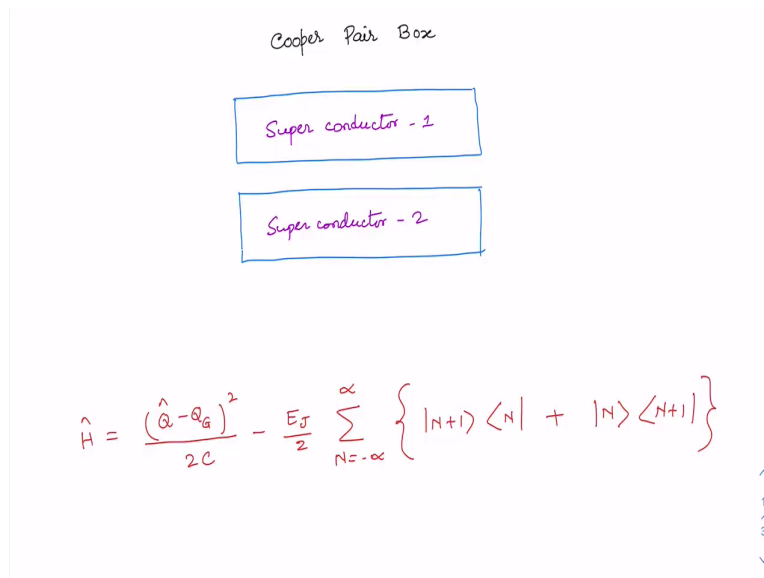


Quantum Technology and Quantum Phenomena in Macroscopic Systems
Prof. Amarendra Kumar Sarma
Department of Physics
Indian Institute of Technology, Guwahati

Lecture – 17
Cooper Pair Box as TLS: Introduction to Transmission Line

Hello, welcome to lecture 2 of module 2. In the previous lecture we got an overview of very brief overview of circuit quantum electrodynamics and we got introduced to the concept of cooper pair box. In this lecture today we learn how to model a cooper pair box as a 2-level system and also you will be introduced to the idea of transmission line which is a must learn topic for circuit quantum electrodynamics. So, let us begin.

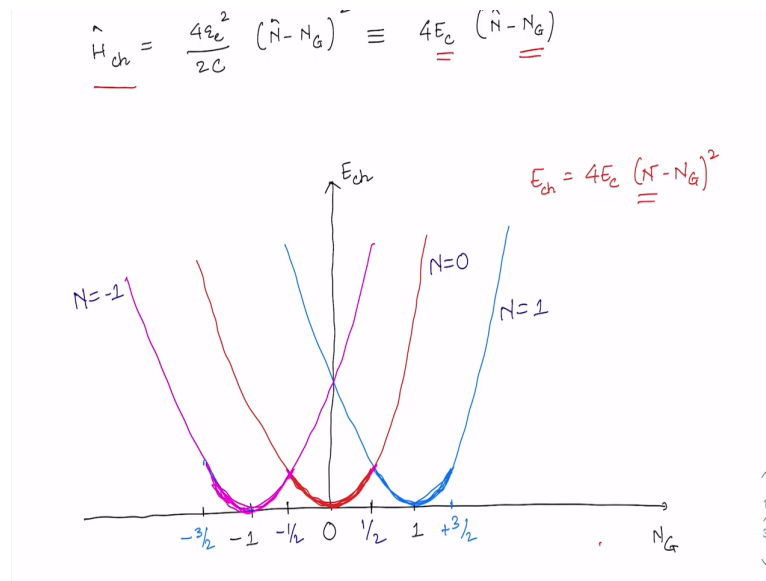
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In the last class we started discussing cooper bear box. A cooper pair box consists of 2 super conducting islands placed close to each other. Typically, a cooper pair box has the dimension in micrometer and the gap between the island is in nanometer. We learnt how to write down the model Hamiltonian for a cooper pair box in the last class. Here, the first term refers to charging energy and the second term refers to or it denotes the energy involved in tunneling of cooper pairs from island 1 to island 2 or from island 2 to island 1.

Q G is the gate source and E J is the Josephson energy then the question was how to solve this Hamiltonian and mainly how to realize a 2-level system out of it. In the first step we took tunneling as a perturbation and ignored it we simply considered this first term first term of this CPB Hamiltonian.

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We wrote the charging part of the Hamiltonian in a more convenient form by expressing this charge operator in terms of the number operator while expressing the gate charge in the similar form. We wrote down the charging part of the Hamiltonian here E_C is the charging energy constant N_G is the controllable parameter as it is related to the gate voltage or the external electric field that is used to tune the Cooper pair box.

We plotted the charging energy and as a function of the controllable parameter N_G and keeping this Cooper pair number of Cooper pair to be fixed. We find that in the absence of perturbation various charging energy states cross each other. For say N_G lying between minus half to plus half the preferred ground state energy state is N is equal to 0 while for N_G lying between say minus say plus half okay plus half to plus 3 by 2 the preferred ground state energy corresponds to N is equal to 1 and so on.

We basically get a set of or array of parabolas for various Cooper pair various number of Cooper pairs.

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$$\begin{aligned}
 N_G &\rightarrow \frac{1}{2} + \delta N_G \\
 (N - N_G)^2 &= \left(N - \frac{1}{2} - \delta N_G \right)^2 \\
 &= \frac{1}{4} \pm \delta N_G + (\delta N_G)^2 \quad (\text{For } N=0) \\
 &= \frac{1}{4} - \delta N_G + (\delta N_G)^2 \quad (\text{For } N=1)
 \end{aligned}$$

Now in the second step we switch on the perturbation that means we consider the second term in the Hamiltonian and we must have charging-energy should be greater than the Josephson energy. Clearly, as we discussed in module 1 the energy states no longer cross each other they simply turn into avoided crossing as I have depicted this in this picture as you see the energy level no longer cross.

So, we get into this is avoided crossing okay this is what I mean by avoided crossing. Now next what we will do is that we are going to focus around one of the degeneracy points. Say for example let us focus around N_G is equal to half then we will restrict ourselves to the energy levels N is equal to 0 and N is equal to 1. As you can see when N_G is equal to half when perturbation was switched off these energy states corresponding to N is equal to 0 and N is equal to 1 cross each other.

So, if I focus around N_G is equal to half then we are actually confining ourselves to the charged energy state N is equal to 0 and N is equal to 1 and another thing you can see that all the other energies will be far off and their effects will be small. Now near N_G is equal to half we can expand the charging energy $4 E_C$ into $N - N_G$ let us expand it near the this near okay let me write here expand the charging energy near N_G is equal to a half.

If we do that, what I mean to say is that let us say N_G replace it by half plus some deviation from N_G is equal to half. So, δN_G is the deviation from N_G is equal to half then we can write $N - N_G$ square is equal to $N - \frac{1}{2} - \delta N_G$ whole square. Now you recall that N this N can be 0 or it can be 1 okay because we are around near N_G is equal

to half. So, for N is equal to 0 we will get 1 by 4 plus delta N G plus delta N G square this is for N is equal to 0.

And if we take N is equal to 1 then we will get 1 by 4 minus delta N G plus delta N G square this is for N is equal to 1. So, this positive this positive sign or plus sign refers to the state N is equal to 0 or you can remember it just like thinking that okay you can as you can see that you are having a positive slope here for N is equal to 0 that is why you are having a plus sign for that and for N is equal to 1 you are having a negative slope for the minus sign okay plus sign refers to N is equal to 0 and minus sign refers to N is equal to here N is equal to 1.

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$$\hat{H} = \underbrace{4E_c \delta N_G}_{-} |0\rangle\langle 0| - 4E_c \delta N_G |1\rangle\langle 1| - \frac{E_J}{2} [|0\rangle\langle 1| + |1\rangle\langle 0|]$$

$$H = \begin{pmatrix} 4E_c \delta N_G & -E_J/2 \\ -E_J/2 & -4E_c \delta N_G \end{pmatrix}$$

We are going to neglect this delta N G square term because we will focus only on the linear term that is the linear curvature around delta N G is equal to half. So, we will just focus near this region only. So, around the linear curvature region and then we can write the charging energy E Ch is as 4 into E C 1 by 4 plus minus delta N G let me emphasize once again that plus correspond to N is equal to 0 and minus correspond to N is equal to 1 or I have E C plus minus 4 E C into delta N G.

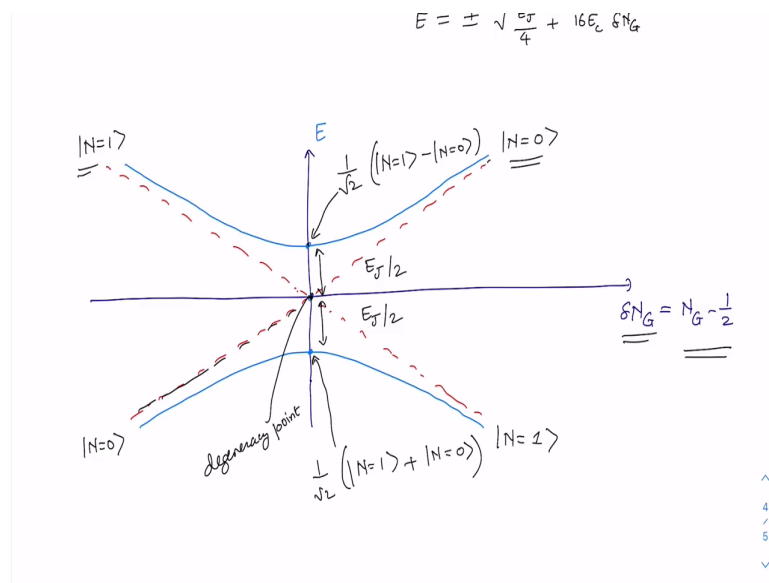
Now this term is basically a constant it is simply energy offset. So, we can ignore this constant term as well. So, if we ignore that then we will have the charging energy as plus minus 4E C into delta N G recall that delta N G basically is the controllable parameter which you can vary. Now clearly for N is equal to 0 state in the absence of perturbation that correspond to this energy eigenvalue for this energy state would be simply 4E C delta N G.

For N is equal to 1 state the energy eigenvalue would be minus $4E_C \delta N_G$ all right. Now let us take N_G this ket state is $|1, 0\rangle$ and N is equal to 1 let me write it as $|0, 1\rangle$ then now let us switch on the tunneling term and in the presence of the tunneling the Hamiltonian can be now written like this. So, we will have $4E_C \delta N_G$ in the basis N is equal to 0 and N is equal to 1 ket N is equal to 0.

Let me write it this way this is for this energy corresponds to N is equal to 0. So, this term and $4E_C \delta N_G$ ket $|1, 0\rangle$ and the Josephson tunneling part is going to give rise to this term ket $|0, 1\rangle$ and ket $|1, 0\rangle$ to this all right. So, therefore immediately we see that we are actually getting a 2-level system Hamiltonian and which can be expressed in the matrix form by this 2 by 2 matrix we have $4E_C \delta N_G$ and here minus $E_J/2$ minus $E_J/2$ minus $4E_C \delta N_G$.

So, that is how we obtained a 2-level system out of a Cooper pair box if we focus ourselves near the degeneracy point N_G is equal to half.

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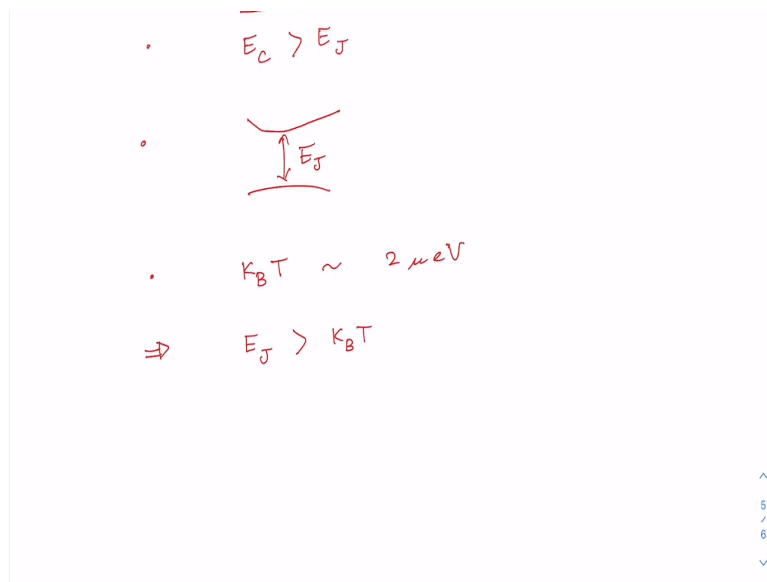
Now we know very well how to solve this 2 by 2 Hamiltonian. The energy eigenvalues we can write easily from our knowledge of module one that would be plus minus will have $E = \pm \sqrt{\frac{E_J^2}{4} + 16E_C^2 \delta N_G^2}$. Just like in the case of 2-level atom or 2-level system that we discussed in module one here also if we plot energy eigenvalue as a function of this controllable parameter δN_G , we will get a typical plot like this one.

So, it should remind you about the 2-level system that we discussed in module one. You see when the perturbation is not there these 2 energy levels cross each other at this point and that is called the degeneracy point N_G is equal to half that is ΔN_G is equal to 0 or N_G is equal to half this is the degeneracy point. The positive slope refers to the eigen state for N is equal to 0 and negative slope refers to the eigen state corresponding to N is equal to 1 state.

When the perturbation is switched on, they no longer cross this plot. I am drawing for near N_G is equal to half okay you just have to remember that. So, and this energy gap that is E_J by 2 this is also E_J by 2 and this particular point correspond to here the state this particular point correspond to the state $1/\sqrt{2}$ N is equal to 1 minus N is equal to 0 and this one corresponds to the state $1/\sqrt{2}$ N is equal to 1 plus N is equal to 0 all right.

So, I encourage and you to actually work out it in the similar way that we have done it for 2-level system and then you will get it. So, please do that. So, we have learnt about cooper pair box also known as charge qubit.

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As I said earlier the typical size of a cooper pair box is in micrometer range and this cooper pair box are fabricated using lithographic techniques. In fact, these same lithographic techniques are used for computer chip production. The typical material used for cooper pair box is Aluminum and you may know that the Aluminum has a transition temperature in the range of 1 Kelvin.

So, clearly this implies that cooper pair box has to be operated at very low temperature and in typical experiments for better or strong superconductivity scientists or experimentalists generally work at the temperature of 20 milli Kelvin which is far below than the transition temperature and T is the operating temperature of a cooper pair box and because at this temperature superconductivity is very good.

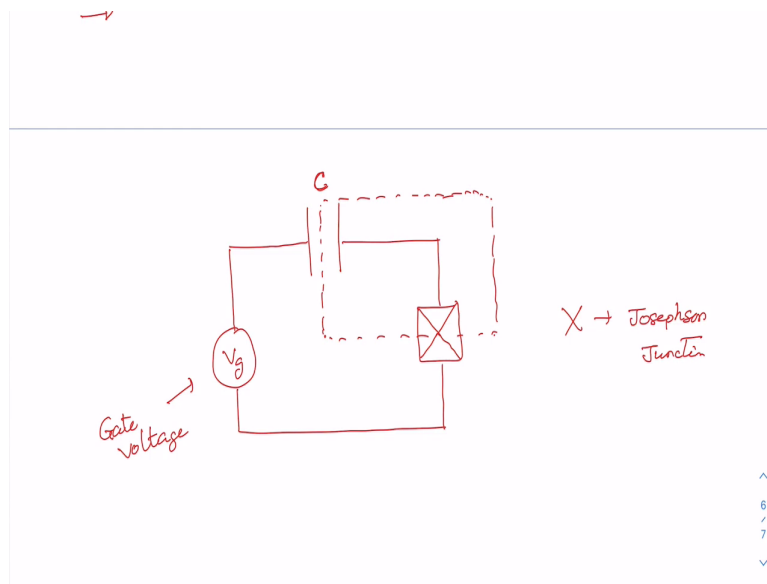
The typical charging energy is of the order of you remember E_C is equal to $q^2 / 2C$. So, this is typically in the range of 100 micro electron volt and if we can we can convert it to temperature as well because you may know that the one electron volt is equivalent to around 10 to the power 4 Kelvin. So, E_C in temperature it would be equivalent to around 1 Kelvin.

On the other hand, the Josephson energy E_J okay is in the range of 20 micro electron volt. So, as you can see here that E_C is 100 micro electron volt and E_J is 20 micro electron volt. So, therefore E_C is greater than E_J and that is needed because as you saw earlier in our analysis that Josephson part in the Hamiltonian we took it as a perturbation. Now interestingly you see the energy gap between the 2 energy levels at this degeneracy point N_G is equal to half is E_J .

So, that is going to be very important. So, this energy gap is E_J . Now the operating temperature is 20 milli Kelvin. So, the thermal energy corresponding to this operating temperature is in the range of if you put the values there this k_B is the Boltzmann constant and you will get it to be 2 micro electron volt. Now this thermal energy is obviously less than this Josephson energy that is this gap between these 2 energy levels.

So, the thermal energy cannot initiate transition between the energy level and that is good because then we can consider this system as a quantum system and thermal energy is not going to play any unwanted role here. Now we can very easily find out what kind of radiation is needed to couple a cooper pair box.

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So, what I mean to say is that what kind of electromagnetic radiation one has to irradiate to this Cooper pair box. So, obviously this radiation energy say $h \times \omega$ has to be equal to this gap that is E_J . So, you just have to solve this equation $h \times \omega$ is equal to E_J you know h is the reduced Planck constant and if you E_J is we see that it is 20 micro electron volt.

So, if you put down all the parameters here. So, ω would turn out to be around 2π into 5 gigahertz okay and just by the way 1 gigahertz is equal to 10^9 hertz. Now you may know that gigahertz frequency is nothing but microwaves in fact one gigahertz is corresponds to actually 30 centimeter you can convert this frequency into wavelength using this relation $\lambda = c / \nu$, ν is here 5 gigahertz.

c is the speed of light in become that is 3×10^8 meter per second if you put down the numbers there you will get that 1 gigahertz is equivalent to 30 centimeter. So, if we are to couple a Cooper box device or charge qubit device to radiation the radiation must be in microwave. So, let me write here if we are to couple if we are to couple a CPB a CPB or charge qubit.

CPB device to radiation has to be the radiation must be a microwave. Before I go further let me quickly give you some important information about Cooper pair box. So, first of all what is the circuit representation of a Cooper pair box okay, let me do it here. So, is Cooper pair box or a charge qubit can be depicted by this kind of a circuit I will explain.

So, you have this gate capacitance here and then you have this Josephson junction this big cross denotes Josephson junction this actually represents Josephson junction and then you have this gate voltage and then you connect it to the Josephson junction and the island is formed by the superconducting electrode let me show it by this dotted line here by the superconducting electrode between these this is the gate capacitor and junction capacitance.

So, this is how a cooper pair box is represented in a circuit. So, this is the gate voltage this is the gate voltage and this gate voltage controls the chemical potential of the island and chemical potential is as you know it is related to the number of cooper pairs. In cooper pair box actually the charge distribution charge contribution to the energy dominates over the magnetic flux as you will see later.

This is why cooper pair box is referred to as charge qubit. Let me repeat again in cooper pair box the charge contribution to the energy dominates over the flux magnetic flux and this is why cooper pair boxes refer to a charge qubit and we will discuss about other forms of qubit or 2 level system in circuit QED later on. Now let us discuss how we can obtain an appropriate microwave cavity.

Because we want strong interaction between the microwave and our 2-level systems for which we need to put the qubit or 2-level system inside the cavity. So, what I mean to say is that in order to enhance interaction between a cooper pair box or 2-level system or the ones that we encounter in circuit quantum electrodynamics, it is better to put the qubit or the 2-level system inside the cavity because that will ensure longer interaction time and also it will enable us to have higher intensity of the microwave radiation inside the cavity.

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• Fabry Perot cavity

partially reflecting mirror

perfectly reflecting mirror

λ

n

d

$$d = n \frac{\lambda}{2}$$

$$\Rightarrow 2d = n \lambda$$

$\omega_n = n \frac{\pi c}{d}$

 ; $\omega = \frac{2\pi c}{\lambda}$

$\omega = n \frac{c}{d}$

But before discussing microwave cavity, let me remind you about optical cavity to which most of you may be familiar for example I am actually referring to the so-called Fabry Perot cavity. You know that the Fabry Perot cavity consists of just 2 mirrors separated by some distance d and one of the mirrors is perfectly reflecting and the other mirror is partially reflecting say this mirror say this mirror is partially reflecting partially reflecting and this one is perfectly reflecting or completely reflecting.

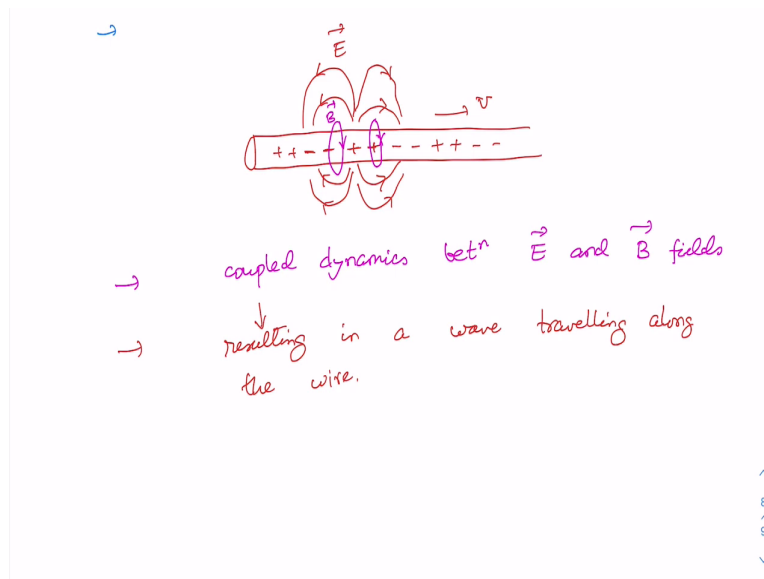
So, say if light is incident on this particular mirror the light of wavelengths λ it can enter into this cavity, then after entering into the cavity it would get reflected from this perfectly reflecting mirror and then it will again hit this partially reflecting mirror and it may actually come out. So, that is the idea here but not all wavelengths can enter inside into this Fabry Perot cavity.

Only those wavelength which satisfy this particular condition say the length of the cavity the distance between these 2 mirrors has to be an integral multiple of the half wavelength which you may know already that means that basically it has to satisfy this condition twice into the cavity length into n into λ and using this you can actually write about the corresponding frequency only certain wavelengths or frequencies are allowed you can easily work out that the angular frequency of this type can only be it would be able to enter into the cavity.

So, this condition or in terms of frequency you may know this well-known expression. So, whenever these conditions are satisfied the light of frequency ν or angular frequency ω would be able to enter into the into the cavity. Just to remind you that ω and wavelength

angular frequency and wavelength are related by this expression suppose the medium inside the cavity is air. So, N is equal to or refractive index is equal to a 1.

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So, by the way we require a bit of familiarity with Fabry Perot cavity while discussing optomechanics in module 3. Now for optical radiations in a typical Fabry Perot cavity the wavelength is of the order of micrometer and typical size of the microwave cavity one dimensional micro cavities of the order of centimeter. In typical experiments people generally shoot neutral atoms.

Let me explain that suppose you have a Fabry Perot cavity like this these 2 mirrors and then say an optical radiation is inside this, this is the envelope of the optical field and the wavelength is in the micrometer regime and people what people do so, this is in centimeter what people does is they throw or shoot neutral atoms into this field okay from any side in the direction neutral atoms to the optical cavity and study their interaction with this electromagnetic radiation.

So, this is our electromagnetic radiation. So, and this is a neutral atom. So, it is thrown into this radiation there or sometimes atoms are kept inside this cavity and then interaction between the electromagnetic radiation and the atom is studied. You may know that in most neutral atoms the usual transition wavelength suppose you consider a 2-level natural atoms, I am talking about a 2-level naturally occurring atom not artificial atom.

The usually the transition wavelength is on the order of optical wavelength or optical frequency say 10^{14} hertz. So, ω is on the order of 10^{14} hertz. Uh So, in that case clearly the Fabry Perot cavity are suitable resonator for starting strong atom light interactions. However, for our case we are interested in the wavelength of the order of centimeter because already I discussed about it.

So, microwave when we talk about microwave its wavelength is on the order of centimeter or frequency is in the microwave that is in the range of gigahertz or that is 10^9 hertz unlike the optical frequency which is 10^{14} or 10^{15} hertz. So, it will be definitely much more convenient to have a microwave cavity and but that microwave cavity should be only directly on a chip microwave cavity we want a microwave cavity on a chip.

Because as you know we already have our Cooper pair box that is embedded in a chip as I discussed earlier and we can actually get a microwave cavity by using the so-called transmission line. So, if we use transmission line, we will be able to get microwave cavity. But however actually to get a good idea it would be better for me to first discuss about wave guides for microwave radiation.

So, let me discuss briefly about waveguides and all of us know a typical waveguide is the so-called optical fiber but that is for optical radiation and in these optical fibers as you know that an optical fiber is basically made up of glass or silica and it some light can travel inside this optical fiber because of the so-called total internal reflection phenomena loosely speaking.

Light is guided inside it and optical fiber is a dielectric but we are going to deal with microwaves and microwaves as you know that they we cannot use glass fibers because microwave would simply would get absorbed inside the optical fiber it will not be able to propagate inside an optical fiber and we know that the microwave frequencies actually can pass through metals.

So, we have to consider metallic waveguide. So rather than dielectrics we have to deal with metallic waveguides if we want to transmit our signal or micro using microwave or if you are interested in transmitting microwave signals better use metallic waveguides. The simplest

metallic waveguide we may consider a simple wire a simple metallic wire. Let us understand how a wave can propagate inside such a wire.

So, we are just considering a single metallic wire let me briefly talk about the physics that means how a metallic wire can support a propagating wave. To understand that let us assume let us assume that we have a metallic wire like this and at some instant of time positive charges are created a collection of charges are basically created positive charges or negative charges like this are created inside the metallic wire by some mechanism.

So, as you can see because of this what is going to happen is that electric field lines would be generated. So, electric field lines like this would be generated these lines would go from positive charge to the negative charge okay positive to negative like this we'll get this field lines. So, we have here plus minus like this I hope you are getting the idea here these electric fields these electric fields would have a tendency to push the charges or charge distribution to spread out.

And this is because of the repulsion you know like charges repel these negative charges will repel each other, positive charges will repel and because of this pushing this charge distribution would spread out as these charges start moving it will result in current and which in turn will result in magnetic fields. Because of that it will result in magnetic field and just this is just it is not rigorously correct just I am giving you an idea. So, magnetic fields are going to be generated because of the generation of this current.

And as these magnetic fields build up this changing magnetic field will create again electric field which will act against this original motion. So, a kind of couple dynamics a kind of couple dynamics between electric and magnetic field okay electric and magnetic fields will occur and this is actually going to result in this is going to resulting in resulting in a wave travelling along the wire.

So, I think you already you actually most of you know how electromagnetic waves get generated right that is basically because of this interplay between the electric field and the magnetic field. So, this is the same thing here only here is that the wave that is getting generated is a microwave. So, it would be result and it will propagate along the wire.

But this kind of actually wave guides are not practical, the reason is we are not going to the mathematical analysis I will just tell you the reason why firstly as you can see from this diagram that this electric field extends actually this electric field extends too far. So, far field would be there, electric fields will extend too far and which may affect the other metal pieces that is actually in the chip and this is going to result in dissipation of energies and which is obviously completely avoidable and we do not want that to happen and this is one reason.

And secondly the other reason is the so-called dispersion what turns out that the speed of the wave is turns out to be dependent on the wavelength of the radiation or it is dependent on the frequency the same thing. So, therefore this is of course a complete completely detrimental as we as long as we are concerned about transmitting signals. So, therefore the question is what kind of waveguide would be suitable?

The solution is actually turns out to be very simple the solution is that instead of 2 one single wire let us take 2 wires parallel to each other and this is actually going to form the so-called transmission line and that is what now we are going to discuss.

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The image contains two hand-drawn diagrams and their corresponding notes:

- Top Diagram:** Shows two parallel cylindrical wires. The top wire is labeled with '+++' and the bottom with '---'. Electric field lines (E) are shown as arrows pointing from the positive to the negative wire, extending significantly outside the space between the wires.
- Notes for Top Diagram:**
 - very little stray fields outside
 - $v = \text{constant}$
- Bottom Diagram:** Shows a coaxial cable with an inner wire labeled '+++' and an outer shell labeled '---'. Electric field lines (E) are shown as arrows pointing from the inner wire to the outer shell, but they are entirely contained within the space between the two conductors.
- Notes for Bottom Diagram:**
 - No stray fields at all outside
 - $v = \text{constant}$

Now if we take 2 wires instead of one wire, the field suppose we create charge distribution like this the field will be strong in between these 2 wires and there would be very little field outside actually. So, very little field I should say very little stray field stray field outside. Moreover, it can be shown that the speed of the signal is not going to be dependent on frequency or wavelength it is going to be a constant.

If you are interested in analysis of this kind of transmission line you can look at any electrostatics or electromagnetism book where transmission line is discussed. Actually, even better if we can use coaxial cables. Say we have a one cylindrical wire inside which is surrounded by a large cylindrical wire outside like this. So, this is the wire inside small cylindrical wire and another one is the bigger one.

And in this case, there would be actually no field at all outside in the ideal case only thing is that the field here would be radially outward field should be radially outward for this positive charge distribution and no stray fields at all outside. Of course, in the ideal case and moreover that the speed of the signal is also going to be constant. In fact, this 3-dimensional version of this coaxial cable transmission line could be realized as a 2- dimensional version of a transmission line on a chip surface.

If we can imagine say a perpendicular a plane perpendicular to this wire suppose it is cutting the coaxial cable okay like this. So, it is going to cut these coaxial cables here, here and here. So, because of this I hope you are getting the idea because of this we are going to get one inner wire and 2 outer wires.

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Circuit representation

Goals

- wave propagation in the transmission line
- quantum mechanics
- transmission line resonator

C, L are linear in a^2

$a \rightarrow 0$
 \Rightarrow finer grid

And this can be actually realized on a chip surface on a chip surface rather than metal we are going to have superconductors. So, this is superconductor this and this is another superconductor these are the 2 outer conductors or outer wire outer conductors and this one you see here this one is the inner conductor this one is the inner conductor and of course we

are going to take superconductor in this case because there will be no loss as currents can flow without any resistance.

This blue color line shows actually the insulating substrate these are this blue one these blue portions here depicting insulating substrate okay. So, this is the 2d version of a coaxial cable transmission line or coaxial cable on a chip surface. However, we want to pick up a model which contains all the essential features of this system that I have described here by essential features, I mean the followings: first of all the model that we are going to build up must contain say charge distribution.

Because here you see we have a charge distribution like this say here the positive charge negative charge positive charge negative charge some kind of accumulation is there and then here you have negative charge positive charge. So, like this and in this other conductor here you have this negative positive negative like this. So, you will have field lines because of that you will have electric field lines on both sides here okay here you will have things like this.

So, first of all charge distribution then field lines okay and because of it of course we are we are going to have currents, so currents. So, the model that we are going to now build up should contains all these things and all these features are already there in our 2-wire transmission line setup that I discussed where we had these 2 wires 2 metallic wire parallel to each other right.

And here as I discussed earlier, we have (say) this positive charge negative charge accumulation if it happens. So, suppose you can create this kind of distribution within these 2 wires then we are going to have field lines between these 2 wires and because these charges are going to repel each other, like charges and because of all these things then current will flow in the wires.

And because of these currents changing currents magnetic field would be there because of the changing magnetic fields electric fields would be generated and that is how a wave will be able to propagate through the wire. Our goal now will be to have a circuit model out of it and this will be basically a discretized version of the problem but with very fine grid. So, now we are going to discuss the circuit representation of this model circuit representation.

And in fact, it will turn out later on you will see that this circuit representation or the circuit model is actually exact in the limit of sufficiently long wavelength or small frequency. Now having this kind of a model here what we will do next is let us divide this transmission line this transmission line into a lot of small cells like this. So, suppose I divide it into very small unit cell or cell size of say length a , each cell size has length a .

Then you can clearly see that we consider each cell as if it is comprising of a capacitor for example if you see this cell here okay. We can just model it by considering it as a capacitor and I think it is easy to understand right because we have this positive charge here and negative charge there. So, we can model it as a capacitor similarly for the next cell if you look at it and that is also a capacitor so you will get a lot of capacitors out of it.

Again, in order to incorporate so this is capacitance of say capacitance C and in order to incorporate the issue of current flowing and current generating magnetic fields and time dependent you know magnetic fields generates time dependent electric field and this all these things can be incorporated by introducing or incorporating it through some kind of inductance say L .

So, that is how we are going to model it. So, we will have this is going to be our circuit representation of the model. Now in principle we should have an inductor down here as also but it actually does not matter as we can choose an effective value of inductance here to get the same result. In fact, please note that this inductance has equal value and So, for simplification purpose we are not going to put any L here.

We will keep it like this. You should note that this capacitance and this inductance this inductance and the capacitance are linear in a , that is the cell size and that is easy to understand because if we increase the cell size then the capacitance also increases and accordingly the inductance also increases or if you decrease the cell size then capacitance decreases or the cell inductance also decreases.

And in fact, some of you may guess that in the end we are going to make a tends to 0 because thereby will get very finer grid very fine grid we can achieve doing that. Now this model already contains this circuit model contains all the essential features that we have discussed.

So, our goal would be to discuss or analyze wave propagation wave propagation wave propagation in the transmission line using this model.

Wave propagation in the transmission line and then do a quantum mechanical treatment just let me put quantum mechanics and then finally what we are going to do that we are going to say cut this transmission line at say 2 points say at 2 points here and here and because of that we will have new boundary conditions and the microwave will then reflect back and forth forming standing waves inside the resonator.

So, finally we are going to discuss transmission line transmission line resonator transmission line resonator. So, we will take up this analysis in the next class. In this lecture we saw how a cooper pair box can be modeled as a 2-level system and also, we got introduced to the concept of transmission line. In the next lecture we will continue with our treatment of transmission line.

And in fact, we will do a quantum mechanical treatment in the next lecture and we will see how a transmission line can be quantized. So, see you in the next lecture, thank you.