Quantum Technology and Quantum Phenomena in Macroscopic Systems
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Lecture – 22
The Jaynes Cummings Model-II

Welcome to lecture 6 of module 2. This is the lecture number 16 of the course in the last class we started discussing the celebrated Jaynes Cummings model. In this class we will continue our discussion on Jaynes Cummings model apart from that we are going to discuss a landmark experiment related to Jaynes Cummings model in the context of circuit quantum electrodynamics. So, let us begin.

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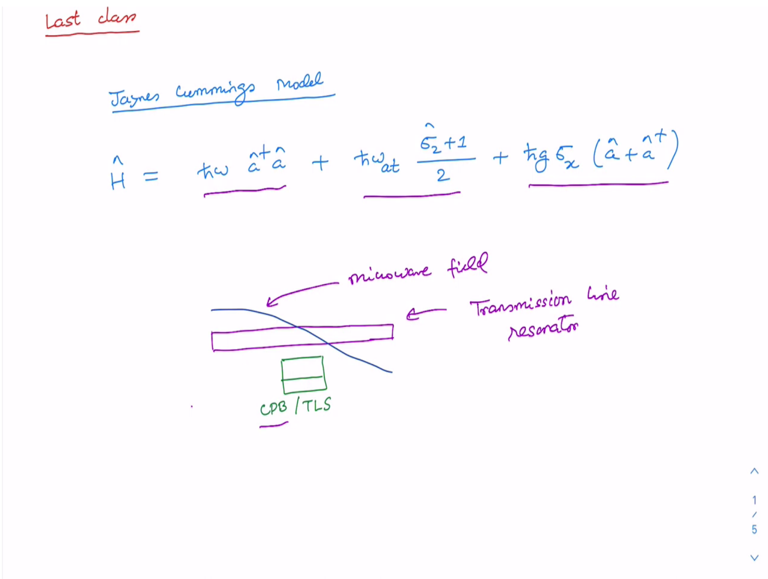

In the previous lecture we started discussing the famous Jaynes Cummings model which described the interaction between a field mode and a 2 level atom or system. In our case the field mode is a microwave field in a transmission line resonator. So, we have a microwave field in a transmission line resonator. On the other hand the atom the 2 level atom is nothing but a cooper pair box.

The first term in the Hamiltonian refers to the field mode energy. The second term refers to the 2 level atom in the coupled to the resonator and while this last term represents the interaction between the 2 level atom and the field mode. In fact the Jaynes Cummings model is pretty universal ,it can even describe interaction of a neutral atom inside ,the placed inside a cavity.

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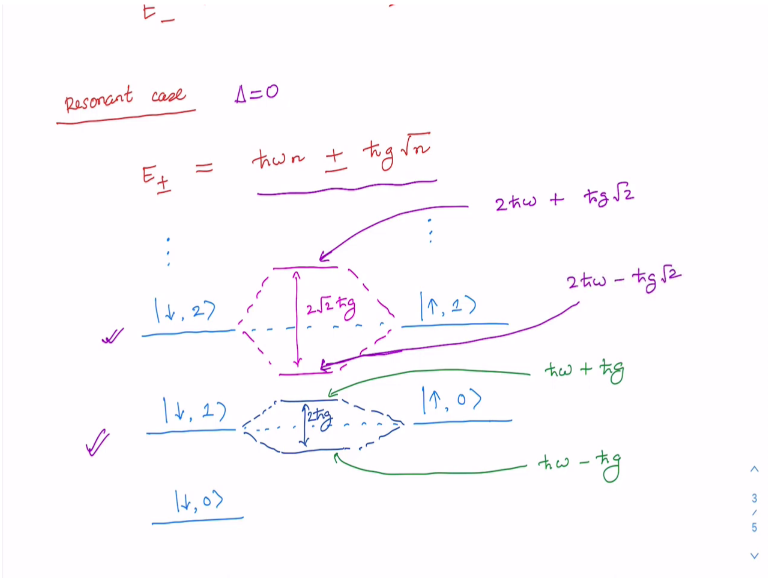

This neutral atom is can be modeled as a 2 level system and this interacts with an optical field. So, this is our optical field one optical field mode inside the cavity and this is our 2 level atom or system and this can also be described using the Jaynes Cummings model and in fact this is the domain of cavity quantum electrodynamics. And what we are discussing in this particular model is actually circuit quantum electrodynamics.

So, but the Jaynes Cummings model can equally be utilized in other domain as well such as cavity quantum electrodynamics. Now this JC model or Jaynes Cummings model cannot be solved analytically. However it can be solved under the so-called rotating wave approximation and in the rotating wave approximation the JC model can be written in this particular form where instead of sigma x we just replace it.

We just as we explained earlier that if we break it up into say lowering operator or raising atomic operator and the lowering atomic operator. And when we multiply we take the whole term because it is has to be multiplied with a + a dagger four terms should be there then out of these four terms we are going to keep only 2 terms because of physical reasons as I explained in the previous class.

Now the label scheme based on this RWA Jaynes Cummings model we discussed here as you see that when they are say coupling is 0 then we can have 2 kind of harmonic oscillator ladder depending on whether the atom is in the ground state or in the excited state and what is the number of photons in the field mode. And we encountered an important parameter called the detuning parameter.

On the other hand when we go over to the case of coupling when this coupling parameter is no longer 0 as well as if this coupling is very weak then under rotating approximation we discuss this only the neighbouring energy levels matters. And we can ignore interaction between the far away energy levels and therefore we can take the basis states to be like this where either the atom is in the ground state with n number of photons in the field mode or the atom is in the excited state with one less photon in the field mode.

And the matrix element that connects these neighbouring energy levels is was given worked out and it was h cross g into the square root of n and then we went on to discuss the matrix form of the RWA Jaynes Cummings Hamiltonian. And we worked out this matrix and also we found out the eigenvalues of this matrix for this Jaynes Cummings Hamiltonian and the resonant case we discussed at the end.

And we found that this for the resonant case that means when delta is equal to 0 we saw that this energy level gets splitted up into 2 say as we see for this particular states here energy this splitting is twice h cross g and for this case we see that the energy levels get splitted by 2 root 2 into h cross g the this excited energy state here would be from this expression you see this would be twice h cross omega + h cross g root 2 and on the other hand this one it would be twice h cross omega - h cross g root 2.

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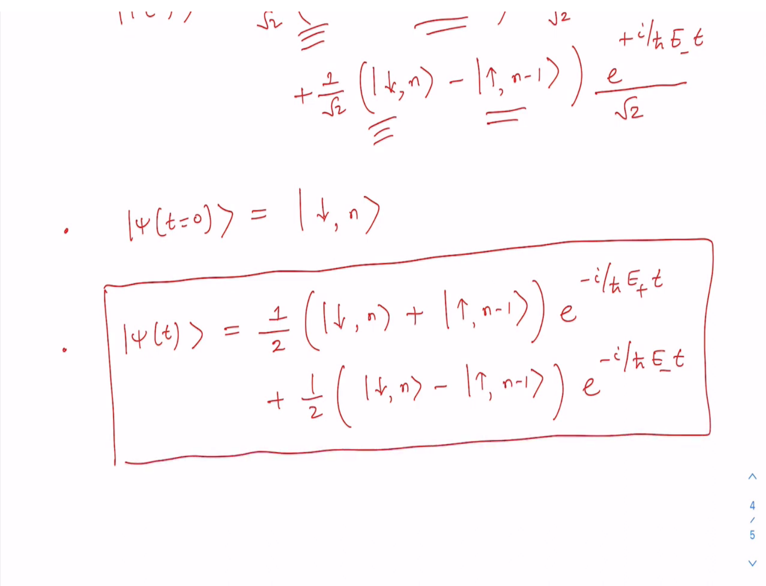

So, that is why the splitting is 2 in into root 2 into h cross g and that is where we actually concluded our class last time. Now what about the dynamics say at time t is equal to 0 the atom is in the ground state and there are n number of photons in the field mode then how the system will evolve in time. Let us say we prepare our the initial state of the system with the atom being in the ground state and there are n number of photons in the field mode. So, this is what we have prepared at time t is equal to 0.

So, this is this state of the system. Now our goal is to find out what is the state at time at some arbitrary time t. It is easy as we already know the eigenstates and their corresponding eigenvalues of the system. Because we already discussed say when we had our energy eigenvalue to be say E + which is we walked out as h cross omega n + h cross g square root of n this corresponds to the eigenstate 1 by root 2.

It is just it is a symmetric superposition of this state where the atom is in the ground state and there are n number of photons in the field mode or the atom is in the excited state and one less photon in the field mode on the other hand E - correspond to h cross omega n - h cross g square root of n and the corresponding state is simply the n anti-symmetric superposition of these 2 states which we already discussed.

We can use this to inform a use these informations to find out what is our state vector at an arbitrary time t and from elementary quantum mechanics it is easy to do. So, we know that one state we have is 1 by root 2. So, let me utilize this information. So, let me write here again we have this symmetric superposition state this is the eigenstate one of the eigenstate of the system and the energy eigenvalue is E +.

So, we have as the state evolve this is one part of the evolution and another one is 1 by root 2 this is the anti-symmetric superposition of these 2 states and this would be e to the power + i by h cross e - t. Now please note that using if this is the correct expression we should be able to get at time t is equal to 0 we should be able to recover our originally prepared step that is the atom in the ground state and there are n number of photons in the field mode.

If you check it we can recover it provided we just put here in the denominator square root of 2 and here if we put in the denominator square root of 2. So, if you check it carefully at time t is equal to 0 you will see this term and this term would get cancelled out and then these 2 this term and this term will add up and you will end up with the required state.

So, therefore let me write that we then obtain that at any arbitrary time the state of the system would be simply one half this symmetric superposition here we have this and here we have e to the power -i by h cross E + t + half antisymmetric superposition of these 2 states and here I have e to the power - i h cross E - t. So, this is the state we obtain.

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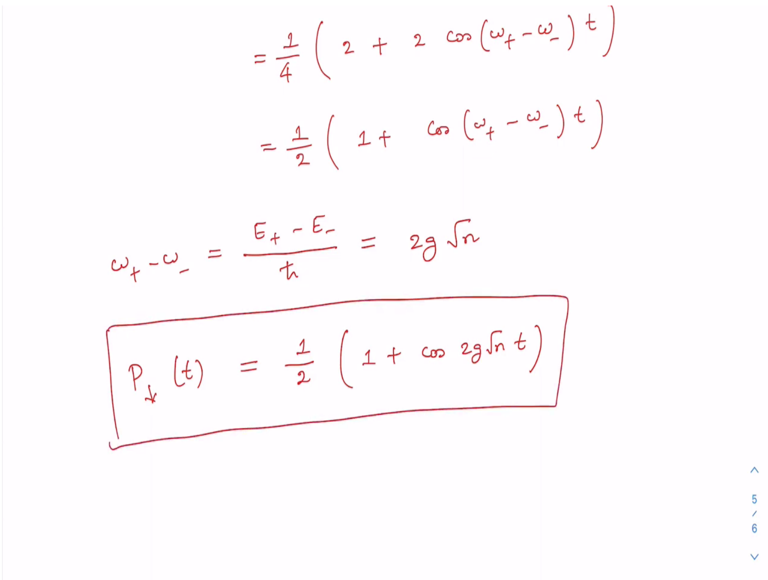

Now the question is very easy actually to work out what is the probability that our you will find the atom in the ground state at a later time. So, how you can work it out what I want to do is this .The question is suppose you were at time at an arbitrary state psi of t. And now you want to go to the state where the atom is in the ground state and n photons in the field mode.

So, what is the probability we know from our elementary quantum mechanics just we have to work out this particular quantity. Now psi of t we have worked out. So, you just take the scalar product of psi of t with this state. So, from this expression I think you can easily work out. So, let us do it. So, if you do that you will immediately see that here inside this bracket you are going to get 1 2 here in the denominator and in the numerator you will have e to the power - i by h cross E + t + e to the power - i by h cross E - t mod square.

And let me write it in the simpler form 2 e to the power - i say omega + t + e to the power - i omega - t mod square here omega + - is basically e + - by h cross I am writing both the expression just by using one x equation. Now if I break it I will have 1 by 4 because it is mod. So, I have e to the power - i omega + t e to the power - i omega - t here into e to the power + i omega + t + e to the power + i omega - t.

So, if you break it up very simple you are going to get 1 by 4 you will get simply I think you please do it, it is very simple you will get 2 + twice of cos omega + - omega - t this I can further simplify it, it is one half 1 + cos omega + - omega - t. Now you know that omega + - omega - is nothing but E + - E - by h cross and that is equal to because we have already this expression you just have to take the difference.

So, you are you are going to get here h cross h cross will get cancelled out and he will have 2g into square root of n. So, finally you obtain this expression the probability of getting the atom or the 2 level system in the ground state would be given by one half 1 + cosine of twice g root n into t .

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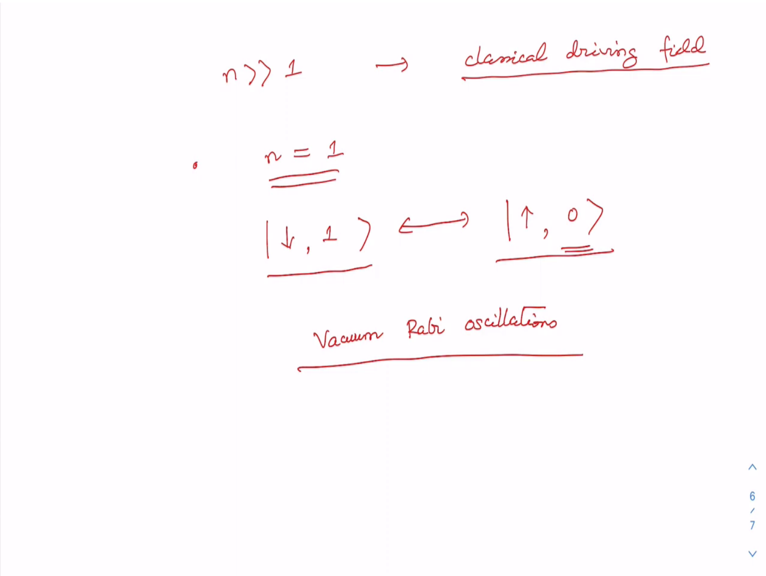

Now let me plot this probability versus time let us plot it. So, probability we are going to plot in the y axis here because it is probability minimum value should be 0 and the maximum it can attain is 1 and in the x axis let us put time. So, this is going to give a clear understanding of what is going on ah. So, at time t is equal to 0 we have prepared our system in the ground state diatom is in the ground state.

So, we will start here and later at some time t this cosine term will become -1. So, we will we'll have this probability to go will go to 0. So, we will have a it will go to 0 . So, when it go to 0 probability of getting the atom in the ground state is 0 means the atom is actually. Now in the excited state and which means that one photon from the field mode is getting converted to one excitation in the atom.

Now as time goes on the excitation in the atom become a photon again and the atom will again go to the ground state. So, it will go to the ground state and. So, as time goes on you will get this behaviour will repeat and you will get oscillating behaviour and oscillation and this oscillation is known as the so-called Rabi oscillation the frequency of this oscillation.

So this is frequency of oscillation is actually given by g into square root of n. Now clearly as this n the number of photons in the field mode is getting larger the oscillation will go faster. So, if n is very large. So, you will get oscillation will also become repeat or very fast in fact this quantity g into root n square root of n play the role of Rabi frequency or field amplitude. So, g into root n this is called Rabi frequency as we discussed in the first module this is related to the field amplitude.

So, as you know for very large n if n is much greater than one for very very large n we actually go over to the we go over to the classical regime classical driving field the driving becomes classical. So, we can consider the driving field to be a classical in nature. On the other hand there is another limit at the other side in the extreme cases say when n is equal to 1 that means you have only one photon in the field mode.

Then in that case as you see from these expressions here either we are going we are going to have actually 2 states either we are going to have the atom in the ground state and there is one photon in the field mode or we are going to have atom in the excited state and there will be no field mode or this would be the eigen states and therefore the oscillations will occur between this particular state and this state where this state is interesting because you see there is no photon in the field mode and that is the so-called vacuum.

And when n is equal to 1 in this case the Rabi oscillation is basically taking place between one proton state of the field and vacuum state of the field. So, that is why this oscillation for the n is equal to 1 case is known as vacuum Rabi oscillations. Now this vacuum Rabi oscillation has no classical counterpart it is a completely it is a quantum mechanical phenomenon.

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$$E_+ = \hbar\omega n + \hbar\Delta + \frac{\hbar g^2 n}{\Delta}$$

$$E_- = \hbar\omega n - \frac{\hbar g^2 n}{\Delta}$$

$$\downarrow$$

$$E_+ = \hbar\left(\omega + \frac{g^2}{|\Delta|}\right)n + \hbar\Delta$$

$$E_- = \hbar\left(\omega - \frac{g^2}{|\Delta|}\right)n$$

Let us consider another important situation when the detuning parameter is very large means when the detuning parameter delta which we defined as omega atom minus omega omega atom is the atomic transition frequency and omega is the field frequency. And we are going to consider the situation when this detuning parameter is very very large than the coupling strength between the atom and the field.

So, this is actually a very useful case we will see that the in this regime when the tuning parameter is very very large then the coupling strength. The frequency of the field mode will turn out to depend on the state of the atom and this actually imply that we can determine the state of the atom just by measuring the frequency of the field without directly making any measurement on the atom.

This case when this detuning parameter delta is much greater than g coupling strength this case is called dispersive case. We will now analyze it in some more details. Now we know that within the rotating wave approximation the Jaynes Cummings Hamiltonian in the matrix form we wrote it already under RWA. It was h cross omega n here and here it is h cross omega n + h cross delta and of diagonal elements were s cross g square root of n h cross g square root of n.

Just recall that we wrote this Hamiltonian in the basis atom in the ground state with n number of photons in the field mode and atom in the excited state and one less photon in the field mode. So, these were the basis states. Now also we walked out the energy eigenvalues of this matrix. So, it was it was E + - is equal to h cross omega n + h cross delta by 2 + - h cross square root of g square n + delta by 2 whole square.

Now let us assume as we have are considering the dispersive case say that the tuning is much greater than the coupling parameter remember that delta can be delta is equal to omega at - omega. So, it can be greater than 0 or it can be less than 0 but for our calculational simplicity we are going to consider delta greater than 0. So, in this case when coupling parameter detuning is much greater than 0 g let me know in that regime let me expand this expression under the square root here.

So, let me just see how I can write it g square and delta by 2 whole square because delta this is much larger than this quantity. So, let me pull it out and if I pull it out then I have delta by 2 here and I can write it let me just put more because delta can be less than 0 or greater than 0 then I have here 1 + g square n divided by delta by 2 whole square. Now which I can write because now this term is smaller than 1.

So, I can do a Taylor series expansion and I can write this actually binomial expansion I can do. So, I have 1 + half and you will have 4 g square n by delta square and then you will have all higher order terms it would be there but as you will see because of the condition that we are having the all the higher order terms we can neglect to a good approximation then we will have the first term would be mod delta by 2 + g square n by mod delta.

And then we have here all the higher order higher order terms in g square by delta. So, now as I said for simplicity we are going to take delta is greater than 0, 0 this detuning parameter then I can write my new energy level E + - I just have to put this expression put the this form here. So, if I put it there then I will get my let me write it pretty clearly here I will get E + - is equal to h cross omega n + h cross delta by 2 + - h cross and then this which is what i am going to have delta + 2 g square n by delta.

So, if I break it up I get 2 energy values one is E + and another one is E - and E + will correspond to h cross omega n + because of this plus sign these 2 terms will add up. So, I will have h cross delta and I will have h cross g square n by delta on the other hand I will have here h cross omega n - h cross g square n by delta. In fact let me rewrite it this I can also write in this form say E + is equal to h cross omega + g square by delta into n + h cross delta and E - I have simply h cross omega - g square by delta n.

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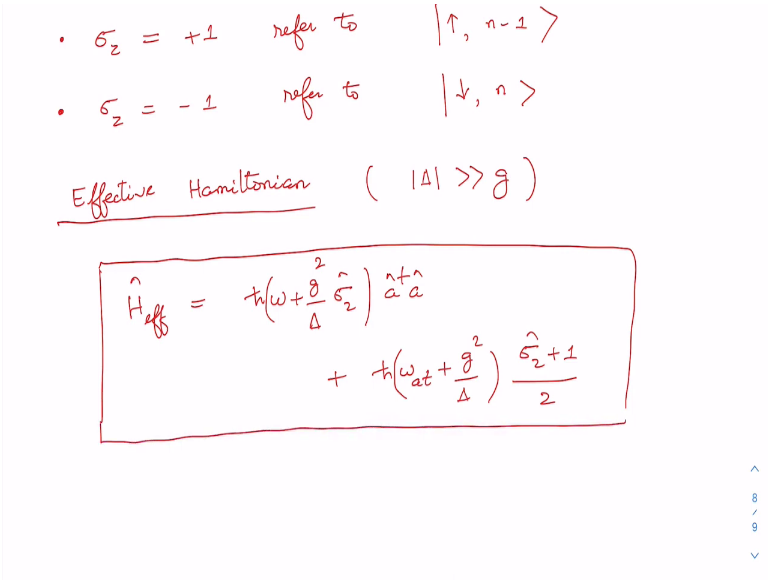

So, clearly if you look at these 2 expressions here you see in reality we are now having in this dispersive regime we are having 2 layers of harmonic oscillator levels which are equally spaced. In one case the spacing is given by omega + g square by delta because you see this is related to the field mode right and you know the field mode is act like a harmonic oscillator.

And now what I am saying is that we have 2 harmonic oscillator where because this is related to field we are having 2 harmonic oscillator and their spacing frequency all and you know that harmonic oscillators are have equal spacing in the energy level. So, spacing here is in one case it is given by omega + g square by delta in another case it is given by omega - g square by delta.

So, now actually it depends on the state of the atom as you know that E + refers to the case when the atom is in the excited state while E - refers to the case when the atom is in the ground state. So, actually you can recall that when we consider the case where g is equal to 0 let us just recall that when we discuss the case g is equal to 0 we had E + is equal to h cross omega n + h cross delta which in fact if you remember we can write it as h cross omega n - 1 + h cross omega atom here.

So, the atom is in the excited state and in the field there are n - 1 number of photons where they are in the field mode on the other hand we had another energy level that was simply h cross omega n. Now here n photons are in the field and the atom is in the in this case the atom is in the ground state atom is in the down state or in the ground state this we already know. So, keeping these things in mind so from let me once again write from E + is equal to h cross omega + g square by delta n + h cross delta and E- is equal to h cross omega h cross omega - g square by delta n.

What we can find is that in the dispersive case when g is not equal to 0 and this detuning is much greater than this coupling parameter the field mode frequency as you can see from these 2 expression let me write here the field mode the field mode frequency. Now you see clearly that it depends on the state of the atom. This is an important result because earlier say when there is no coupling nothing else was there suppose the field frequency was omega.

Now because of the coupling and when the detuning is very large. So, now the free frequency becomes omega + - g square by delta. So, here this plus sign plus refers to atom in the excited state. So, atom in the excited state what it mean is that if you make a spectroscopic measurement. Suppose you measure the field frequency if you find the field frequency to be larger because of the plus sign then the original field frequency omega.

Then you will automatically know that the atom is in the excited state on the other hand the minus case the lower case refers to the fact that the atom is in the ground state. That means if you make the spectroscopic measurement that means you make the measurement of the field frequency then if the field frequency is lower than the original frequency of the field mode then you will automatically know that the atom is in the ground state.

So, this is going to be this is actually a very useful tool to determine the state of the atom. Without making any measurement directly on the atom is a very useful spectroscopic tool and it is. Now it is routinely used in circuit quantum electrodynamics. Now all these effects that we discussed for the dispersive case could be summarized in the form of an effective Hamiltonian.

In order to write the effective Hamiltonian let us use this sigma g to sigma g is equal to say + 1 this is the z component of the Pauli matrices. Let us say sigma z is equal to +1 refer to the atom in the ground state and with n - 1 number of photons in the field mode while sigma z is equal to - 1 refer to I mean actually the eigenvalue of sigma z you know it is either it has 2 eigenvalues +1 and -1. So, I am writing the +1 refers to the case when the atom is in the excited state.

And there are one less photon in the field mode that means n - 1 number of photons in the field mode. On the other hand when eigenvalue of sigma z is equal to -1 we have the atom in the ground state and there are n number of photons in the field mode. So, then we can write an effective Hamiltonian for the system for the dispersive case. So, we can write an effective Hamiltonian nd we are writing it for this dispersive case.

Now you recall that earlier what we had for the field mode in the Jaynes Cummings model in the field mode we have this field frequencies h cross omega and then we had this a dagger a was there. Now because as you see the field frequency. Now starts depending on the state of the atom to incorporate that we have to put this field frequency as we have here from here you see because of this we have to write it as in the operator form we can write it g square by delta.

Let me write it this way and then sigma g. So, this I am we have an operator here and then we have this a dagger a which represents the photon number. On the other hand for the atom case again from the Jaynes Cummings model we know that in the case of the atom we had this and we had this term sigma z + 1 by 2 where if sigma z is equal to 1 then we know that the we have h cross omega atom that means the atom is in the excited state and if sigma z is equal to - 1 then the atom is in the ground state.

Now to get the correct energy values that we have worked out here you have to add you can actually verify it you have to add this term here g square by delta then you are going to get the correct energy eigenvalues for the system. So, this is the effective hamiltonian for the model in the dispersive case. In fact this Hamiltonian could be written in some other different form as well but I will not go into those models or those form of the Hamiltonian.

And now I think it is time for us to discuss some experimental work regarding this Jaynes Cummings model in the context of circuit QED. So, let us study or actually just discuss some landmark work regarding experiments on the Jaynes Cummings model in circuit quantum electrodynamics. The standard setup for circuit quantum electrodynamics which we already discussed earlier is as follows.

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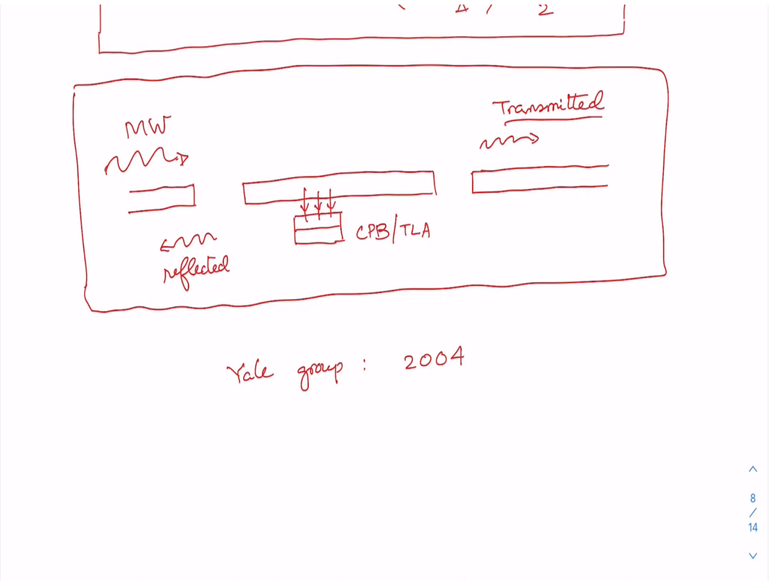

We have a we have a transmission line resonator we know how to get a transmission line resonator. Suppose we have a transmission line infinite transmission line then we break it at 2 locations and then we get a transmission line resonator and then we have a cooper pair box or a 2 level system which is getting coupled to the transmission line resonator at some location.

And the coupling strength between the coupling strength between the transmission line resonator and this cooper pair box is dependent on the location that where the cooper pair box is put on the transmission line and this thing we discussed earlier. In fact so far I discussed about only one type of 2 level system system. So, in the next class I am going to discuss about some other kind of 2-level artificial system which is routinely used in circuit quantum electrodynamics experiments or setup.

Now anyhow at the moment let us discuss say we have a cooper pair box that is put on the transmission line resonator like this. Now what is done is a microwave drive signal is incident on the transmission line resonator or the system here like this. And this microwave signal gets is some part of the microwave signal that is incident on the resonator is getting transmitted it is get transmitted.

And a part of the signal here may also it may also get reflected and what is done in experiment is that by looking at the intensity of this transmitted beam the people infer the experimentalist infer about the state of the cooper pair box or the 2 level atom and other things about the system. In fact this scheme was experimentally realized in laboratory by a Yale group long back in 2004.

And it is considered to be an landmark experiment in the area and this was published in the journal nature.

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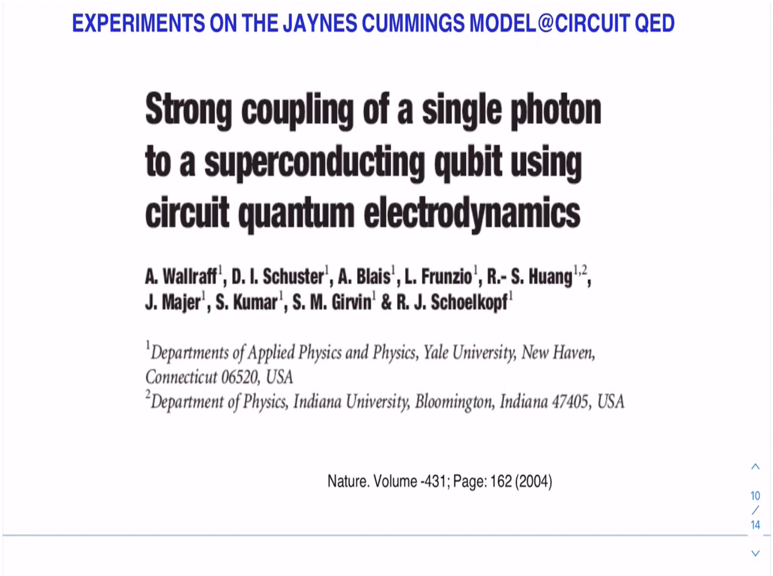

The title of the article was this strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics. So, it was primarily it was the Yale group and one person was actually associated with Indiana university as well. So, it is primarily it was done in yale and this was published in the journal nature. And it may be useful to read the very insightful abstract of the paper.

Let us read it, it says the interaction of matter and light is one of the fundamental processes occurring in nature and its most elementary form is realized when a single atom interacts with a single photon. Releasing this regime has been a major focus of research in atomic physics and quantum optics for several decades and has generated the field of cavity quantum electrodynamics.

Here we perform an experiment in who is a superconducting 2 level system playing the role of an artificial atom is coupled to an on-ship cavity consisting of a superconducting transmission line resonator. We saw that the strong coupling regime can be attained in a solid state system and we experimentally observe the coherent interaction of a superconducting 2 level system with a single microwave photon.

The concept of circuit quantum electrodynamics opens many new possibilities for studying the strong interaction of light and matter. This system can also be exploited for quantum information processing and quantum communication and may lead to new approaches for single photon generation and detection.

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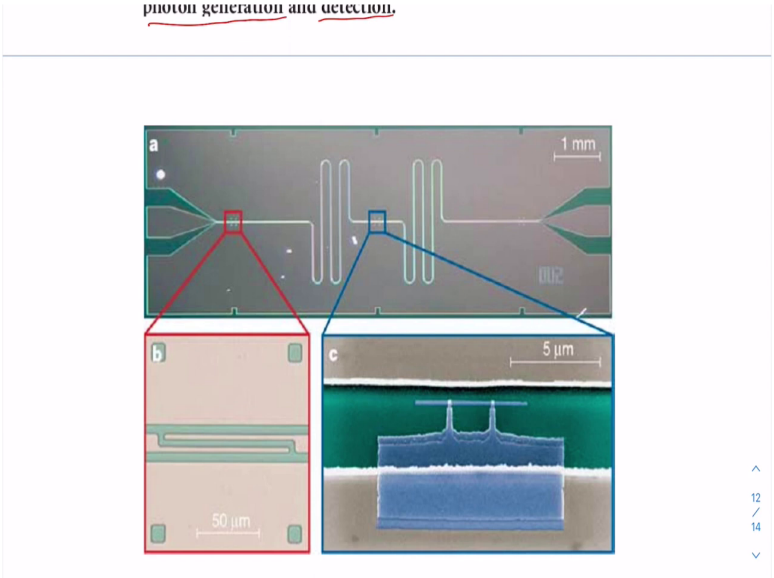

So, this is a very important work and the experimental real transmission line resonator and the cooper pair box is actually shown in this diagram I would not go into the details actually what you see here it is the input end of the transmission line resonator and this is the output end here and in middle here this cooper pair box is placed. I would like to encourage interested people to read the article for experimental details and fabrication.

Actually this on-ship cavity that is shown here is made by patterning a thin superconducting niobium flame deposited on a silicon chip and you may go to the read the article for the experimental details and also the about the fabrication. So, the essence and the idea of the experiment is already I have described. So, in a sense this is basically this is the setup or this is the scheme as I already explained.

What I will do now is that I will try to explain the key findings of this landmark paper based on what we have learned.

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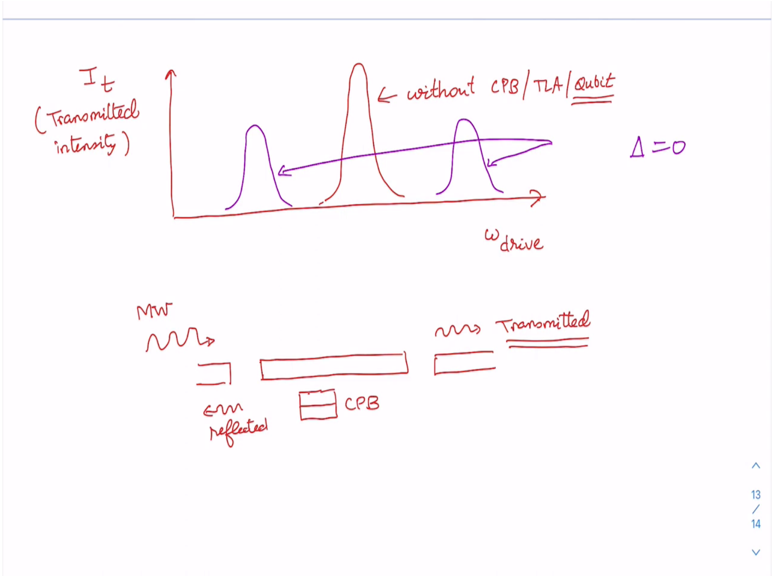

So, far now what the group observed was the intensity of the transmitted beam they plotted the intensity of the transmitted beam as a function of the drive frequency. So, in the y-axis they plotted the transmitted intensity of the signal or the beam here the transmitted intensity and it was plotted as a function of the frequency of the weak oscillating microwave drive.

Let me just again explain what I actually telling you you remember the essentially the experiment the scheme is like this we have this transmission line resonator is there and this transmission line resonator is getting coupled to a cooper pair box placed somewhere here say and a microwave and weak oscillating microwave drive is incident on the transmission line resonator.

Then a part of the microwave is getting transmitted and a part of it may get is getting say reflected. So, this is the scheme and what they observed they basically measured the intensity of this transmitted beam and when they did the experiment first of all they observed that when there is no cooper pair box or the qubit they observed one single peak like this. So, this is without this is without the cooper pear box or 2 level atom or now onwards we can also say it to be qubit.

Because ultimately qubit is nothing but a 2 level system and this is one of the important application that they had in mind as you have read in the abstract that their quantum information processing. So, now onwards let me tell 2 level atom or cooper pair box simply as qubit. So, this is what they had without this qubit and by the way once we have a setup like this or we have done the fabrication we cannot get rid of the cooper pair box right.

But then how they got it this result it can be done if we draw if we detune the qubit or detune the drive frequency from the atomic transition frequency significantly then the influence of the qubit would no longer be there. So, when I say that without qubit or cooper pair box I mean to say that it is actually without having it influenced. Now when this drive frequency matches the atomic transition frequency that means; when delta the tuning parameter delta is equal to 0.

So, what they observed is they observe 2 peaks they observe 2 peaks instead of this single peak. So, and this is the case when detuning is equal to 0 and the spacing between these 2 peaks was found to be 2 into g where g is the coupling strength between the microwave cooper pair box and the microwave signal right. So, this is what they observed experimentally. Now we will try to understand this particular result.

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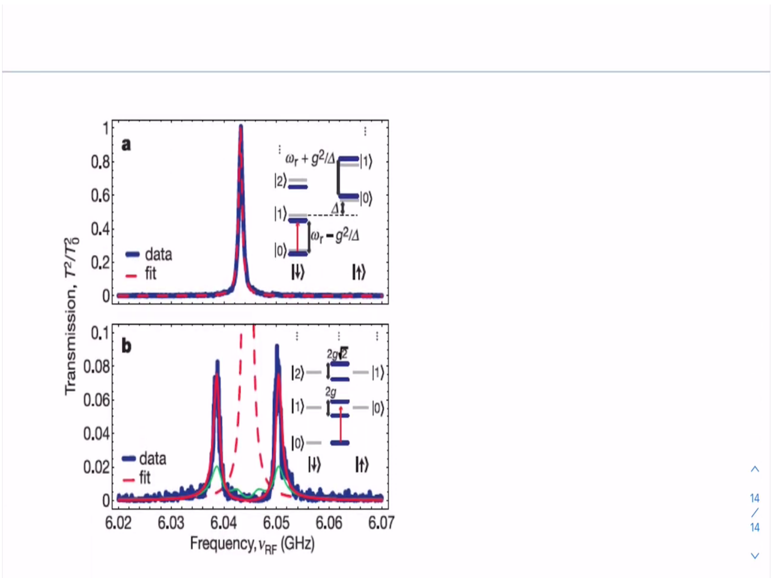

So, the question is in fact let me before I we try to understand it let us let me give you the experimental what the in the sort in the experiment you see here in this first plot they first observe the only single peak when this qubit is heavily detuned that means it when it had no influence as I explained already and when in the detuned case when delta is equal to 0 they observe these 2 peaks. Let us now try to understand actually what is going on.

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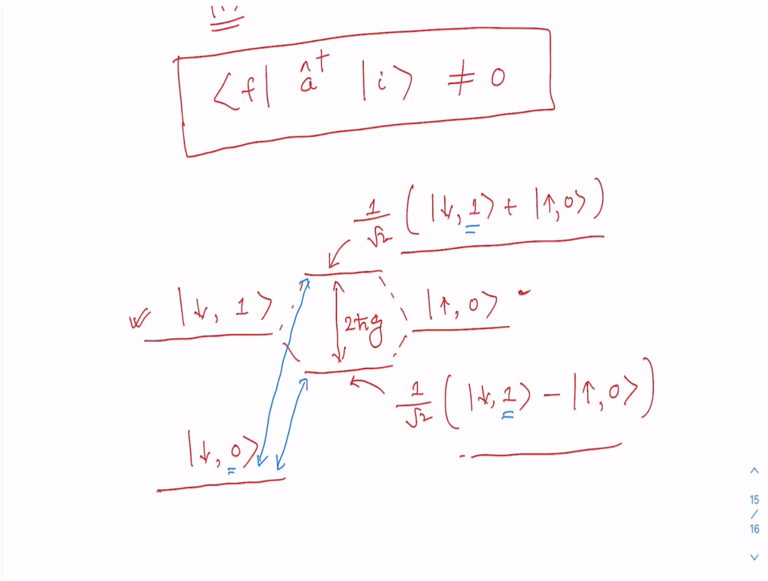

So, the question that we have to ask is at what frequency omega at what at what frequency say at what frequency omega of the omega drive a microwave photon can enter a micro wave photon can enter into the resonator and thereby excite the atom there or the qubit. So, clearly that can happen only if when this frequency matches this atomic transition frequency that we know from elementary quantum mechanics.

So, the point is that initially what we had initially initial state of our system is the atom who is prepared in the ground state and there is no field there no photon on the field mode. So, this is what we have initial this is our initial state and then finally what we had is that we had a final state I will discuss about it what it is because in the now in the final state obviously what we have we have one photon is getting created inside the system.

So, this initial state and the final state is basically connected by a creation of a photon inside the system and this matrix element just for our understanding I am writing it like this. So, it means that this matrix element connecting the initial state and the final state who is connected by the creation of a photon inside the system is no longer 0 and let us understand this can be you know very well understood the origin of these 2 peaks just by going back to our Jaynes Cummings model level scheme.

So, you recall that what we had in the Jaynes Cummings model the atom initially is in the ground state and there is no photon in the field mode. And we had this excited state here energy state when atom is in the ground state and for one photon in the field or when at resonance we have this situation the atom may be in the excited state and there is no field photon. But because of the coupling between the atom and the or the qubit or the microwave these degenerate states become non-degenerate.

They become hybridized and we have these 2 energy levels and this splitting happens and this splitting as we already discussed is equal to twice h cross g right and where g is the coupling between the microwave in the qubit or the atom and this particular energy state is this state is simply is a superposition of this state. And this state symmetric superposition we have this symmetric superposition of these 2 states.

On the other hand this particular state is a anti-symmetric superposition of atom in the ground state one proton in the field mode and atom in the excited state no photon in the field mode right this is what we have. Now, as I said that we have a photon is getting created inside the system. So, if you look at these 2 hybridized states here this state as well as this you will observe that in both this contribution while we started with no photon initially.

Now here we have one photon a contribution is there similarly a contribution is there. So, clearly 2 transitions are possible one transition would be this one and another transition would be this one right. And therefore we can say from this it is easy to see what is the reason behind observing these 2 peaks this is the reason because of this these 2 possible transitions where we get 2 peaks.

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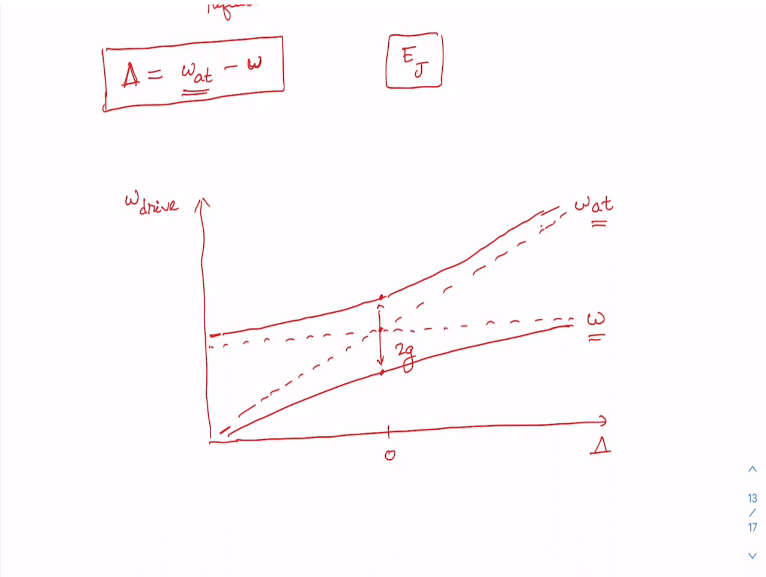

Another study the group did was to change the detuning parameter delta by changing the atomic transition frequency by the way here this results that is plotted is for the tuning parameter delta to be equal to 0 and as you remember that delta is equal to omega atom transition frequency of the atom minus the bear cavity frequency and once we fabricate the system actually it is bare capital frequency small omega gets fixed and only option that we have if we want to sense this detuning parameter we can we have to change the atomic transition frequency.

Now, because we are using cooper pier box or artificial atom, so, this changing this atomic transition frequency is easy we just have to change this deception coupling energy here easy and that can be done by changing the gate voltage or the gate charge. So, what is done is that in one axis we can just plot this we have this say the tuning parameter while in the another axis that that is in the say y axis here y axis we have this omega drive that is the driving frequency of the microwave.

So, now say delta is equal to 0 is somewhere here as we already know at delta is equal to 0 as you can also see from this plot here the omega drive we have 2 peaks. So, omega drive has 2 values here and we have 2 peaks 2 points. So, we have one peak here and say we have another peak is here right. And let us say our bare cavity frequency which is fixed that is equal to omega this is the bare cavity frequency and we are as we are the tuning parameter is getting changed we just have to change the atomic transition frequency and that is something like this.

So, this is what our omega atomic that is how it is what atomic transition frequency is getting changed and this is the point of crossing or the degeneracy point where omega is equal to this bare cavity frequency is equal to omega atomic frequency or the drive frequency is exactly equal to the omega here and this is the point and this difference is as we know that is equal to twice g.

Now when the coupling is there when g is not equal to 0 these energy levels no longer cross and we have these bare energies here and here also we have this bare energy levels are there and we encounter what is called avoided crossing right. Let me draw it properly. So, what we encounter is called atomic avoided crossing and this already we discussed in great details in module one.

As well as here as you see in the context of Jaynes Cummings model we this is observed in the Yale groups experiment they have experimentally demonstrated this kind of behaviour.

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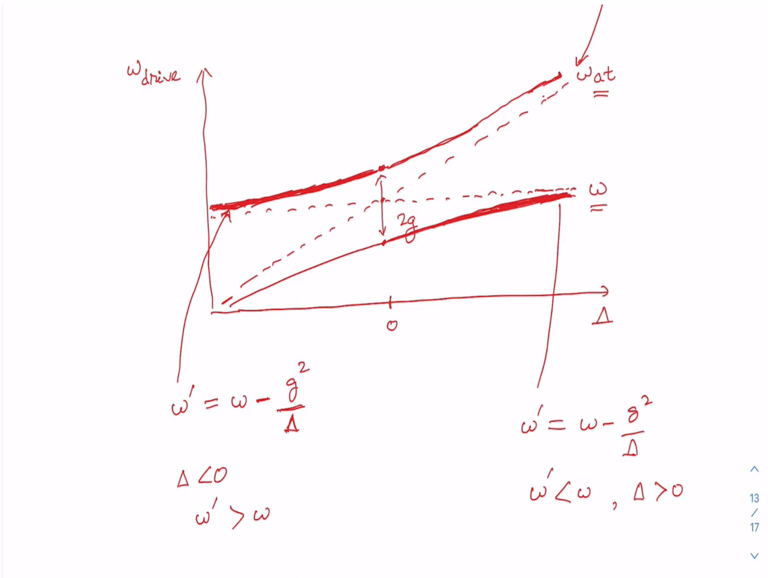

Another issue that we can discuss is what about the intensity of these peaks. Now when delta is equal to 0 that means at the detuning this peak intensity is actually related to as I discussed earlier this is related to this matrix element connecting the initial state and the final state and that is connected by generation of a or excitation of a photon inside the cavity. Now we as we discussed earlier that this is the situation when the contribution.

I think here we discussed it, yes here we discussed it the contribution is when a photon is we have actually started with no photon in the field mode and. Now a photon is created inside it contribution is because of this term as well as this term and both these terms has this equal weightage is 1 by root 2. So, when delta is equal to 0 the intensity would be actually the same here and we will the both the peak would have the same intensity.

But the question is that what happens if we go away from this detuning parameter or detuning delta is equal to 0. The answer is simple here because you see what we are going to have here is suppose we are our omega drive frequency is here that is exactly equal to the atomic transition frequency. But at this point you see we are away from this br cavity that means there is no photon inside here inside the system.

So, intensity the matrix element this would be equal to nearly would be equal to 0 right. So, it would be the situation here. So, on the other hand if we are at this end when our omega drive is nearly equal to omega then there would be a generation of photon inside the cavity. So, we'll have this would be non-0 we will have a finite contribution from here. So, there will be intensity similar argument goes to this point.

So, you will get a intensity would be high here near the bare cavity frequency when omega drive is equal to bear this frequency bare cavity frequency similar is the case here it will go the intensity would be high up to this point then as we go away from this point intensity would get lower and lower I hope this point is clear. So, this is what is observed experimentally as well.

Now another thing is that what about the dispersive case that we discussed as you see from this plot as well as this plot the dispersive cases we are aware from the detuning. So, here you see the this omega freak frequency that is your omega drive is actually not exactly matching the bare gravity frequency and that is different from the bare cavity frequency by omega is by z square by delta that is what we discussed.

And here also the same thing that is also given by omega dash is equal to omega - z square by delta because of the fact that we always start you see this minus and refer to the fact that the atom is actually in the ground state and that is what we always start right in this experiment diatom or the qubit is always in the ground state. Now in this case because delta is less than 0 in this part here I am talking about this part.

So, your omega dash is greater than omega because delta is less than 0 and in this case you have omega dash is greater oh sorry it would be smaller than omega as you can actually see from the diagram also omega dash is equal to omega bare cavity frequency because delta is greater than 0. So, all these things you see s per our model everything all the experimental results is very nicely explained by the analysis that we did with the Jaynes Cummings model.

Let me stop for today in this class we completed our discussion on Jaynes Cummings model and we saw how we can understand the experimental results obtained by the Yale group based on our discussion on Jaynes Cummings model. Now so far we have not discussed mass about the Josephson junctions and this is what we are going to take up in the next class. We are going to discuss in some more details about Josephson junction in the next class. So, see you in the next class, thank you.