Quantum Technology and Quantum Phenomena in Macroscopic Systems
Prof. Amarendra Kumar Sarma
Department of Physics
Indian Institute of Technology, Guwahati

Lecture – 28
Pure-dephasing and Dissipative Bloch Equations

Welcome to lecture 11 of module 2 and this is lecture 21 of the course. In this lecture we will learn about the phenomenon of Pure-dephasing and we will see how Pure-dephasing can be incorporated in the Linblad quantum master equation. Apart from that we will also discuss about the dissipative block equation in the context of a qubit that means when the qubit is there in the presence of various relaxation process including the dephasing. So, let us begin.

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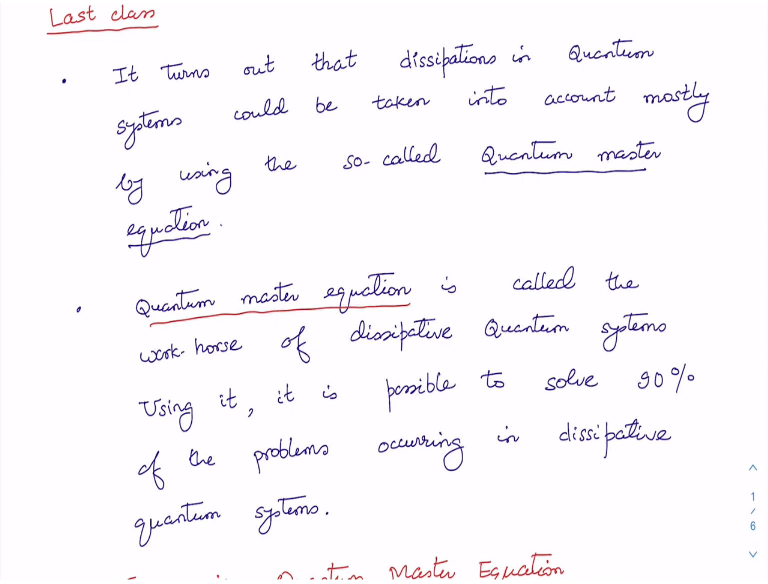

Last class

- It turns out that dissipations in Quantum systems could be taken into account mostly by using the so-called Quantum master equation.
- Quantum master equation is called the work-horse of dissipative Quantum systems. Using it, it is possible to solve 90% of the problems occurring in dissipative quantum systems.

Master Equation

In the last class we discussed the so-called quantum master equation. As I said in the last class that the quantum master equation is the workhorse for dissipative quantum system almost 90% of the problem in quantum dissipative system can be taken into account by this equation and it is also known as the Linblad Markovian Quantum Master equation. It is Markovian because this equation which takes into account how the density operator changes with time rate of change of the density operator.

So, each point in time the time derivative is given by the present state. So, rho here refers to the density operator corresponding to the present state only there is no memory and this is why this master equation is more aptly termed as markovian, Linblad markovian quantum master equation. And it has 2 parts one part takes coherent evolution part of the system takes into account.

And another part refers to the dissipation here L is a operator and this operator operates on another operator that is the density operator and L is this is known as the super operator because it operates on an operator or also it known as the Linblad operator. And this operation has a particular structure where this is the structure that we discussed. Here this operator A is an arbitrary operator through which interaction between the system and the environment takes place.

And in this equation this gamma refers to the relaxation decay rate of the relaxation process a operator a there may be many many relaxation processes in a particular system quantum system depending on the situation. So, here this sum it is all the processes has to be taken into account that is why this sum is there. So, gamma j refers to the decay rate corresponding to the relaxation process say discussed by or referred by the relaxation operator A j.

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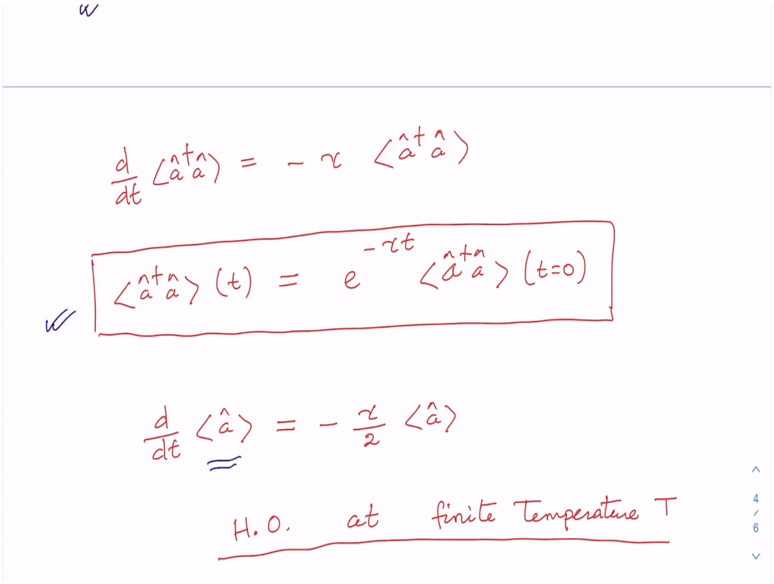

$$\frac{d}{dt}\langle \hat{a}^{+}\hat{a}\rangle = -\gamma \langle \hat{a}^{+}\hat{a}\rangle$$

$$\boxed{\langle \hat{a}^{+}\hat{a}\rangle (t) = e^{-\gamma t}\langle \hat{a}^{+}\hat{a}\rangle (t=0)}$$

$$\frac{d}{dt}\langle \hat{a}\rangle = -\frac{\gamma}{2}\langle \hat{a}\rangle$$

H.O. at finite Temperature T

Now this particular structure is extremely critical because this particular structure guarantees that this density operator remains Hermitian regardless of how we switch the decay rate gamma and the relaxation operator A and trace of rho is equal to 1. So, probability remains conserved and rho remains semi-positive definite then we went on to discuss some examples.

First we discussed the 2-level system relaxing via the so-called spontaneous emission and we were able to derive the rate equation appropriate equation for the probabilities rate of change of the probabilities but in addition to that we also got to see that how the coherences decay and coherences decay attended gamma by 2. And we also discussed the case of thermal excitation of a 2 level system where the atom is going from the ground state to the excited state due to the thermal excitation.

This also we discussed what it turns out that whether we have spontaneous decay or thermal excitation the off diagonal elements still decay and it decay at a rate gamma by 2 and we discussed the so-called damped harmonic oscillator at zero temperature and in this case only one process is there that is the downward transition and we worked out how the probability of finding n photons.

The rate of change of probability of finding n protons we worked out using the quantum master equation and also we saw how the average number of photons changes with time. So, and apart from that we found that the exp the expectation value of this annihilation operator decays at the rate of gamma by 2.

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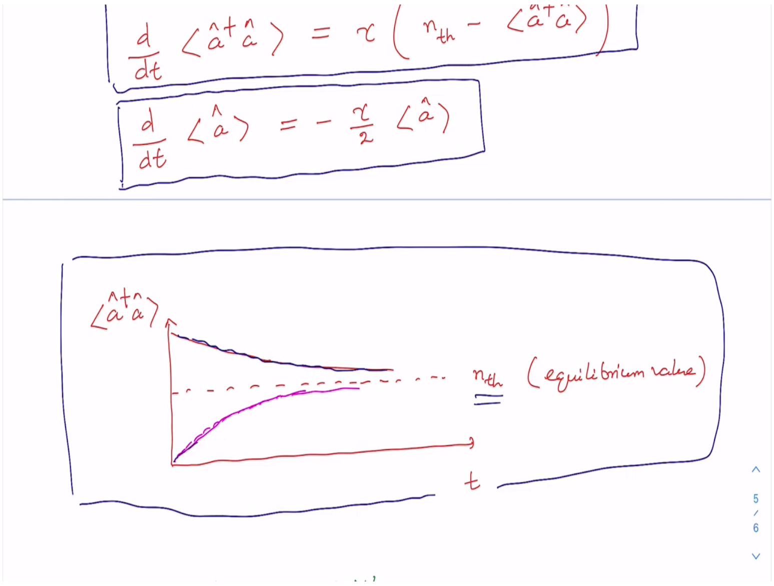

$$\boxed{\frac{d}{dt}\langle \hat{a}^{\dagger}\hat{a}\rangle = \gamma\left(n_{th} - \langle \hat{a}^{\dagger}\hat{a}\rangle\right)}$$

$$\boxed{\frac{d}{dt}\langle \hat{a}\rangle = -\frac{\gamma}{2}\langle \hat{a}\rangle}$$

We discussed the harmonic oscillator at finite temperature and here 2 processes may be there one is the downward process for downward process the operator a this relaxation operator can be taken to be represented by the annihilation operator where the upward process is by the creation operator. And here we found we saw that this decay rate downward decay rate is basically modified by a factor of n thermal this is n th which is the average number of thermal protons.

And this is given by this particular equation. On the other hand the upward transition rate is given is modified by this n th gamma n th. In fact at zero temperature there will be no thermal excitation. So, n th is zero there and. So, we will have only one process that is the downward process downward decay process. And the expectation value of the average number of photon the rate of change of expectation value the average number of photon is given by this equation.

Here also we found that the expectation value of the annihilation operator it decays at the rate of gamma by 2. And in fact this solution is represented by this particular diagram here and you see that depending on where we start suppose the initially we have the average number of photon is gathered in the thermal equilibrium value which is n th it is then it will decay and approach the thermal equilibrium below.

On the other hand if the average number of photon is zero initially at time t is equal to zero then the as time goes on it will approach the equilibrium value.

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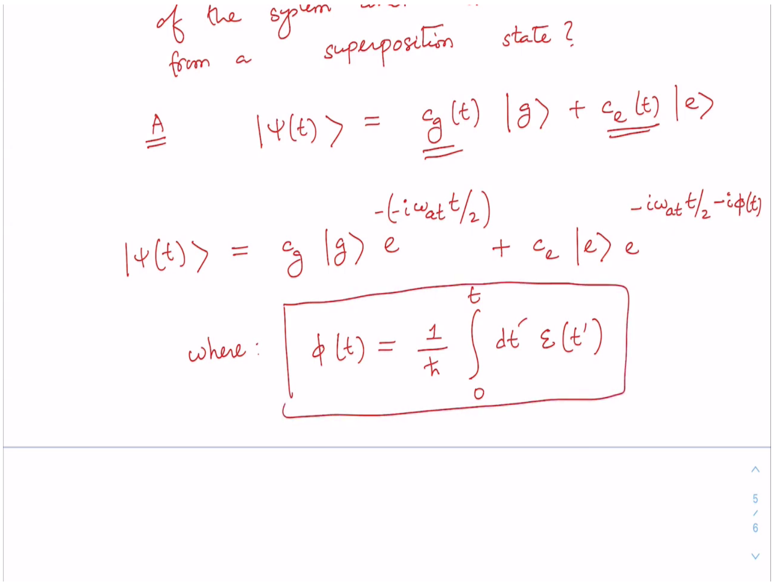

of the system

from a superposition state?

$\underline{\underline{A}}$ $$|\psi(t)\rangle = \underline{\underline{c_g}}(t)\,|g\rangle + \underline{\underline{c_e}}(t)\,|e\rangle$$

$$|\psi(t)\rangle = c_g\,|g\rangle\,e^{-(-i\omega_{at}t/2)} + c_e\,|e\rangle\,e^{-i\omega_{at}t/2 - i\phi(t)}$$

where: $$\boxed{\phi(t) = \frac{1}{\hbar}\int_0^t dt'\,\varepsilon(t')}$$

So, now let us go back to the case of transmon or the qubit system in the context of circuit QED and I want now to discuss and very important decoherence phenomena and that is called Pure-dephasing. So, this is what we are now going to discuss Pure-dephasing is a process where we do not jump between energy levels say we have this 2 level system this is our ground state and this is our excited state and let us say the energy difference between these 2 level is h cross omega atom.

Let us say this one has energy h cross atom by 2 while the ground state has energy say h square minus h cross omega atom by 2 and this is what energy. And we in Pure-dephasing we are not having upward transition that means say going from ground state energy to the excited state energy or from the excited energy state to the ground state energy we do not have such kind of transitions.

Rather what happens is that we it turns out that that even though there is no such kind of transitions the off diagonal elements in the density matrix for example this term or say this term which are known as the coherence terms in the density matrix which we already know these terms actually decay. And this is what this decay of coherences is basically known as dephasing or Pure-dephasing.

Because you know that this off diagonal elements in the density matrix gives the phase relation between the amplitudes of the amplitude coefficient of the ground state and the excited energy state. Let us explain it further let us understand it properly let us say one of the energy levels let us say this upper energy level is fluctuating in it time. So, this is a fluctuating with time that means we are not having a very sharp energy level sharp width rather it is fluctuating with so with time.

And the ground state is of course it is intact let us say the question is now what happens. The question is what happens if or what happens to the state vector or the wave function wave function of the system when we say start from when we start from a superposition state why superposition state. Because as our discussion it is important for us to have this kind of state because we are interested in the phase relation between these amplitude coefficients ground state amplitude coefficient and the excited energy state amplitude coefficient C e t this one as well as this one.

So, we want to know what is actually going on if one of the energy level is fluctuating with time if the energy levels remain eigen levels that is if they are still remaining as the eigen state of the Hamiltonian all the time then the wave function at any arbitrary time we can write it as say C g this is ground state and it would have say e to the power i omega atom t by 2 because you see this plus I am getting because minus of minus okay that is why I am getting plus.

You remember that you have e to the power minus i by h cross e this is what we have this coefficient varies like this. So, actually this is what we are you know writing here for the first term and in the second term I have C e e and I have e to the power. Now I will have a term minus i omega atom t by 2 and in addition to it I will have a i phi of t where phi is the phase and in fact that is the energy level excited energy level which is fluctuating with time. So, phi of t is 1 by h cross.

So, because this is fluctuating with time so, this is basically integral over the fluctuations. So, this is the energy fluctuation term. So, this is the phase term.

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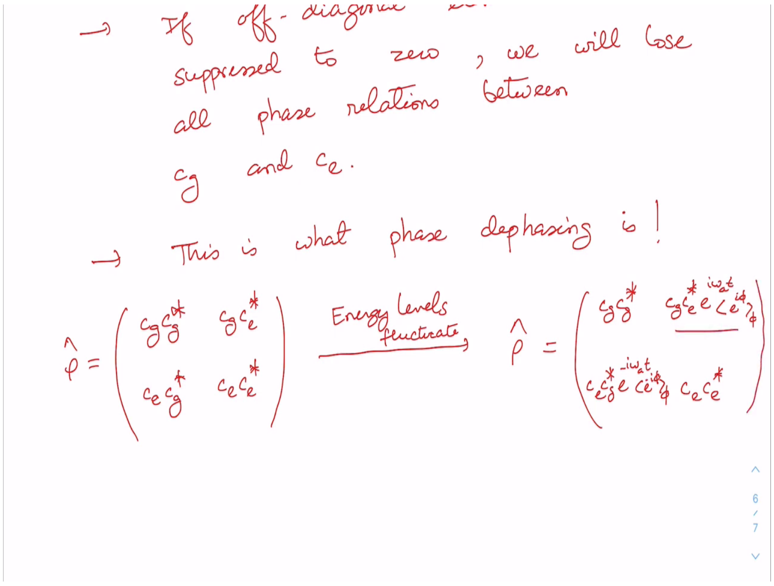

→ If off-diagonal

suppressed to zero, we will lose all phase relations between c_g and c_e.

→ This is what phase dephasing is!

$$\hat{\rho} = \begin{pmatrix} c_g c_g^* & c_g c_e^* \\ c_e c_g^* & c_e c_e^* \end{pmatrix} \xrightarrow[\text{fluctuate}]{\text{Energy levels}} \hat{\rho} = \begin{pmatrix} c_g c_g^* & \underline{c_g c_e^* e^{i\omega_a t} \langle e^{i\phi} \rangle_\phi} \\ c_e c_g^* e^{-i\omega_a t} \langle e^{i\phi} \rangle_\phi & c_e c_e^* \end{pmatrix}$$

However as we know that to get a more appropriate picture of the situation because in real experiment many many repeated measurements are going to be made we need to look at the density matrix. So, to get a real picture; so we have to basically look for the density matrix, so density matrix would be psi of t like this but which has to be average over all the fluctuation.

Now in fact what we will get if we do it this straightforward calculation starting with this wave function you should be able to show that this coherence term say e g rho e this matrix element you can show that this would be simply C g C e star e to the power i omega a t and then we have this e to the power i phi of t and this is e averaged over the fluctuation. Clearly we do not have transition between the energy levels that means when we are not having transition between the energy levels.

However the off diagonal elements has a free time evolution that is given by e to the power i omega a t and in addition to that we have a fluctuating phase factor. So, this is our fluctuating phase factor and in fact this represents free evolution. We know that if we take the average of a complex quantity we obtain a result that has a magnitude less than 1. So, this implies that the magnitude of the off diagonal elements.

So, if you take the magnitude of this term e to the power i omega a t this is what we have this one and apart from that okay this is what I have here. So, if I take the modulus of magnitude. So, what it turns out that this would be less than this guy would be actually less than 1 and so therefore you will find that this is overall less than this quantity C g C e star. So, what it means that instead of having a pure state because the density matrix this is actually now getting decayed overall it is less than 1 because of this is less than this quantity.

So, we will have a mixed state this implies that we will have a have a mixed state rather than a pure state because the we are losing information about the initial phase because as you see that because of the energy fluctuation we are having this particular term because of the fluctuation this object elements its magnitude is getting suppressed and in fact if the off diagonal component is suppressed to zero completely.

Suppose this off diagonal elements is made to be this whole thing gets zero. Then we will not know any information about the phase relationship between the complex amplitude C g and C e. So, if say off diagonal if off diagonal element is or component is suppressed to zero we will loss all phase or initial will not have any idea about the initial phase information or we loss loss all phase relation between the amplitude coefficient C g and C e.

So, this is very important and in fact this is what phase dephasing is. So, this is what phase dephasing is. I hope this is so clear let me once again repeat suppose the energy levels are sharply defined there is no energy flow there is no energy fluctuation in any of the energy levels then the density operator here would be having the elements like this say C g C g star C g C e star C g C e C g star and we will have C e C e star but when one of the energy level is fluctuating energy level fluctuate.

Then this these elements would remain no problem with this of diagonal element diagonal elements but these elements. So, you will here as I already we saw that this would be C g C e star and then along with it you will have e to the power i omega a t and because of the this particular term okay which is due to the fluctuation of the energy level similarly here you will have that similar kind of term.

So, that would be basically let me write it. So, here we will have terms like say C e C g star e to the power minus i omega a t and you will have e to the power i phi okay or whatever minus. So, this is what we will have and these terms would be less than this term. And because of that this particular state is no longer going to be this one is no longer going to be pure rather it is mixed on the other hand this is pure state.

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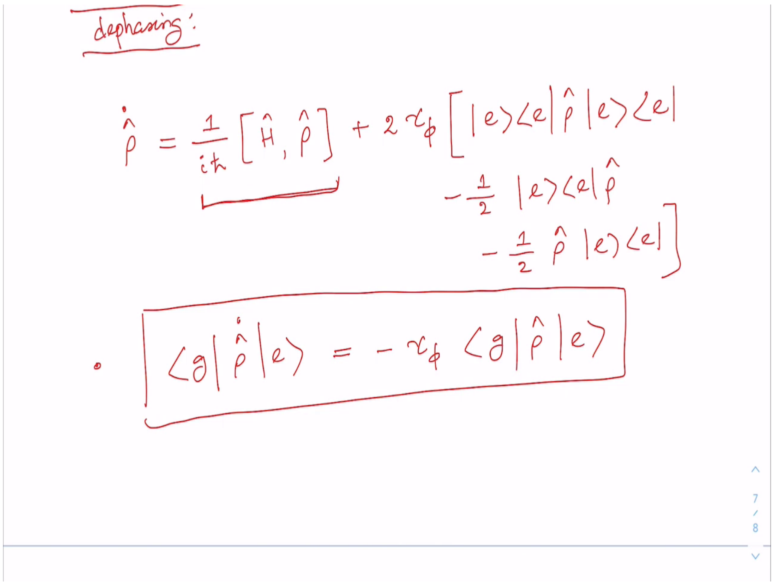

dephasing:

$$\dot{\hat{\rho}} = \underline{\frac{1}{i\hbar}[\hat{H},\hat{\rho}]} + 2\gamma_\phi \Big[|e\rangle\langle e|\hat{\rho}|e\rangle\langle e| - \frac{1}{2}|e\rangle\langle e|\hat{\rho} - \frac{1}{2}\hat{\rho}|e\rangle\langle e| \Big]$$

$$\boxed{\langle g|\dot{\hat{\rho}}|e\rangle = -\gamma_\phi \langle g|\hat{\rho}|e\rangle}$$

Now I will not go into the details but in many practical cases the ones that we generally encounter in circuit QED or in quantum optomechanics which is the topic for the next module or in quantum optomechanics mostly this particular fluctuating term e to the power i phi of t when we take average overall the fluctuation is approximated is approximated as e to the power minus gamma phi of t.

So, this actually this term decays exponentially with time and here this particular decay rate gamma phi is known as the dephasing rate this is called dephasing rate. This kind of quantum noise when we make this kind of approximation this is comes under something called Gaussian noise. So, let us not bother too much about how it came in all these things but this is I think for our course this is this one is sufficient a good approximation and this is enough for us to know.

Now the question is can we build up a relaxation operator in the Linblad quantum master equation which can take this phenomenon or the noise into account. So, this is what we can now answer and of course the answer is yes in the in the quantum master equation let me write it as QME quantum master equation. What we expect? We expect that our this coherence term would it should decay at the rate gamma phi.

So, we are expecting. So, this is basically g rho dot e we expect that we will have a some kind of say some term would be here and that would be the usual bare dynamics usual bare dynamics and apart from that there will be a term referring to the decay that will be gamma phi rho g e. So, as you can see. So, this is going to give us the required relation and similar expression you can get it for rho e g.

So, similarly for rho e g you will get some terms here and then here you will have gamma phi rho e g. Now we want an operator. So, we are given the decay rate to be this one that is the dephasing rate gamma phi now what about the relaxation operator. The operator which takes us because there is no actually relaxation what we having is this kind of a situation we are in the ground state and here this is the excited state we have this 2 energy state but there is no transition.

But fluctuation is happening in the energy level here upper energy level. So, as if we are going from this energy level and going back to it okay. So, this is what is we are having. So, this is actually taken we can guess it as we are going from excited energy state again to the another excited again to the same excited state and then this is typically given by these relation okay this is what the relaxation operator in this case for the Pure-dephasing case.

So, if we have this then we can easily write down the Linblad master equation for pure phasing case. So, Linblad quantum master equation Pure-dephasing for Pure-dephasing let us now write it. So, we are given if I and the decade is there. So, this would be time rate of change of the density operator the first term gives us the coherent evolution and then we are having the decay rate that is gamma phi and in fact because root 2. So, you know the master equation. So, let me write the master equation for the dephasing case.

So, that would be we will have terms like this e rho e e please verify it and you have minus half e e rho and minus half rho e e. So, this is what the quantum master equation actually you can very easily obtain from this Linblad master equation you can show that you will get g rho dot e would be equal to minus gamma phi g rho e. In fact this particular term because coherent evolution if the Hamiltonian is diagonalized.

Then this term is going to be zero this contribution from this term would be zero and you will obtain this equation and you see this clearly shows that the coherence actually decays at the rate exponentially it decays at gamma phi.

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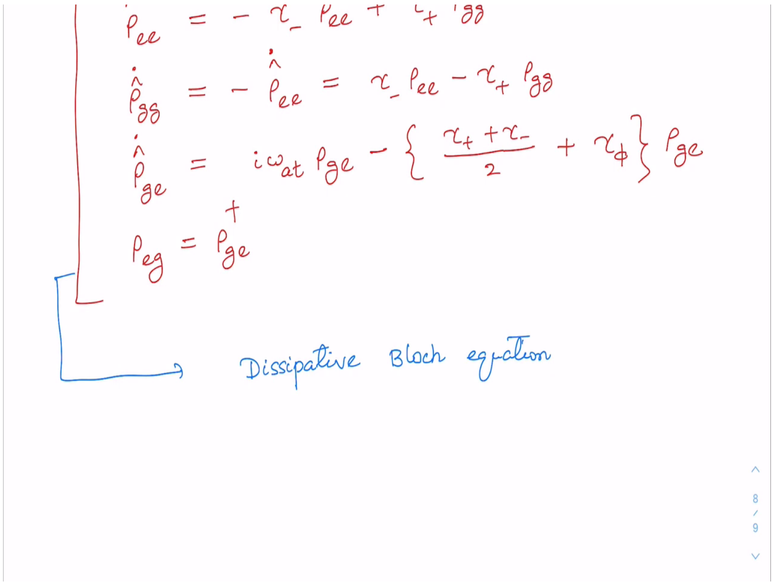

$$\dot{\rho}_{ee} = -\gamma_- \rho_{ee} + \gamma_+ \rho_{gg}$$

$$\dot{\hat{\rho}}_{gg} = -\dot{\hat{\rho}}_{ee} = \gamma_- \rho_{ee} - \gamma_+ \rho_{gg}$$

$$\dot{\hat{\rho}}_{ge} = i\omega_{at}\rho_{ge} - \left\{ \frac{\gamma_+ + \gamma_-}{2} + \gamma_\phi \right\} \rho_{ge}$$

$$\rho_{eg} = \rho_{ge}^{\dagger}$$

Dissipative Bloch equation

So, now let me discuss one important issue suppose we are having this transmon qubit or any or any 2 level system in the presence of relaxation and dephasing in the presence of relaxation and dephasing. So, what I mean by this is this. So, this is a very common situation and often this kind of situation is encountered when we are dealing with artificial 2 level quantum system.

We have processes like this suppose there is a downward transition which is happening at the rate gamma minus and there is thermal excitation which is happening at the rate gamma plus and apart from that there is this dephasing is also happening because suppose this energy level is not fluctuating with time or some kind of a phase decay is there. So, then how one can take this into account now we have actually taken all the cases separately.

So, therefore we can combine all of them and then we can write down the Linblad master equation for example we can write down the relaxation process for the downward transition is this we are going from the excited state to the ground state and which is basically your atomic lowering operator and this is happening at the rate gamma minus then we have this process A plus where we go from the axis excitation we are going from the ground state to the excited state.

This is atomic raising operator and this is happening at rate gamma plus and then in addition to the all these things we are having this dephasing where we are going from the excited state to the excited state and this is happening at the rate gamma phi. Using this one can very easily set up the Linblad quantum master equation and from it you should be able to obtain these density operator equations.

The density matrix equations basically for all the elements for example very straightforwardly you should be able to get it. So, please do that you will get time rate of change of this term would be equal to minus gamma minus rho e e + gamma + rho g g and then you will get rho dot gg this is for the time evolution of the ground state probability that is basically minus of this term because the total probability is equal to 1. So, you can easily get it.

So, therefore this would be simply gamma minus rho ee minus gamma plus rho gg and this coherence terms decay if we can get it will be from the master equation you can. So, that you will get terms like this it will be rho g e minus you will get gamma ++ gamma minus by 2 and you will have plus gamma phi. So, these are total decay and then here you will have rho ge the other equation is straightforward to get because you have rho eg is equal to rho ge this is the Hermitian conjugate.

So, these are actually called dissipative Bloch equation. Earlier we have studied block equations without presence of any noise. So, now also noise or dissipations are taken into account. So, these are called dissipative Bloch equations. And these equations are capable of describing the dynamics of an artificial qubit or transmon in the presence of quantum noise.

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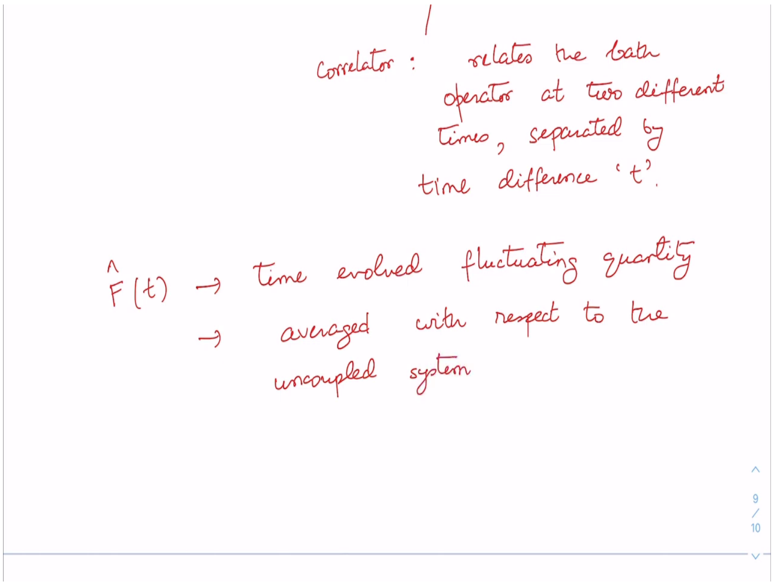

Correlator : relates the bath operator at two different times, separated by time difference 't'.

$\hat{F}(t)$ → time evolved fluctuating quantity

→ averaged with respect to the uncoupled system

Finally now let me give you a quick prescription on how to work out the decay or relaxation rate we can work out the decay rates using the. So, called Fermi golden rule and this rule you must be familiar with because you may have studied in your quantum mechanics course may be in the chapter called time dependent perturbation theory. Let me explain the rule say we have the system and environment.

Suppose we have this system and this is surrounded by some environment this environment is also called bath or bath and this environment and the bath is say it is getting coupled to each other by some coupling parameters say g and the interaction between them or the coupling between them is responsible for all the relaxation process. So, coupling between the system coupling between the system and environment or bath is responsible is responsible for all relaxation process.

All relaxation process that means whether upward transition or the; downward transition processes in the system. So, we know this let us say the interaction Hamiltonian has this generic form the interaction Hamiltonian for the bath and the system has this generic form say A refers to the system let me put A s that refers to the system part and then we have the another part refers to the environment.

So, this is the system part and this is the environment or bath. So, we have this let us say the system is initially in the state say ket i and its final state is say f or any state suppose it is going from the cis state it transiting from the state ket i to the state ket f by emitting an excitation it may be in atomic case it would may be a photon. So, according to Fermi golden rule, so, in this case the transition is happening from initial state to the some state say f.

And according to the Fermi Golden rule the relaxation rate when we are going from the state i to the state f or which i can simply write it as gamma f i this is given as per the Fermi rule by this I will explain it first of all this interaction is happening within the; transition is happening within the system via this system part of the Hamiltonian it is going from the state i to the state f this is the transition probability.

If I take the mod square, so and this is caused by the environment and or the bath and this is evaluated and this is called I will explain what this guy is ff and this is evaluated at the frequency omega. Let me write it here this is evaluated at the transition frequency of the system say omega is equal to e i energy of the ket i and f e f is the energy of the ket f divided by h cross this is evaluated at this particular frequency.

And this is basically the spectrum because this is evaluated at frequency omega this is the spectrum of the fluctuating bath operator. Spectrum of the fluctuating bath operator but operator evaluated at omega. And f is basically it is a noise or a f is a noise or perturbation we can take it like a perturbation. Perturbation due to the environment and the noise spectrum this quantity is basically the noise spectrum and this is nothing but the Fourier transformation of the correlator.

So, this is the bath operator f of t at a time t and this is evaluated at time t is equal to zero and then if we take the averaging of these 2 quantities product of this 2 quantity and then we take the integral and this is the Fourier transformation we are having here it is going from say minus infinity to plus infinity. So, this particular quantity is known as the correlator, this is called the correlator.

What it gives, it basically relates the bath operator at different time and separated it by by time t. So, this correlator relates the bath operator or the environment operator bath operator at 2 different times separated by time difference as you can see it is simply t here now this quantity f of t which is the time eval fluctuating quantity.

It is actually time evolved fluctuating quantity representing the bath fluctuating quantity and this is important to remember that as per the Heisenberg picture this one this is averaged when we take the average it is averaged with respect to the with respect to the uncouple system with respect to the uncoupled system that means when we are averaging it we are not considering the coupling we are just considering the basis state of the uncoupled system and then we are calculating the expectation value.

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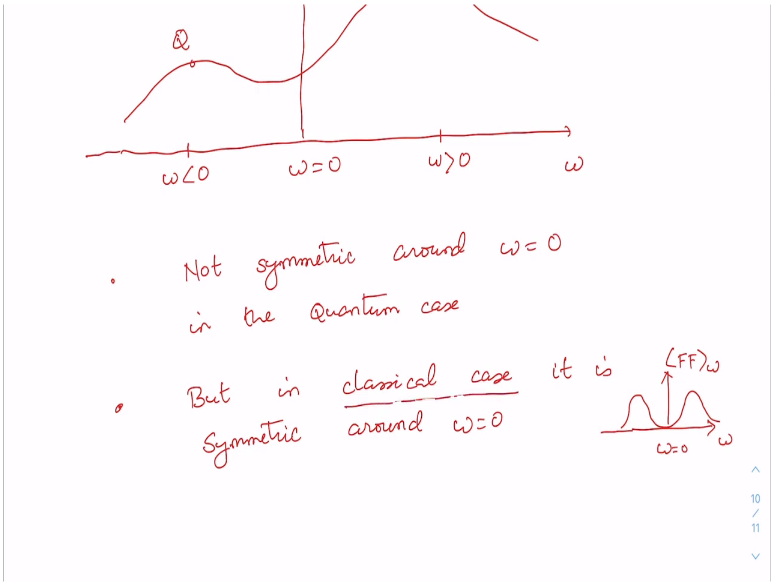

- Not symmetric around $\omega = 0$ in the Quantum case
- But in classical case it is Symmetric around $\omega = 0$

So, basically what I am saying is that when we are writing this quantity or working out this parameter this term here this is the expectation value. The expectation value is taken this is important to remember is taken with respect to the unperturbed with respect to the unperturbed unperturbed that is not yet coupled but okay with respect to the unperturbed.

But okay in fact it turns out that this quantity or the noise spectrum when you evaluate this it is real valued and it is non-negative or it is positive and is non-negative that means greater always greater than zero. A typical spectrum generally looks like this we will see some example later on. Suppose we plot this noise spectrum here with respect to frequency typically we will get a plot like this.

So, this is in the quantum case suppose this is your omega is equal to 0 and this is not symmetric around omega is equal to zero as you can see suppose we have this 2 point here say P and then you see Q and as you can see this point P here. Here omega is greater than zero at the point q the omega is less than zero and you see they are not both sides are not symmetrically located it is basically not symmetric in the quantum case.

However this is not symmetric around omega is equal to 0 in the quantum case in the quantum case. However in the classical case and but in classical case and this has many physical implication in the classical case it is symmetric it is symmetric around omega is equal to zero that means if we will get the spectrum to be something like this it would be let me draw it properly.

So, it would be symmetric suppose this is ff in the classical case if it is omega is equal to 0 this is omega then it will be symmetric in the classical case okay. This asymmetric behaviour as we have in the quantum case this is asymmetric this spectrum this is asymmetric about omega is equal to 0 in quantum mechanical case and these are some physic interesting physics for example if we look at the point p you see at the point p as i said at the point P we have omega is greater than 0.

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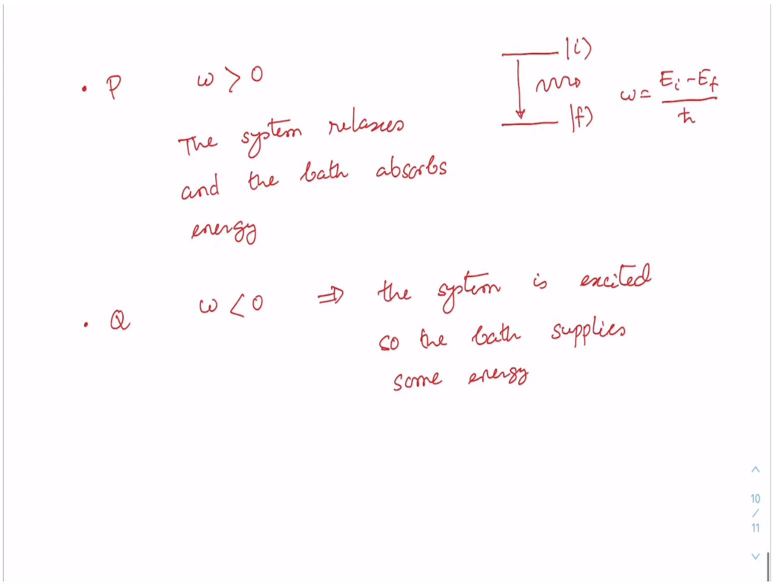

- P $\omega > 0$

The system relaxes and the bath absorbs energy

$|i\rangle$

$|f\rangle$

$\omega = \frac{E_i - E_f}{\hbar}$

- Q $\omega < 0$ $\Rightarrow$ the system is excited so the bath supplies some energy

Now what it physically mean the spectrum when omega is greater than 0 you see omega we are defining as e i minus e f divided by h cross that means the transition is taking place from the state i to state f and this is system is going from the state i to state f by emitting some kind of an excitation here. That means the system when omega is greater than zero the system is releasing we can say our system better let us say system relaxes the system relaxes and the bath and the bath absorbs energy.

The bath in the process gets energy absorbs energy okay on the other hand if you look at the point q where omega is less than 0 this side here. Then in this case as you can make it out the system is excited the opposite things happen the system is excited and so the part supplies some energy the bath supplies some energy to the system. So, in this case the system is going from the lower state to the say excited state and in the process the bath is supplying this energy.

So, this is what the physical implication is in fact what happens in thermal equilibrium as you see in thermal equilibrium there is always a larger tendency for the environment to absorb some energy then to supply some energy to the system.

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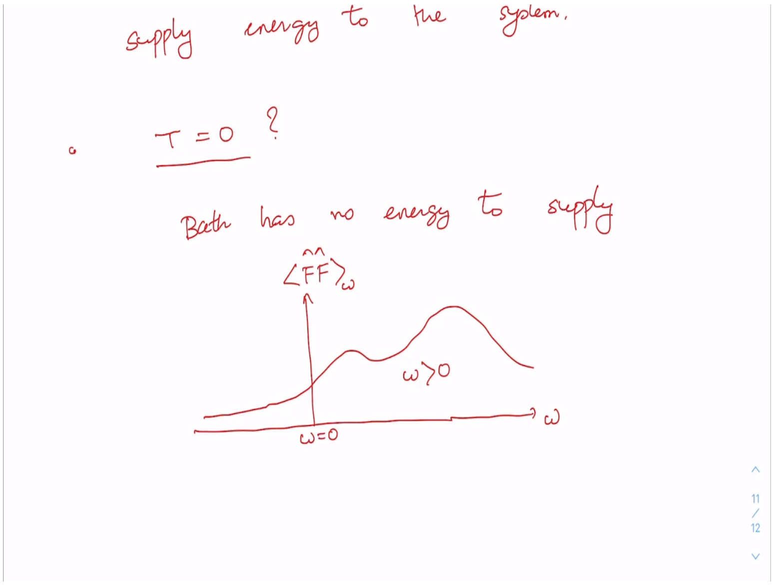

supply energy to the system.

- $T = 0$?

Bath has no energy to supply

Let me write here in thermal equilibrium the there is there is always a larger tendency there is always a larger tendency of the for the environment or the part for the environment for the environment to absorb to absorb some energy from the system some energy from the system then to supply then to supply energy to the system. But interesting things happen at t is equal to zero what about at t is equal to zero case.

In this case you see at t is equal to 0 the bath has no energy the bath has no energy to supply while okay at absolute zero temperature bath has no energy to supply and you see omega less than zero refers to the case the other side of on the left hand side of omega is equal to zero that represents the part where the environment is supplying energy. So, as you can see in the absolute zero temp at t is equal to zero temperature the spectrum would look like this.

So, here we will have a situation like this suppose at t is equal to 0 the spectrum will be of this type. Here omega greater than 0 and this is omega and this is your omega is equal to 0.

So, for example if we have a qubit or an atom it can easily emit a photon into the empty vacuum field empty vacuum fill we can consider as the bath and atom is the system at zero temperature and this is why in general you will we know in circuit QED or even the topic that we are going to discuss in the next module of quantum optomechanics we will get into this kind of picture again and again.

As you know that whether in circuit QED or in this kind of system or quantum optomechanics the system is always kept at a very very low temperature nearly equal to zero temperature. So, the spectrum there will always get like this. Let me stop here for today in this lecture we learned about the phenomenon of Pure-dephasing and we saw how quantum Linblad master equation can be used to incorporate this phenomenon.

And after that we discussed the dissipative Bloch equation in the context of a qubit where both relaxation processes and pure phase dephasing are taken into account. Finally very briefly we discussed about the Fermi Golden rule which is used to calculate various decay rates. In the next lecture we will start discussing quantum optomechanics which is another potential platform which is exploited nowadays for various quantum technological applications.

In fact we will see there that some of the tools that we have learned in this module as well as in this lecture is going to be very useful. So, see you in the next class, thank you.