is basically integer into c by 2L which is nothing but the free spectral range. So, immediately we get a relation between the quality factor and the finesse.

That would be Q is equal to integral multiple of this finesse. So, both of them are basically essentially gives us the idea whether it is quality factor or finesse. It essentially gives us the idea about the number of oscillation a photon or light makes inside the cavity. We have now learned enough about Fabry-Perot cavity. Let us now go back to our cavity optomechanical setup.

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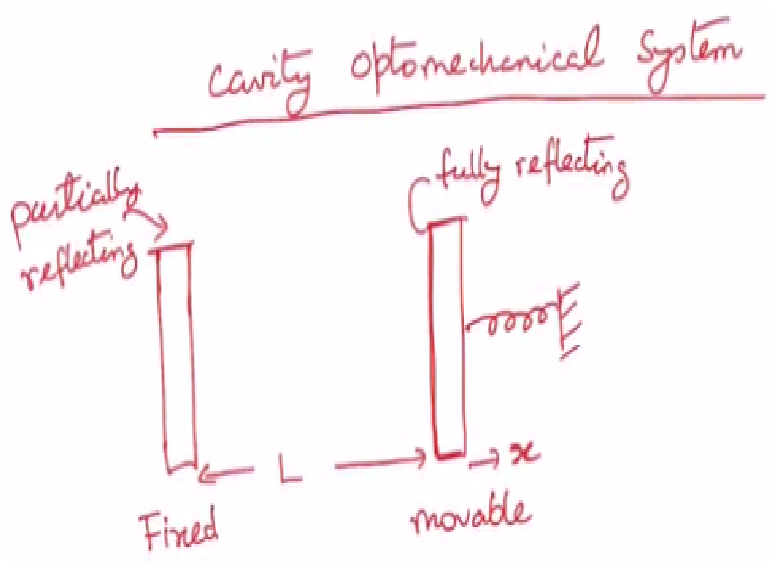


In the cavity optomechanical setup, we have one mirror fixed and the other mirror is movable. Let us say the other mirror is attached to some spring. This mirror is fixed. And, this mirror is movable. And, as you can see, if this mirror is not movable, then this is the typical Fabry-Perot cavity. And, the length between these 2 mirrors is say L while this particular mirror is also fixed, but it can of course, displaced by some amount say x.

And, this is the typical cavity optomechanical setup. Or, I can just say cavity optomechanical system. This is a prototype. One of the mirror say this one is or this one. Let us say this one is fully reflecting. Say, this is fully reflecting. And, this particular mirror is partially reflecting partially reflecting or transmitting so that light can enter into the cavity. As you will see later that this displacement x of the movable mirror turns out to be an very important variable. Now, let us guess the Hamiltonian.

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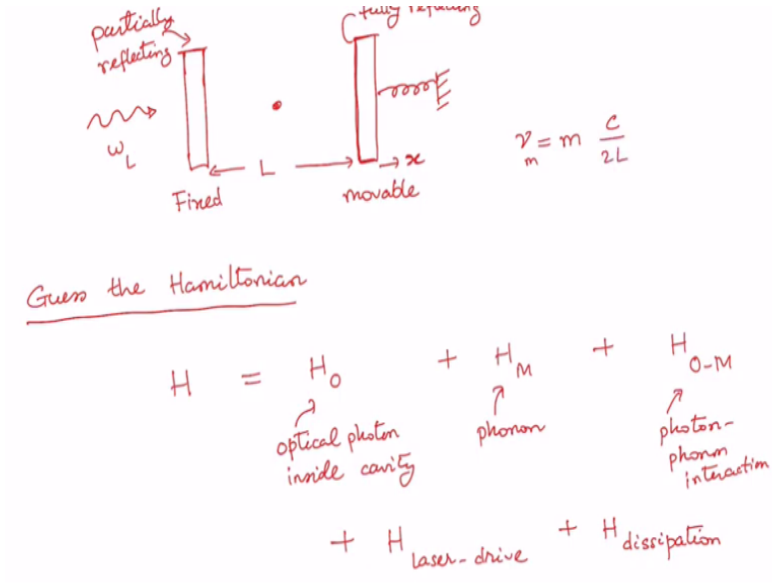


So, now, we will do the guessing work and later on we will of course study this Hamiltonian in great details. Let us now guess the Hamiltonian which is not very difficult because you know that the total Hamiltonian of the system. Let us say it is say H is due to contribution of various parts. Let us say one contribution is going to come from the optical photon inside the cavity. So, say, optical photon inside cavity.

The other contribution may be from for phonon in the mechanical oscillator which is the movable mirror. So, this is due to the phonon. And, there will be interaction between the photon and the phonon. So, this is photon phonon interaction term would be there. This would be photon Hamiltonian corresponding to photon phonon interaction. Apart from that this cavity, we have to throw a laser light.

So, or, that means we have to drive the cavity by laser light. So, therefore, there will be a term which should be in the Hamiltonian total Hamiltonian and that would be there to represent the laser drive. And not only that, we will have dissipation of this photon will eventually decay from the cavity or even the phonons in the mechanical oscillator will also decay.

So, there will be a Hamiltonian which will, the term which will be responsible for all dissipative processes. So, let us say that is H dissipation. So, these are the contributions that will have, now, let me explain these terms one by one in little bit. Of course, we need to note one thing that this optical cavity contains infinity of normal modes of electromagnetic field.

However, this laser light that is getting incident on the system would have a fixed frequency will have some fixed frequency.

And, it will be only that light will be able to enter into the cavity which will be close to one of the optical modes. As you can, as you know that the, this cavity is going to support many modes because we already see that nu is equal to for Fabry-Perot cavity it is integral multiple of c by 2L. So, therefore, many modes are possible, but we will consider only that particular mode which will be close to the laser frequency incident laser frequency.

So, we will focus only on a particular optical mode. Now, the optical mode as we know is a harmonic oscillator. So, we can write that for the optical mode or a cavity mode.

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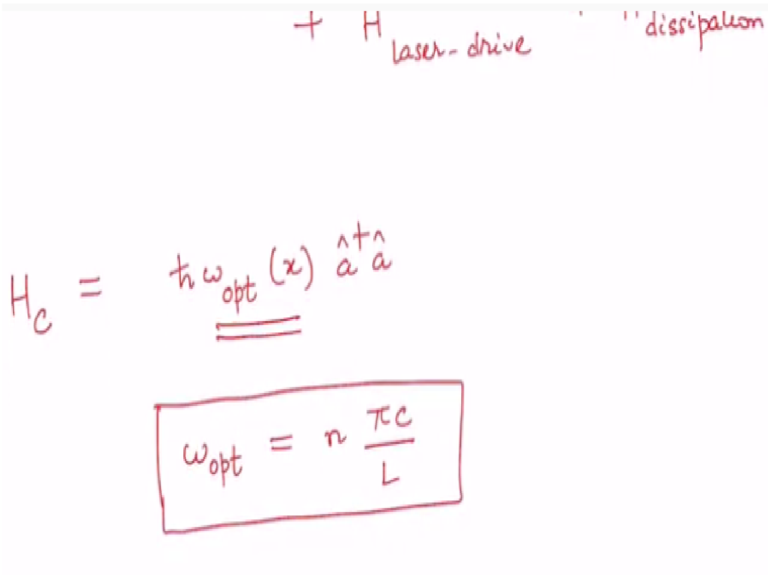


$$H_c = \hbar \omega_{opt}(x)\, \hat{a}^{\dagger}\hat{a}$$

$$\omega_{opt} = n\frac{\pi c}{L}$$

H c is equal to the energy. This is the harmonic oscillator energy. Say, optical frequency is angular frequency is omega opt. And, this is going to be a function of this displacement. I will explain that. This is the energy of an optical photon. And, that will be the number of photons having the same frequency. So, this is the Hamiltonian. But, optical mode frequency now here, this quantity depends on the displacement of the mirror.

If the cavity has fixed length, then we know that this resonance condition that would be some integral multiple let us say now integer is n into pi c by L. This is omega not nu. So, this is the resonance condition for a cavity with fixed length. Now, due to the small displacement of this n mirror, this mirror this movable mirror I can write omega opt.

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$$\omega_{opt}(x) = \frac{n\pi c}{L + x} \approx \frac{n\pi c}{L\left(1 + \frac{x}{L}\right)}$$

$$= \frac{n\pi c}{L}\left(1 - \frac{x}{L}\right)$$

$$= \omega_{opt}\left(1 - \frac{x}{L}\right)$$

This is a function of x now. I can write it as n pi c divided by L. There is a small displacement. So, I can write it like this. If I expand it because x is very small I can write this quantity in this way, n pi by L into 1 + x by L. Now, this quantity is very small compared to 1. So, therefore, I can write it as n pi c by L. I can expand it as 1 minus x by L. And, this quantity is nothing but the omega optical when with a fixed length. And, the whole thing then I can write it as 1 minus x by L.

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$$H_c = \hbar\omega_{opt}\left(1 - \frac{x}{L}\right)\hat{a}^{+}\hat{a}$$

$$= H_O + H_{O-M}$$

$$H_O = \hbar\omega_{opt}\,\hat{a}^{+}\hat{a}$$

$$H_{O-M} = -\frac{\hbar\omega_{opt}}{L}\,x\,\hat{a}^{+}\hat{a}$$

So, this will enable us to write this Hamiltonian for the cavity photon. That is H c is equal to h cross omega opt 1 minus x by L a dagger a. And, this I can further write and divide it into 2 parts. One I can write it as the optical photon H o. And, another one would be the interaction between the optical photon and the mechanical oscillator which is actually a, here in the quantity x. x is coming from the movable mirror or the mechanical oscillator.

So, this particular term would be responsible for the photon phonon interaction. So, in fact, H o is, as you can see, you can read it from here that from this expression, it would be h cross omega opt a dagger a. And, the introduction term would be minus h cross omega opt by L into x a dagger a. So, these are important quantities. It is easy to see that the radiation process what about the radiation pressure force.

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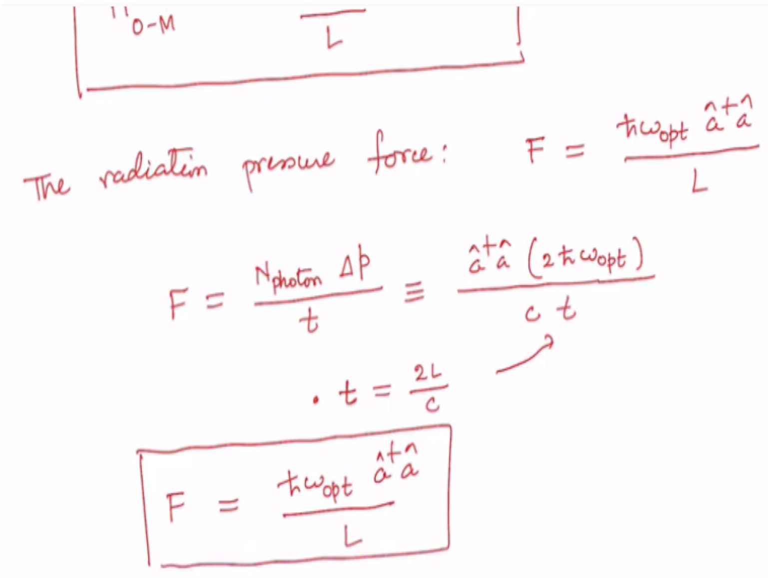


So, the radiation pressure force, it is easy to see that the radiation pressure force would be h cross omega opt a dagger a divided by L. Let me explain it. Earlier, we got this expression that the forces basically rate of change of momentum. And, if there are n number of photons are getting incident and for each photon the change in momentum after it is getting reflected from the mirror is say delta p.

And, this is the time. Now, what we have here N photon, the number of photon is a dagger a and delta p is twice h cross k which I can write it as twice h cross omega optical the that resonance frequency divided by c and this time t. But, this t is basically the round trip time. And, t is equal to say 2L by c. And therefore, because x is very small, so, I can take the round trip time to be like this.

So, therefore, the force expression if I put this here, then immediately you will see that we are going to get this expression h cross omega t omega opt a dagger a divided by L. So, this is the radiation pressure force.

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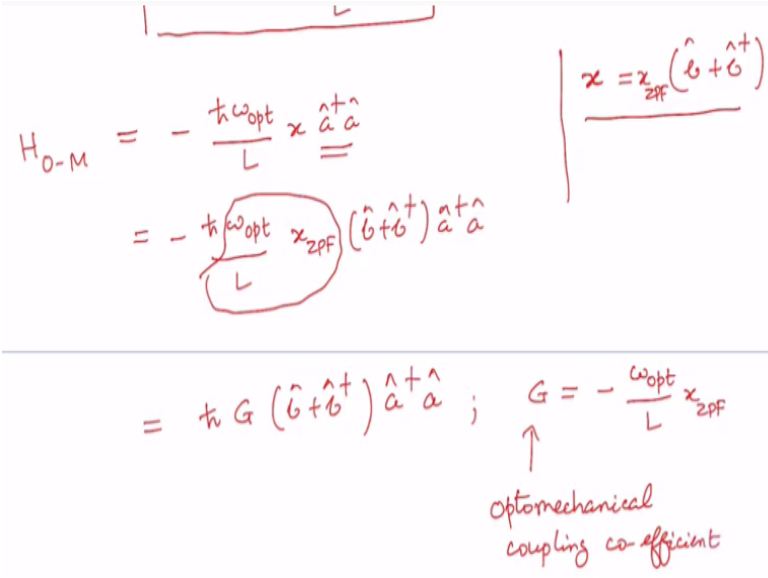


Now, the, this particular term mean the interaction term can be written little bit in a different way in a more familiar form actually. That would be h cross omega optical by L x a dagger a. This I can write as, because, you see this is x is the displacement of the movable mirror. So, this quantity we can write because I have already used this a and a dagger. So, for the, this is associated with a mechanical oscillator.

So, let us say the annihilation operator these things we discussed. So, we can write it as b + b dagger multiplied by the so called zero point fluctuation. So, if I use this expression here, then what I can do? I can write it as minus h cross omega optical by L. And here, I write x zero point fluctuation b + b dagger. And, we will have a dagger a. This whole thing because this particular quantity, this is the dimension of frequency.

I can write it as minus or in fact rather I will write it this way. I will write it as h cross capital G b + b dagger a dagger a where I am taking this capital G as minus omega optical by L x zero point fluctuation. So, this G is called the optomechanical coupling coefficient. So, therefore, what I have?

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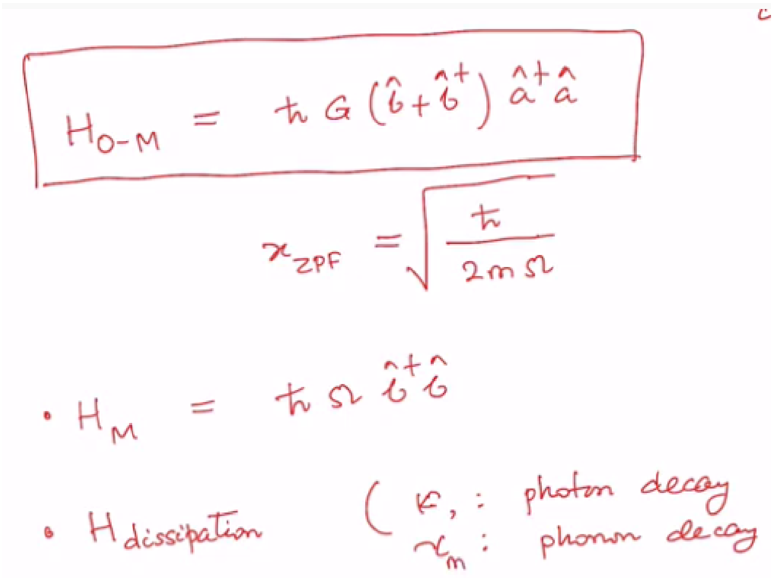


I have this optomechanical interaction term. I can simply write it as h cross G b + b dagger into a dagger a. So, this is a very important term important component of the total Hamiltonian we have got. And, just to recall that this x zero point fluctuation, this is h cross divided by twice m omega. m is the mass of the mechanical oscillator and this is typically around say femtometer, 10 to the power minus 15 meter.

And, there are other terms in the total Hamiltonian. For example, this one the mechanical the component due to the mechanical oscillator and mechanical oscillator is also a harmonic oscillator. So, here this capital omega here is the oscillation frequency of the mechanical oscillator. So, here H M would be it is a harmonic oscillator. This would be h cross omega b dagger b. b dagger b is the number of phonons.

Apart from that, we have this dissipation terms. And, we will talk about this later in the course in some more details. These terms has contribution from first of all the photons inside the cavity you know coupled to the outside world from the partially reflecting mirror and eventually it decays photon decays at a rate kappa. So, this photon decay rate is a kappa. Again, the phonons also decay. This is photon decay.

Contribution would be there in the Hamiltonian. And, phonons also decay. And, that is generally this decay rate is given by gamma. That is for phonon. Phonon actually decay because the cantilever or the mechanical oscillator which is generally attached to some kind of a substrate. So, the vibration of the beam may be transferred to the substrate. In a way, photon phonons leak away leak out of the localized vibrational mode.

And, radiate it away to the substrate. This process is actually called mechanical damping. And the rate of mechanical damping is denoted by gamma. I can put a m here to emphasize that I am talking about the decay of the mechanical or say mechanical photon phonons or quantum of mechanical oscillator.

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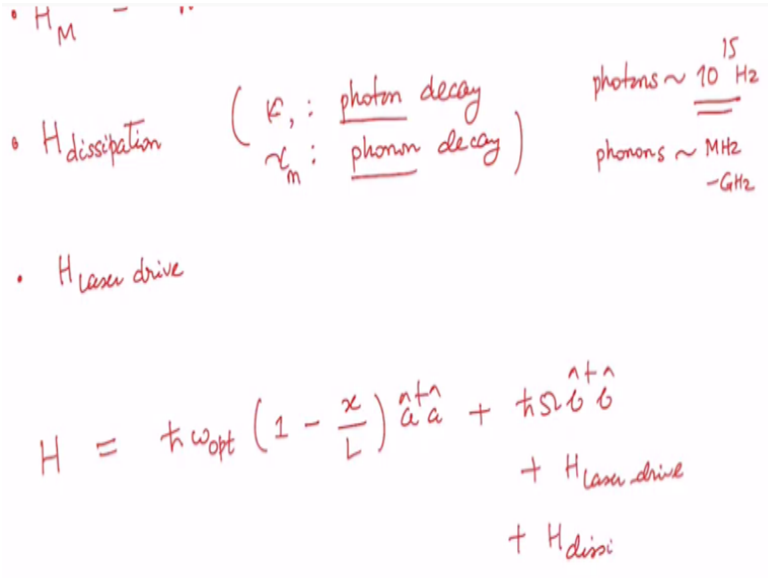


There is a, however, a one important difference between photon and phonon is there. Because you know typically photons have a frequency in the range of photons have frequency of oscillation in the range of 10 to the power 15 hertz. And so, the (()) (37:55) to which they are getting couple effectively are considered to be at 0 temperature because the oscillation frequency is so high.

On the other hand, phonons oscillates which we discussed or I said earlier phonons oscillate in the frequency regime say megahertz to say gigahertz 10 to the power 9 hertz or 10 to the power 6 hertz. So, there are always some kind of thermal phonons around. And it is quite possible that sometime thermal phonons will enter into the calculate cantilever from the outside as well. So, apart from that, as I said there is the laser drive is also there.

And, we will also talk about it later. So, overall, to conclude, therefore, we have the total Hamiltonian will have these terms. One would be h cross. I can write it in this way. Total Hamiltonian is h cross omega opt 1 minus x by L a dagger a. And then, we have h cross omega b dagger b. That is due to the mechanics mechanical oscillator then we have this laser drive. And then, we have all kinds of dissipation processes.

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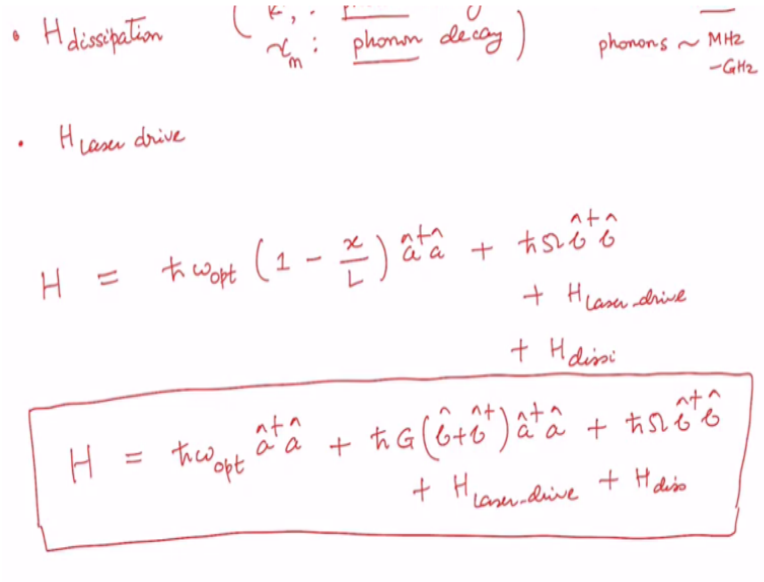


Or, rather, let me write it in a, this way also h cross omega optical a dagger a. That is due to the photon inside the cavity. Then, I have this photon phonon interaction term. That is h cross G b + b dagger a dagger a. And, this is from the mechanics mechanical oscillator. And then, we have this laser drive and then dissipation. So, we have actually guessed, what is the Hamiltonian of this cavity optomechanical system? Now, we are going to discuss the basic physics of optomechanics in a qualitative manner.

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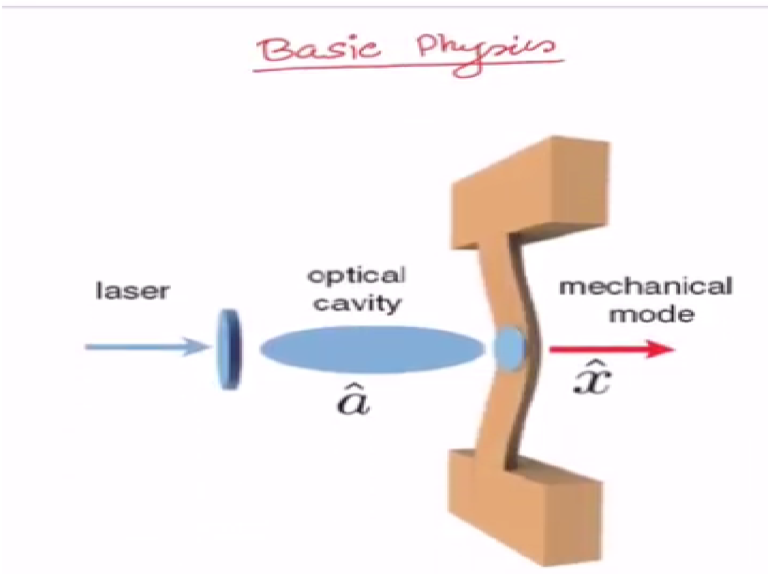

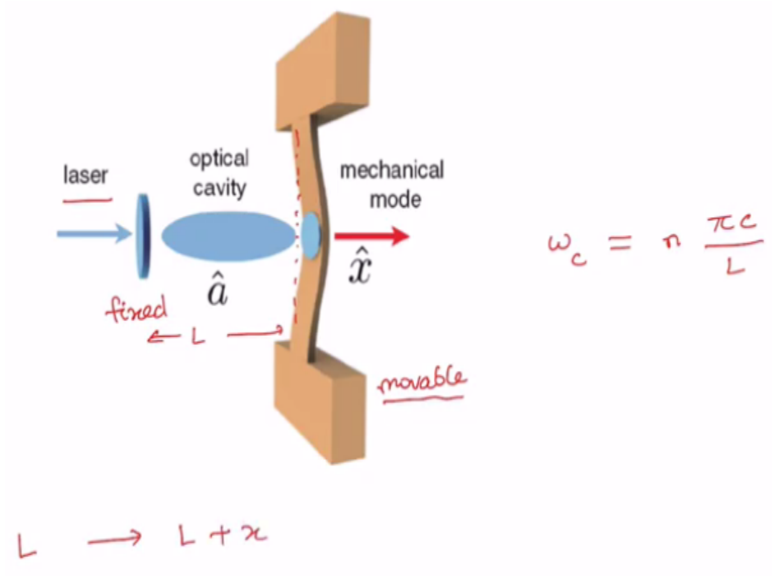


Let us consider this cavity optomechanical setup as shown in this diagram. Here, this left hand mirror is fixed and right hand mirror is attached to a mechanical oscillator or it is a cantilever. So, this mirror is movable. Now, if this mirror is not moving, then say the length of this cavity is L. In that case, as we know from our lesson on Fabry-Perot cavity that light can enter.

This laser light will be able to enter into the cavity provided this resonance condition is satisfied where omega c is equal to n into pi c by L. However, because this right hand mirror is movable, this length changes from L to say L + x.

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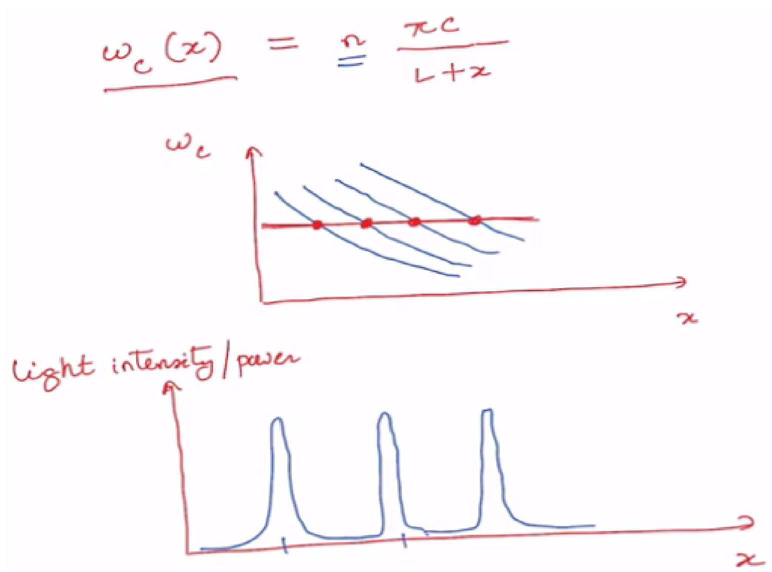


And accordingly, this resonance condition is also getting changed. And, resonance frequency would be n into pi c by L + x. Now, if x is a variable quantity, then the resonance frequency is also getting changed with distance. Now, in fact, if we plot the resonance frequency versus this variable x, the plot will look like this. For a given mode, it will simply look like this. Let me draw it by a different color. So, it will look like this.

In fact, there is not one resonance. There are multiple resonances in such cavity and because if I take various n and then if I plot all these resonance frequencies as a function of position, we will get something like this for various modes or resonance frequencies. It will, for a given mode, resonance frequency will decrease with a variable x like this. And of course, all of them are equidistance. So, do not know these should be equidistant.

And, if a laser with fixed frequencies incident then we will have various positions of the mirror. Suppose the laser frequency is somewhere here. So, this is fixed laser frequency. Then, we will have various positions of the mirror. This is one position. This is another position. This is another position. Like this where the laser frequency will satisfy the resonance condition.

This means that the light power or the in light intensity circulating inside the cavity as a function of position. We saw a typical resonance behaviour of Fabry-Perot cavity. So, if I plot say light intensity or light power in the y axis and here, this is the displacement of the mirror, then we are going to have various resonance peak separated equidistantly like this to various peaks we are going to obtain.

It has very sharp spikes when the resonance condition is satisfied. Now, we know that the light power is actually proportional to the radiation pressure force. So, clearly this diagram is also a diagram of force versus position. So, rather than light intensity or power, we can simply say that this is actually the radiation pressure force.

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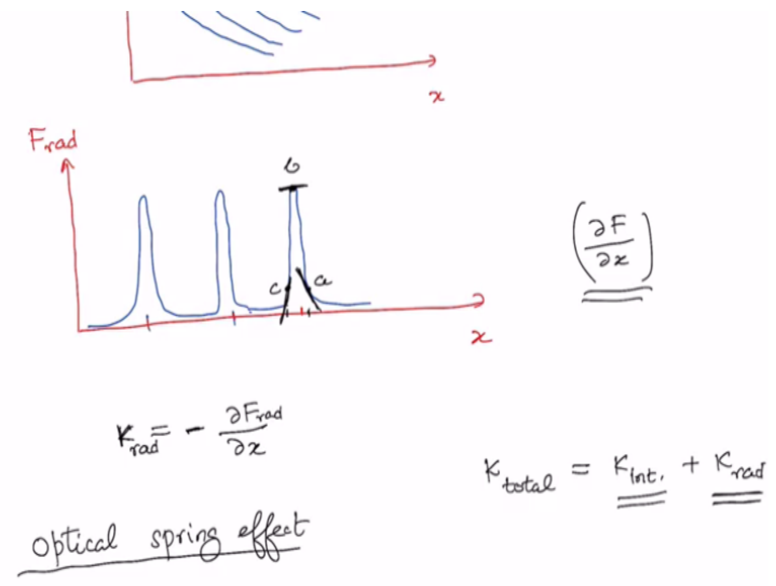


So, say, let us say, let me write radiation pressure as F rad radian radiation pressure force. One thing it immediately implies is that we have a strong behaviour of force at the resonance positions. The force is very high. Another thing you can notice that this from this diagram is that at any given point, the force has a certain slope. For example, if let us look at this particular point say this point this point.

You see that if I draw a tangent, you see, the force has a slope here. That means, we have a finite del F del x. That is the gradient force gradient. So, what does it mean? We know that in this case here as you see the force is position dependent, then also it is clear from this diagram that the force is position dependent, then the slope gives an effective spring constant say k is equal to minus del F del x.

So, the force gradient with a minus sign will give us the spring constant. Now, as the mirror is moving towards the right, we have a force pointing to the left. So, you see the force is pointing to the left. And, this force linearly increases in amplitude as we move the position. So, this is exactly the effect of a mechanical spring. Now, the overall spring constant of the mechanical mirror is increased due to the radiation pressure force.

As the mechanical spring already has an intrinsic spring constant and this effect is known as optical spring effect. This is called optical spring effect. Now, this particular spring constant that we are getting, in fact, the total spring constant of the mechanical oscillator would be due to the intrinsic spring constant and this is because of the radiation pressure force spring constant due to the radiation pressure force.

So, we are going to have an effective spring constant and that will come due to the contribution from the one is the intrinsic one and another one is due to the radiation pressure force. And in fact, this effect can be very strong and sometimes it may be so strong that it may this increase in the spring constant maybe even exceed this intrinsic spring constant of the oscillator or the mirror.

However, as you see that, if the mirror is at another position, for example, say this is at this position. So, at this position, then we have zero spring constant due to the radiation pressure force or we can even have reduced spring constant. Suppose, here, our position of the mirror is here, then the, as you see the slope is positive now. If the slope is positive, the spring constant will be negative.

And, on the other hand, at this position, the spring constant the slope is negative. So, therefore, the spring constant will be positive. So, let us say this is my position a. This is my position b. This is my position c of the mirror. At the position a, the, will, the effective spring constant of the mirror or increase at position b, it will remain the same because contribution from the radiation pressure force would be 0.

On the other hand, at the position c of the mirror, the effective spring constant will decrease. In fact, we can understand the optical spring effect from another perspective as well. So, let us understand that because we know that is basically from the concept of potential.

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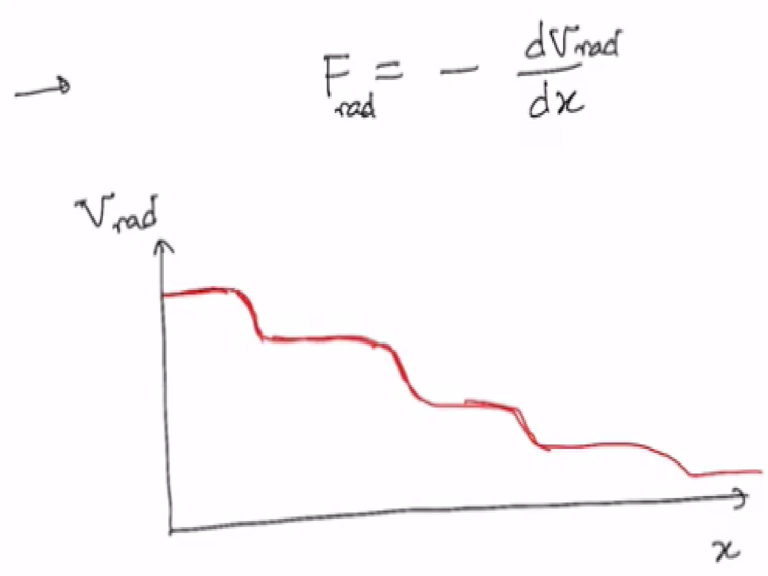

We know that the force is gradient of potential. That is minus dv dx. That is how we know if for a position dependent force and force is negative gradient of the potential. So, if we are given the force, then we can easily obtain the potential just by doing the integration. Then, we can obtain in our present case a type of potential versus position curve. Let me first draw it and then I will explain.

In fact, this is I am talking about radiation pressure radiation potential. So, potential due to the radiation pressure force and if I plot radiation pressure force versus position, we are going to get a plot like this. It will be fine. Staircase kind of a potential we are going to get. So, whenever there is a resonance peak, for example here, as you see, whenever, as you can see when there is no force, the potential would be constant.

But, when there is a strong force, the, or, at the resonance of force would be very strong and will have a, this stair would be there. And, we will have another step. And again, we will have another step and so on. Whenever we are having a resonance, we are going to have this step here. So, dips these steps are actually rounded as the peaks are not infinitely sharp. This is only radiation induced potential.

We need to if we want to know the total potential of the mechanical oscillator or the movable mirror or the mechanical oscillator here, then we need to add the intrinsic potential that describe the motion of the mechanical object.

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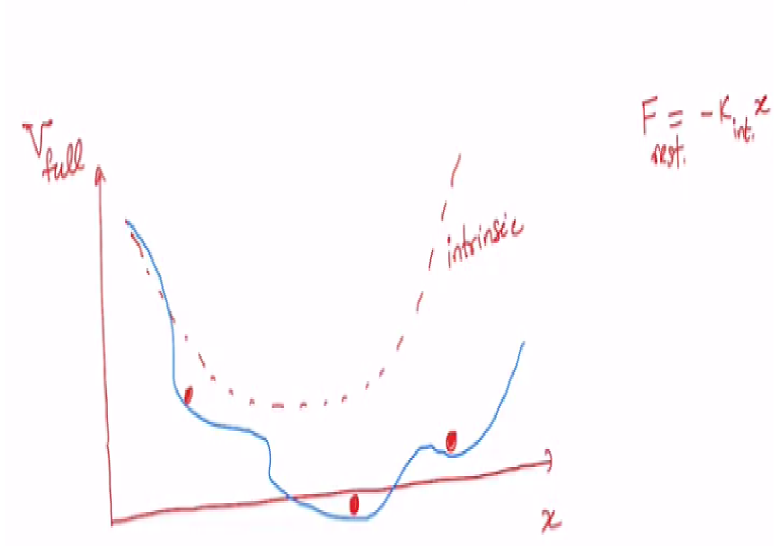

And, you know the mechanical oscillator oscillates simple harmonically. So, therefore, we will have a parabolic potential like this so that intrinsic potential would be a parabola like this. So, consider it as a parabola. So, this is intrinsic and on top of it I have to add the, this staircase potential. Then, I will have the full potential. Then, full potential versus position of the mirror or the variable x we can plot.

And, that plot will look something like that if I add this intrinsic potential as well then the plot will look like somewhat like this. So, what is the significance of this result and potential? If you look at it, then you will observe that there are several local minima here. For example, you have, you can have a, you have a minima here, you have a minima here. You have a minima here and so on. You can have a minima here. There are several minima.

So, this means that we have several equilibrium positions. The mechanical object can be at either one of these positions and is completely stable. In these positions, the radiation pressure force, exactly balances the restoring force brought about by the intrinsic spring constant because you know for a mechanical oscillator, a harmonic oscillator the force restoring force is minus k say this is intrinsic into x.

So, this relation you know. This is directly proportional to the displacement. So, this will balance out the radiation pressure force. So, having these multiple stable position is known as multistability.

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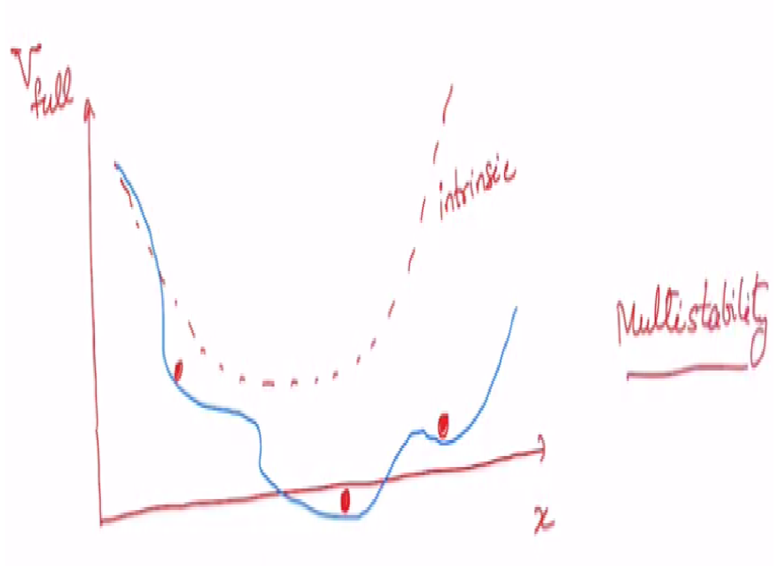

This is called multistability. At typical plot of a restoring force in a cavity optomechanical system, say the restoring force is, say this is my position.

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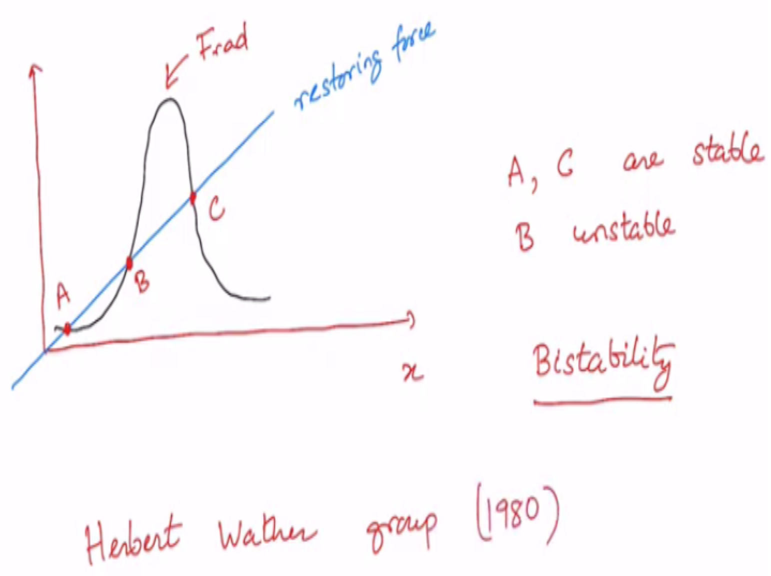


Restoring force is directly proportional to position. So, you will have this. This is the restoring force and the radiation pressure force would be like this. So, as you see, there are 3 points. Say, this is one point A and another point B and this point C. This is due to the F radiation pressure force. At this position, the radiation pressure force balances the restoring forces. So, at point A, B and C, restoring force and radiation pressure force are equal.

However, out of all these 3 points, only A and C, the point A and C are stable and B is unstable. Why? Because the rate of growth of the restoring force at point A and point C is faster than that of the radiation pressure force. So, this is the phenomena of bistability. So, there are 2 points at which this cavity optomechanical system would be stable. So, this is the phenomena of bistability. It is called bistability.

Please note that optomechanical spring effect is already experimentally demonstrated way back by a group of Herbert Walther. Herbert Walther group in 1980s, they experimentally demonstrated this effect. Let me stop here for today. In this lecture, we try to understand the basic physics of a cavity optomechanical system assuming that the light reacts instantaneously to the position of the movable mirror.

That is when the mirror moves very slowly. In the next lecture, we will examine the dynamical case. And then, we will start discussing cavity optomechanical system quantitatively first by examining the classical regime. So, see you in the next class, thank you.