Quantum Technology and Quantum Phenomena in Macroscopic Systems
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Lecture – 16
Introduction and Basics of Superconductivity

Hello ,welcome to lecture 11 of this course and this is the first lecture of module 2. In this lecture I will briefly introduce circuit quantum electrodynamics, then I will remind you some elementary concepts of superconductivity. Then we will start discussing the so-called cooper pair box which is the first artificial quantum system that we are going to encounter in this course. So, let us begin.

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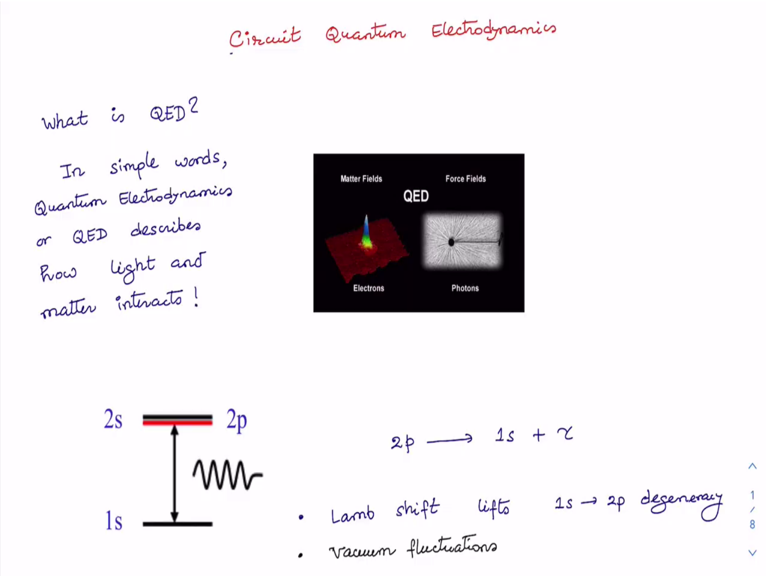

In this module we are going to study about circuit quantum electrodynamics one of the most important platforms for quantum technology. Before I say what is circuit QED, let us very briefly understand what is quantum electrodynamics. So, what is QED? In simple words QED basically describes how light and matter interacts at the quantum level. Here one studies how atoms get coupled to photons resulting in phenomena such as spontaneous decay or spontaneous emission as it is depicted.

In this schematic here as you can see that photons are getting emitted spontaneously when a transition is happening from 2s or 2p level to 1s level and this phenomena is attributed due to the vacuum fluctuation. Also as you can see that the degeneracy between the level 2s and 2p is now getting lifted and that is also happens that is known as Lamb shift and it happens due to vacuum fluctuation.

And when we trap the photons as discrete modes inside a cavity, it gives rise to a field of study called cavity quantum electrodynamics.

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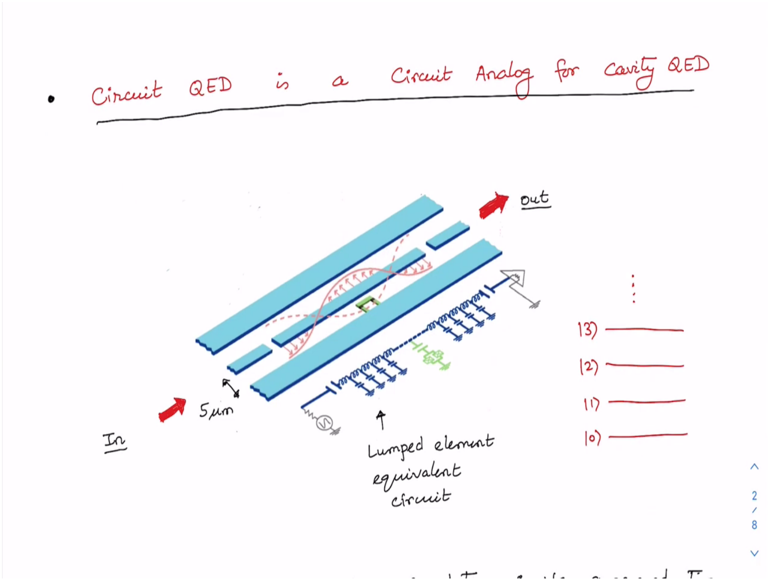

For example ,here you see we are putting an atom and atom is put inside a cavity and this is the domain of cavity quantum electrodynamics .On the other hand circuit QED is a circuit analog for cavity quantum electrodynamics. Circuit QED or circuit quantum electrodynamics is the study of electrical circuits that operate at microwave frequencies. These frequencies are the ones that is used in our cell phones.

And these electrical circuits are superconducting circuits, they are cool very close to absolute zero. So, that there is no resistance and electrical resistance or dissipation of energy is not there. The goal of circuit quantum electrodynamics is to make these electrical circuits exhibit quantum behaviour, just like atoms that has quantized energy levels. We know from quantum mechanics that atoms have discrete energy levels like this.

And when an atom make transition from energy one energy level to another, either they either emit or absorb photon. In circuit QED, electrical circuits having quantized energy levels are made to transit from one energy level to another and the change in frequency corresponds to microwave frequency. So, say ,when artificial atom is transiting from level 1 to level 0, it is going to emit frequencies and which are in the microwave regime that means there would be around 5 to 10 billion oscillations per second would be there.

In circuit QED, the circuit elements for example here this particular circuit element which we'll understand later in more clear terms, is a, is a kind of small antenna about a millimeter long that splits into 2 halves and these 2 halves are connected by a small circuit element called Josephson junction. So, these are artificial atoms. So, I am actually explaining it more here.

So, this particular section as you see here this one it is an artificial atom and it is made up of actually 2 superconductors generally made of aluminums and there is a gap between them and there is a insulating layer and it is made of aluminum oxide and this is 2 ways tunneling can happen. And tunneling happens in terms of cooper pairs because the electrons in superconductor always exist in pairs known as copper pairs.

And these copper pairs can tunnel from one island to the another island. Another thing that should be kept in always in mind is that generally the quality factor of the structures are on the order of 10 to the power 3 to 10 to the power 6. What does that mean is that that means the photons the microwave photons can travel up to 10 kilometer while in the resonator.

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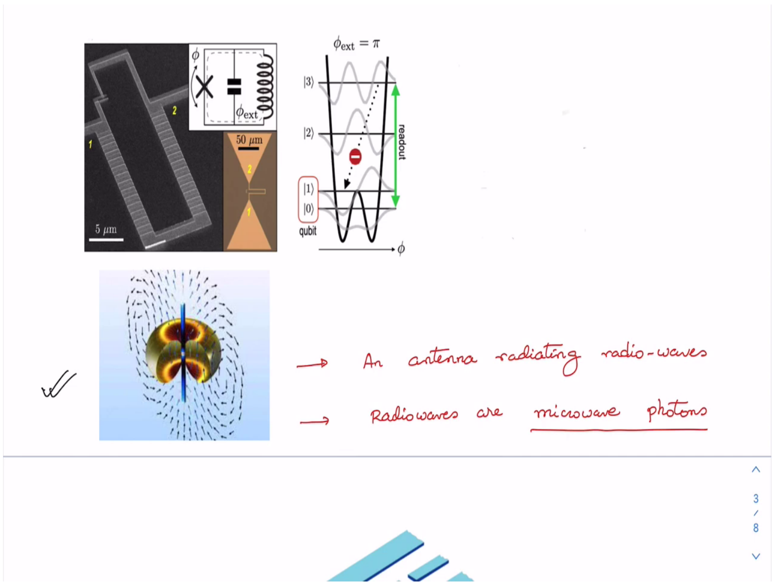

Now in a superconductor as I said, electrons travel in pairs called cooper pairs without any electrical resistance and the Josephson junction is a small barrier that facilitates tunneling, here this is the Josephson junction here that facilitates tunneling of electrons from one half of the antenna to another half due to quantum mechanics. In the ground state of these artificial atoms ,there is very little of this back and forth motion between from one island to the another island.

However in the excited state there is more and more back and forth motion .In this artificial atom, in this electrical circuit back and forth motion of the cooper pairs correspond to excitations and when we have an antenna you know that the charge is moving back and forth means that current is flowing in the antenna just as is shown in here in the schematic and this implies that it can radiate radio waves just like the antenna in our cell phone.

These radio waves are photons. They are actually microwave photons and they can travel to another part of the circuit and be absorbed by another artificial atom which is basically another Josephson junction.

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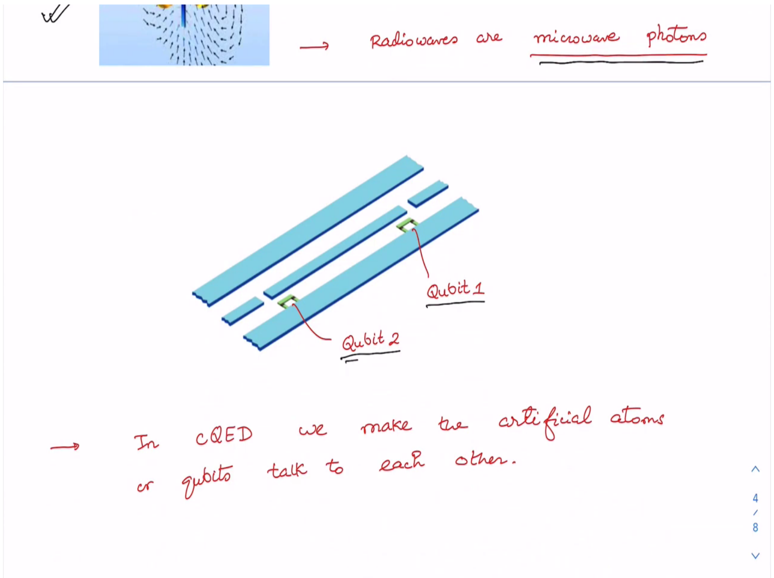

As it is shown here that this qubit 1 and qubit 2 there are 2 Josephson junctions and microwave photons can travel from one part to the another and in circuit QED ,you, this is I have just shown only 2 qubits, there will be numerous number of such qubits should be there and in circuit QED we get these artificial atoms talking to each other by exchanging microwave photons and so we can communicate quantum information from one place to another.

Moreover we can create states which are in a superposition of ground state and excited state at the same time using these electrical circuits or artificial atoms.

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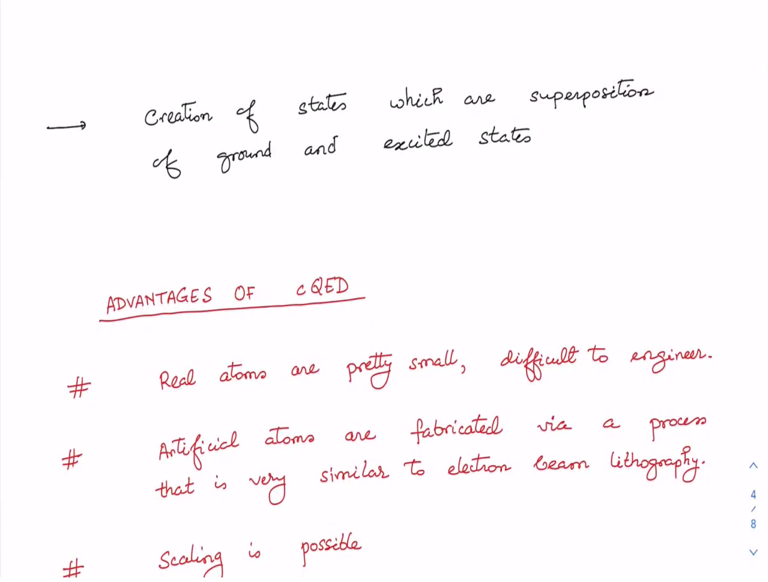

And we know that in a quantum computer a quantum bit can be either 0 or 1 0 and 1 which is the essence behind the power of quantum computer over classical ones and we can transfer superposition of 0 and 1 to another atom in the circuit or the artificial atom by use of this exchange via this emission and absorption of micro photons. And there are numerous advantages of circuit QED, for example you know that real atoms are pretty small they are hard to see and hard to hold on to.

If you hold them ,you perturb their properties , at the same time they have lot of great properties because of which they are using atomic clocks .They have very precise frequencies ,one can put them in superposition states for long times but unfortunately for technological applications they are very difficult to, difficult entities because they are not engineerable at all so, very difficult to engineer.

On the other hand artificial atoms are made by evaporating aluminum which is the superconductor, you know aluminum is a superconductor and these artificial atoms are made by evaporating aluminum onto a substrate by a process that is very similar to the so-called electron beam lithography. Very similar to the same process that is used to make the computer ships that maybe you are using in your laptop.

So, in principle, well it is not yet in practice ,we can imagine scaling up of this process and manufacturing structures with first say tens and then hundreds and thousands of these qubits by manufacturing methods that that are relatively straightforward. And the qubits are sort of entities, artificial atoms I am talking about they can be glued down to the substrate and they do not fall under gravity the way that natural atoms do.

We can engineer their properties as per our requirements and clearly they are more much more engineerable than ordinary atoms or ions. So, these are the major advantages of circuit QED.

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Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation

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We propose a realizable architecture using one-dimensional transmission line resonators to reach the strong-coupling limit of cavity quantum electrodynamics in superconducting electrical circuits. The vacuum Rabi frequency for the coupling of cavity photons to quantized excitations of an adjacent electrical circuit (qubit) can easily exceed the damping rates of both the cavity and qubit. This architecture is attractive both as a macroscopic analog of atomic physics experiments and for quantum computing and control, since it provides strong inhibition of spontaneous emission, potentially leading to greatly enhanced qubit lifetimes, allows high-fidelity quantum nondemolition measurements of the state of multiple qubits, and has a natural mechanism for entanglement of qubits separated by centimeter distances. In addition it would allow production of microwave photon states of fundamental importance for quantum communication.

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You can have a actually detailed discussion ,you can find in this particular paper about the theory which anyway we are going to build up in this module and this is a landmark paper. In the meantime, all of us already know that superconducting circuits are one of the most can promising candidate for quantum computers because of this recent IBM as well as Google computer in particular the Google computer all of you are aware off.

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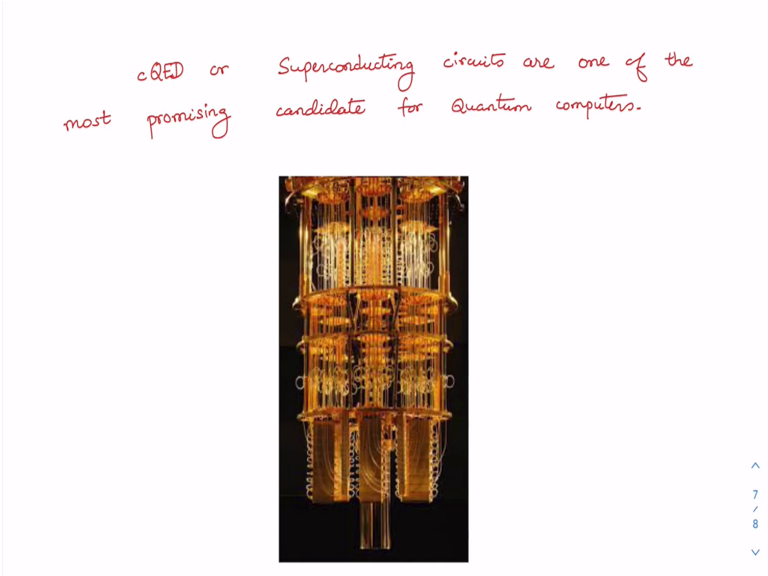

So, now let us have a very brief discussion on the on this particular topic and the role of superconducting circuits in this new kind of quantum computers. In 2019 scientists at Google claimed that they have achieved the so-called quantum supremacy .That is, they have a they have made a quantum machine which could perform a calculation that would be practically impossible for a classical machine.

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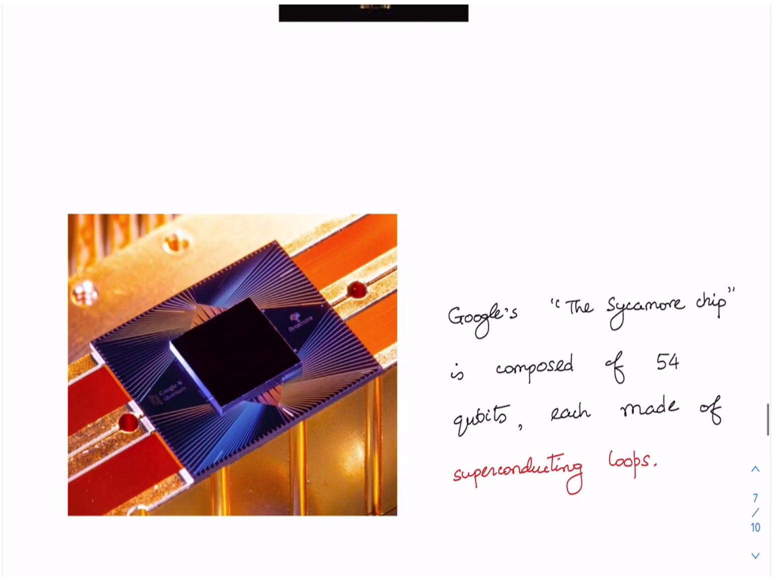

Google's algorithm runs on a computer chip composed of 54 qubits and each qubit made of superconducting loops. However there is a caution, by the way, that this is a tiny fraction of the 1 million qubits that could be needed for a general purpose machine, still it was considered to be and hyped as a great achievement and indeed it was a great achievement. Anyway you know that quantum computer works in a fundamentally very different way that from the classical machines a classical bit is either 0 or 1.

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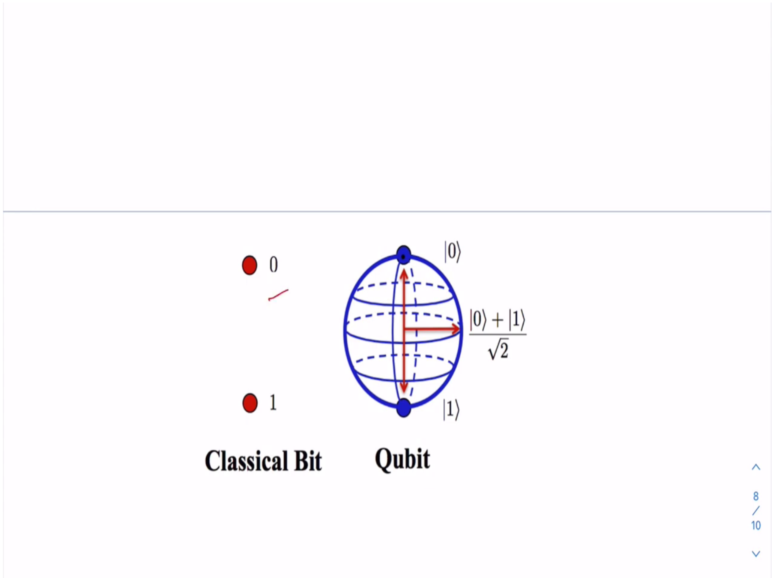

But a quantum bit or a qubit can exhibit or it can exist in multiple states at once at the same time. When qubits are intricately linked ,physicists can in theory or in principle exploit the interference between the wave like quantum states to perform calculations that might otherwise take millions of years.

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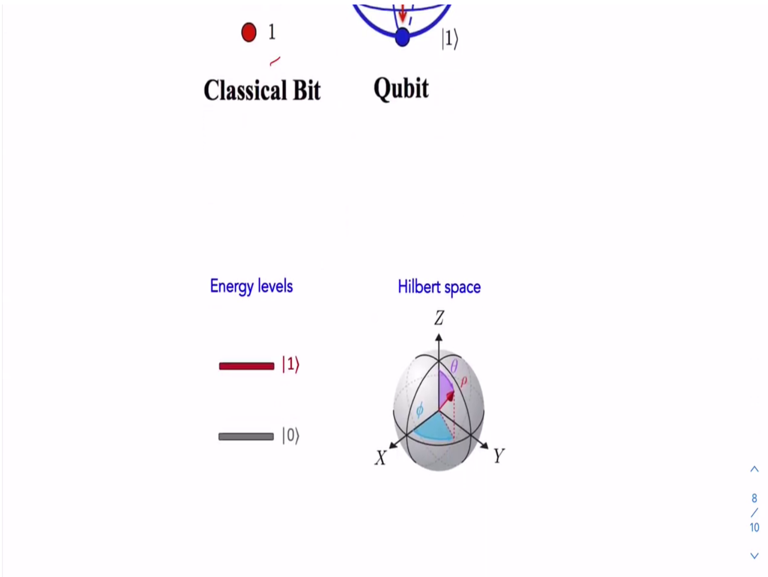

So, the crux of the matter is that ,realizing a qubit in real world and then appropriate architecture is of primary importance and circuit QED platform exactly provides that. Now, all of us know that a qubit consists of 2 energy levels denoted by ket 0 and ket 1. Ket 0 referring to the ground state while ket 1 denoting the excited state .We learned in the previous module that a qubit can be represented in the so-called Bloch sphere in the Hilbert space.

Now the question is how does a real qubit look like ?Because so far what we have discussed qubit in the abstract Hilbert space only, but how it look likes in real laboratory.

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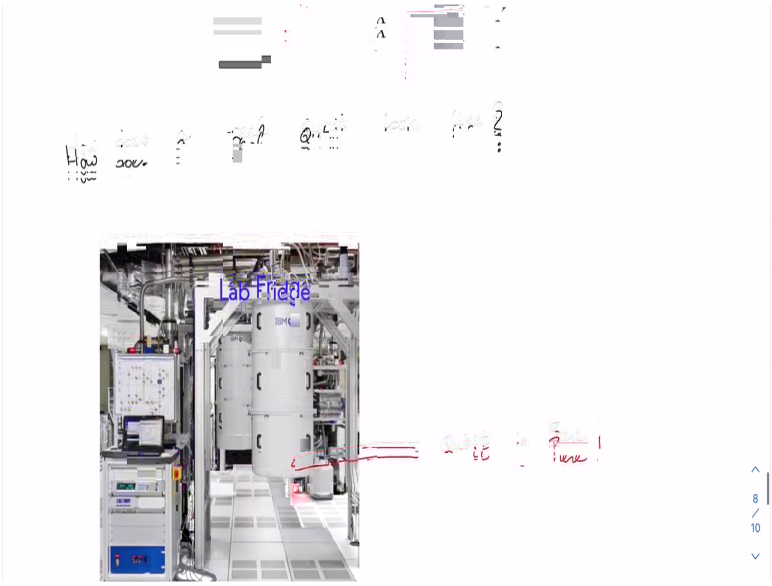

So, here is a picture of a real device made by the company IBM .This is actually a very low temperature freeze where the actual physical quantum bit resides at the bottom of the freeze ,at the very bottom of the freeze.

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And in a recent time these freeze actually look like something of this type and now the question is what is inside this freeze. If we zoom in and unveil the inside of it by removing the outer cane ,one can get a picture which appears to be of this type ,which is now, it must be familiar to you by now.

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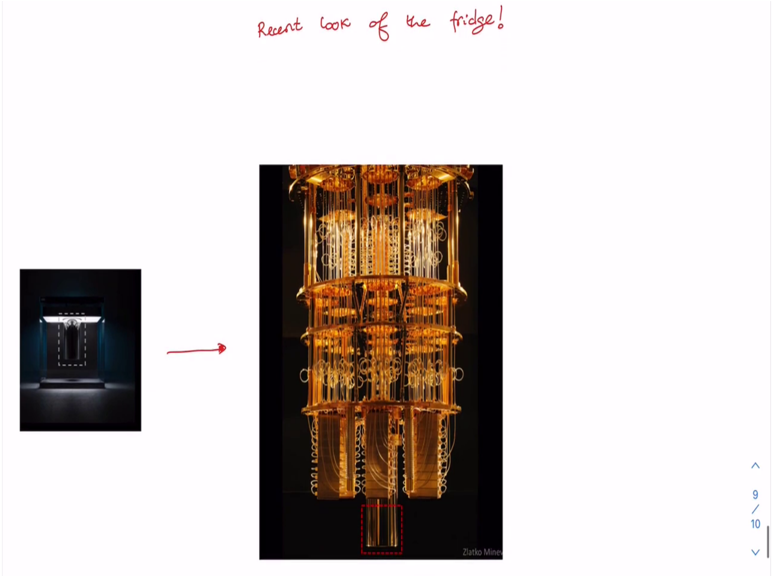

So, this is what it will look like if we remove the can .As you can see this is a actually it is a superconducting dilution freeze, as you can see that there are a lot of cables going in and out which are gold plated .However everything that is quantum is at the bottom here at the very bottom of the freeze and this operates near absolute zero ,actually, 15 this operates at the temperature around 15 milli kelvin which is minus 273 degrees centigrade or celsius ,okay ,this is such a low temperature.

Now the next question is what is inside this shield ,okay, what is inside this shield .Unveiling it further would actually reveal some superconducting Chips.

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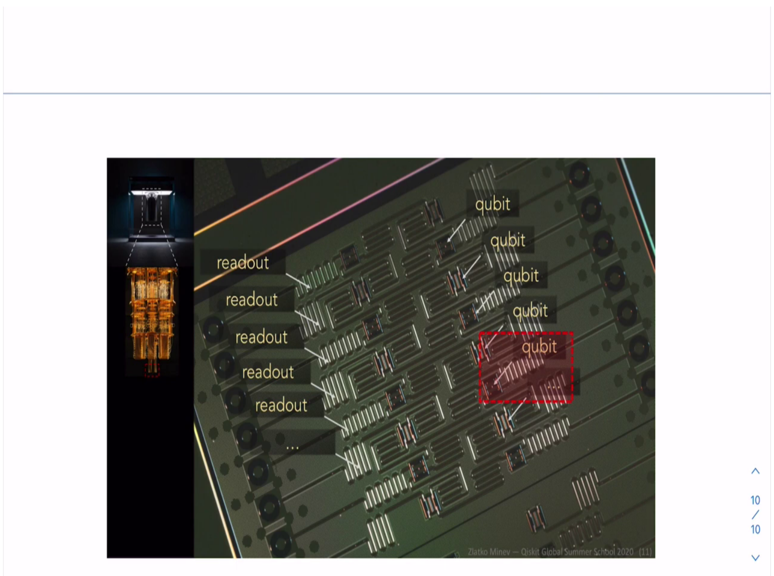

As it is shown in this picture here and what you see is a device having quite a number of qubits like this .These are a lot of qubits you can see here and these are housed in a substrate which can be, this substrate can be silicon or it can be sapphire or it can be any other some other dielectrics. And apart from that there are a number of readouts are there these readouts are again superconducting and they are superconducting coplanar waveguides and they indicate readout resonators.

In fact if we zoom in on a particular qubit like this if we zoom in, we shall encounter some other structures just like here.

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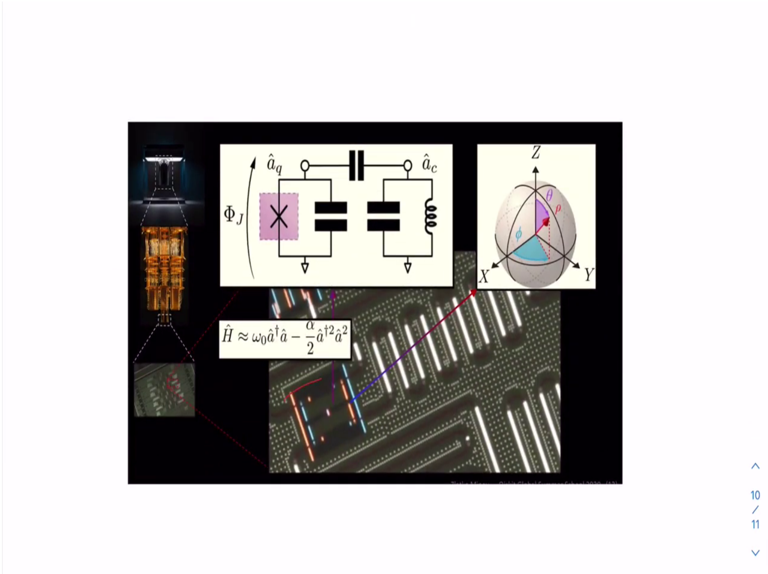

We'll encounter a structure of this type as you can see and you may recognize that it consists of 2 superconductors separated by an insulating layer. So, that is a Josephson junction and this is the one that supports the physical picture represented by this Bloch's sphere. At the end of the day, these are typically actually really these are microwave electromagnetic microwave oscillators and they could be represented by circuits as it is represented by this diagram here.

And in fact they are going to be quantum circuits and we will see later then later that we can write a Hamiltonian to represent the system. This is what we are going to work out later in the module. Now let me pause for a moment and ponder why we need the circuit to be superconducting in the first place. Well we know that when we have something on a chip we need to deal with currents and voltages.

And if we have a superconductor, we will not have to worry about losses and we can hope to study quantum mechanics in such systems. We would have coherent quantum evolution. By the way we do not need the knowledge of superconductivity at all in this course however having some idea still may be very satisfying.

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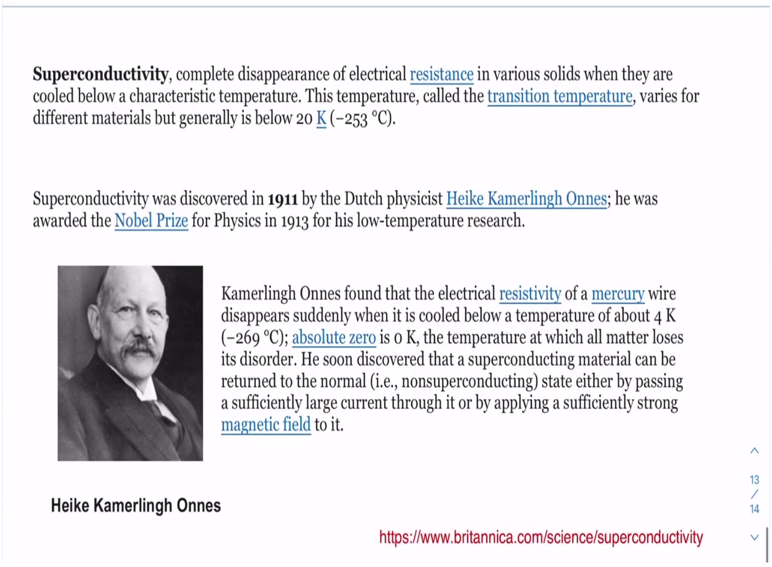

So, superconductivity basically implies complete disappearance of electric resistance in various solids when they are cooled below a critical temperature known as the transition temperature and this transition temperature varies from material to material. And superconductivity was discovered by Kamerlingh Onnes he was a Dutch physicist, he discovered superconductivity in the year 1911.

And he got the Nobel Prize for his work in 1913 for his low temperature research. What Kamerlingh once found was that the electric resistivity of a mercury wire disappears suddenly when it is cooled below temperature of about 4 kelvin and you know that absolute zero is at 0 kelvin temperature and he also discovered that superconducting material can be returned to normal if you apply a sufficiently strong magnetic field or a very sufficiently large current through the material.

So, you can read about basics of superconductivity in Wikipedia or I have taken this from Britannica dot com. Actually in Onnes', time scientists already learned to liquify different gases and they actually they actually use these coal gases to cool other things and Onnes asks ,Onnes asked this question that what happens to the resistance of a metallic wire if temperature is reduced .This is the fundamental question he asked.

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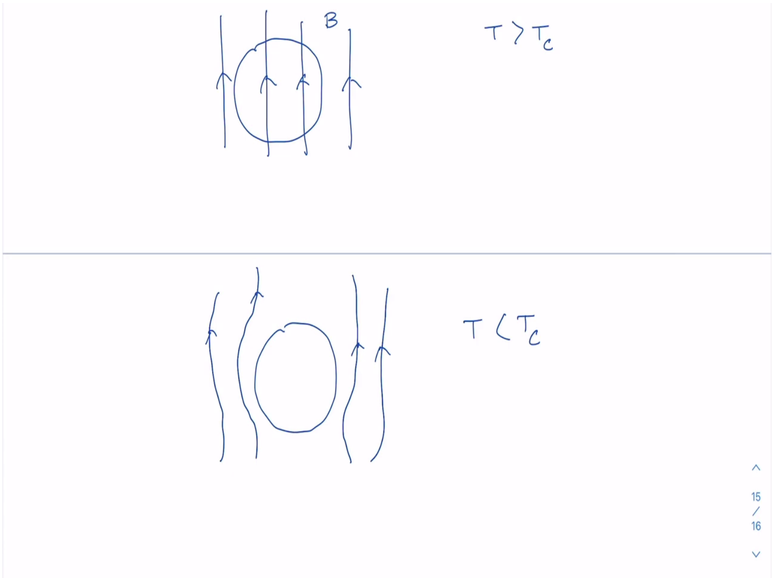

So, if I just say, plot resistance versus temperature, resistance in the vertical axis temperature in the horizontal axis. So, if say, temperature is getting reduced then if I start from a high temperature and go to a lower temperature then the resistance decreases. So, let me just use a different colour here. So, say the resistance decreases and this actually happens due to the reduction in the lattice vibration as the metal is cooled and there is significant decrease in the electron phonon interactions.

In some materials or metals this resistance however saturates actually, this resistance saturates to a finite value and if it has impurities in it because of which scattering of electrons occur and these occurrences are actually temperature independent, however what can Kamerlingh Onnes observed was that in his experiment that the resistance is getting absolutely zero at a okay.

So, resistance is getting absolutely zero at a particular temperature known as the transition temperature. And later on ,it was found that this is actually a phase transition and this zero resistance thing is maintained even below T less than T c and this is called and then we call the metal as superconductor. So, below here we have R is equal to zero, alright. So, this is actually what Kamerlingh Onnes observed in his experiments.

And one of the most intriguing or important or interesting feature of superconductor is that they can completely expel magnetic field and I know you know it this is called the so-called Meissner effect or I do not know how it is pronounced maybe Meissner effect. So, what happens here is that suppose you place a normal metal like this in a magnetic field ,if it is a normal metal that means your temperature is greater than the critical temperature or the transition temperature.

Then the magnetic lines of force is going to penetrate, the penetrator metal and magnetic field simply penetrates the metal above T c transition temperature. Now the question is what happens if temperature is lowered below T c then what was observed experimentally was that the magnetic these lines of forces are completely getting expelled from the metal when it becomes a superconductor.

That means so this is what happens if we go below T less than T c ,okay, pardon my diagram. So, I hope you are getting the idea. So, this happens owing to the fact that electric field is associated with magnetic field, suppose is changing magnetic field and which gives rise to some kind of Eddy currents in the sample the surface and because it is a superconductor and there is zero resistance this currents will be very strong and they will be always in a direction such that they will oppose any seeds in magnetic field or that give rise to the magnetic field.

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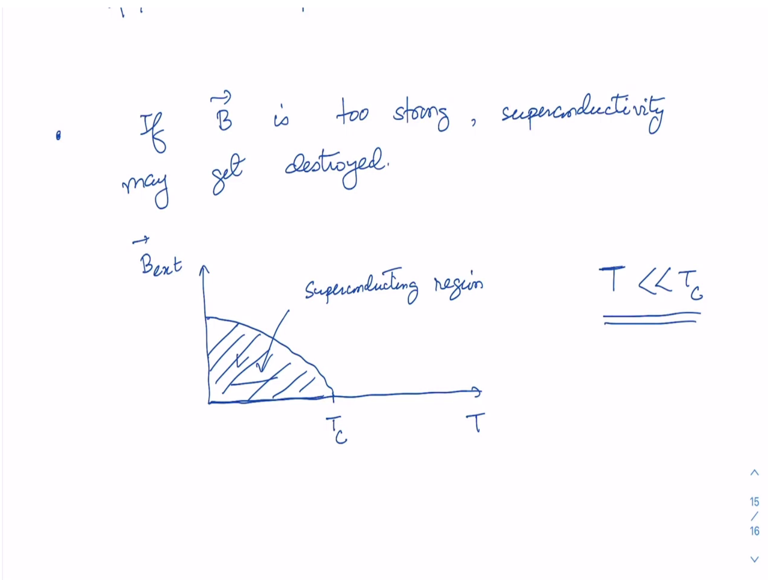

In other words, the superconductor acts like a perfect diamagnetic material and actually either you can have a metal which is already cooled below T less than T c and then you switch on the magnetic field or say, you already have switched on the magnetic field and you have this metal but then now if you just cool it. If this is suppose this is a normal metal now ,it is T greater than T c.

But say, you now ,you reduce the you know the temperature of this metal ,then it becomes a superconductor and when it becomes a superconductor this magnetic lines of force is now going to get expelled. So, because, that means the final state of the metal is independent of the path actually, whatever whether you switch on the magnetic field first or you cool the metal first, okay, oh vice versa whichever the path it does not matter.

So, this behaviour is going to be exhibited and that is why it is called a real phase. However, I think please note that the superconductor can withstand magnetic field up to a certain limit only that means if the strength of the magnetic field is too strong. Let me write here if B is too strong then superconductivity, may actually conductivity, may get destroyed. So, the metal will no longer be a superconductor that means the magnetic field will be able to penetrate the metal and it will no longer be a superconductor.

So, that means that there is a critical magnetic field beyond which superconductivity cannot be obtained which I can actually show it in a diagram. So, as you can easily understand that suppose I plot temperature versus this external magnetic field here ,okay, critical magnetic field. So, at T is equal to T c superconductivity in the material would be weak. So, it would be easy to destroy it.

So, but if we go far below T c right in this region here; the superconductivity will be very strong and you will require a very, very strong magnetic field to destroy it. So, therefore the plot would look like this. So, you will need a strong magnetic field to destroy superconductivity. So, in this particular region here the it is actually called the superconducting region. So, so what you need, more stronger magnetic field is required if you this is what is your superconducting region.

So, more and very strong magnetic field is needed if you want to destroy superconductivity and far below this temperature far below the critical or the transition temperature and the quality of the superconductor is better at very low temperature. So, but these are the general stuff we know about superconductivity but for this particular course we just need to a little bit of knowledge of the BCS theory actually not the full theory we just need to know some of the results.

So, let me now talk about BCS theory who is gives the game the first explanation of a rigorous explanation of actually what goes on a superconductor.

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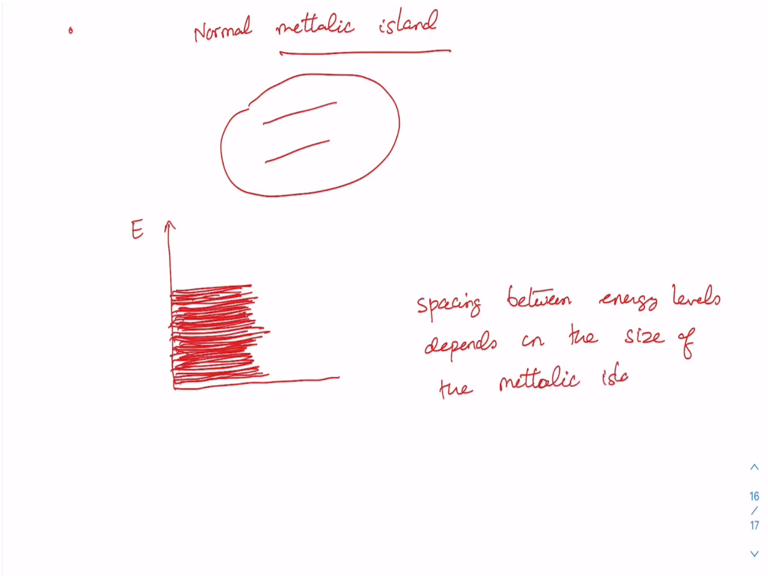

So, now I will briefly discuss the so-called BCS theory and as you know that the superconductivity was discovered experimentally in the year 1911. So, 1911 experiments but theory was there but the rigorous quantum mechanical and correct theory was given in the year 1957 only and this theory is called this BCS theory and it was given by 3 people ,3 scientists there Bardeen, Cooper and Schrieffer, okay, these 3 gentlemen.

And they got the Nobel Prize for this their work in the year 1972 they got the physics Nobel Prize. I will just tell you the key observations of this BCS theory. Firstly the cooper actually realized that even though the electrons have coulomb repulsion there is a slight tendency towards attraction between 2 electrons due to the presence of phonons and you know that these phonons are basically quanta of vibrations.

And because of phonon interaction 2 electrons are glued to each other 2 electrons, this is key observation, 2 electrons are glued to each other due to phonon interactions phonon interactions primarily and thereby they form a pair and this pair is called cooper pair and because you know this actually now these electrons travels inside the superconductor in the form of cooper pair and as it is a collection of 2 electrons the charge of a cooper pair is twice that of the charge elementary electron charge.

So, they form a pair like this. So, it charges twice a minus electron source and when a charge is transmitted from one superconductor to another superconductor ,it get transmission in the, transmitted in the form of a cooper pair. And obviously some energy is required to break up these cooper pairs and this actually going to result in the energy gap in a superconductor. Let us understand it a little bit .For example if we considered a normal metal.

So, let us consider a normal metal or metallic island okay some size is of, this size this is a metallic island. And if we look at the energy spectrum of this metal then suppose this is energy. So, if we consider a fixed number of particles in the metal we are always going to get a ground state and due to numerous interactions the energy spacing between energy levels in the metal would be really small and therefore what we are going to get is we are going to get continuum of energy.

So, we will get actually continuing continuum of energy. So, it would look like though energy levels are actually discrete but because of. So, many interactions so it will look like as if we are having continuum of energy.And the size you know, the this basically the spacing between energy levels are dependent on the size of the metallic island. So, this is important .Spacing between energy levels between energy levels depends, depends on the size of the metal, size of the metallic island.

So, metallic island in fact as you go on increasing the size of the metal quite clearly the interactions are also going to increase. So, therefore this gap between the ground state and the excited state is also going to decrease. So, it will look like a continuous continuum.

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On the other hand ,for so this happens for temperature greater than the critical temperature or the transition temperature where the superconductor would be a metal. But if we now consider a superconductor, the things are quite different there because again ,of course now let me talk about superconductor here or superconductive islands, superconductors. So, see this is what you have and again you will ,you are going to get a ground state.

You are going to create a ground state of energy for a fixed number of electrons . However due to the fact that you need energy to break a cooper pair, we will get a finite energy gap. So, this is the ground state. So, this is what we have is our ground state for a fixed number of electrons and then because we need energy to break a cooper pair, we, the first excited state will be ,will have a gap and that gap is conventionally denoted by this 2 delta S 2 delta.

And interestingly this gap ,this gap does not depend on the size of ,the size of the island. This is extremely important ,this is, and then beyond this gap there is closely spaced energy levels and they for most purposes to a good approximation they basically are look like a continuum. This 2 delta, this if you convert it to temperature this is actually in the Kelvin temperature, in the order of Kelvin temperature.

One can be in this ground state provided one cools down to Kelvin temperature and then thermal excitation will not be able to take us up to this excited level. So, we will remain in the ground state only if the we cool to a far low temperature. Now the question is how this gap ,okay, how this gap depends on temperature. Now quite clearly the full gap would be, okay let me plot, say ,we plot temperature versus this gap.

Quite clearly this gap would be 2 delta at absolute zero temperature. So, but as we approach this critical temperature or transition temperature this gap will decrease. So, therefore what we get, we get this kind of dependence as per the BCS theory. So, this is obviously intuitively also it should be correct because the critical temperature be aborted, we are going to heat the metallic state.

And this critical temperature and this energy gap 2 delta are the properties of the material and one of the significant success of BCS theory is to predict the relation between this energy gap 2 delta and this critical or transition temperature or as per BCS theory this energy gap at absolute zero is equal to 7 by 2 or 3.5 K B T c this is the transition or critical temperature .This is a very important result and this is this can be experimentally verified and it was verified and because of that only they got the Nobel prize by the way.

And typically T c this transition temperature is in the range of Kelvin or below Kelvin but there are some other kind of superconductor nowadays available they are critical temperature may be on the order of 100 Kelvin and these are known as so-called high T c or high super conducting materials.

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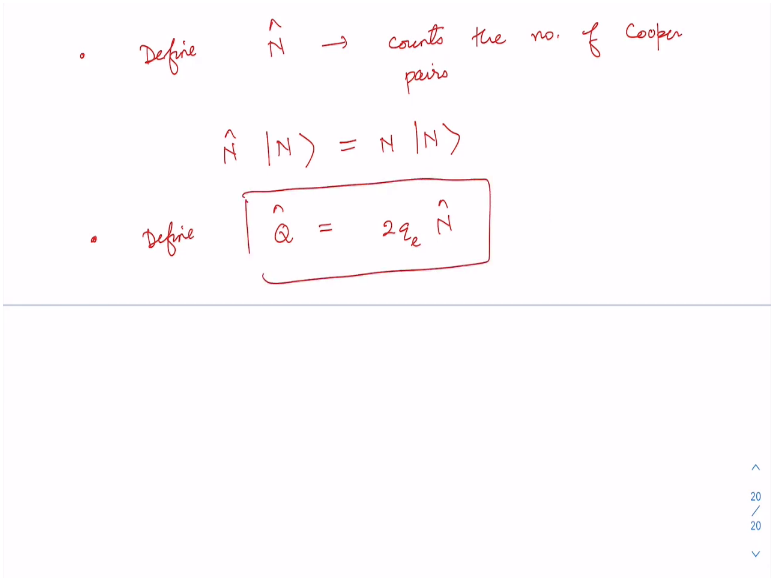

Now you see as we are in the, at low temperature ,as we are in the low temperature, we'll always be in the ground state and the trouble is that is that it is only a single energy level. So, it is not a 2 level system at all. So, it is of no use for us. So, we need to get a 2 level system and we can get uh we can ,we can actually get to that by putting another super conducting,so we have this super conducting island here.

And if we put another superconducting island nearby this one our original one then cooper pair may tunnel from one superconductor to another superconductor giving us a new degree of freedom and this actually brings us to the idea of cooper pair box this gives us the cooper pair box which is of extreme importance for us and we are going to discuss it now. So, cooper pair box or in short notation will say CPB is basically 2 superconducting island as I said.

Two superconducting islands separated by some finite distance .Generally, this size of a copper pair box is on the order of say, one micrometer and this gap between 2 superconductor is typically in the range of one nanometer or so. Say these 2 superconductors ,initially these 2 superconductors are neutral but when they are brought close to one another then tunneling can happen .

Say cooper pair so, this cooper pair can tunnel from this upper island to the lower island if it does so, what happens because 2 electrons are now getting transferred to the lower island. So, upper island would have now excess of 2 positive charges and the lower island will have excess of 2 negative charges but because these are superconductors ,the charges are going to be distributed smoothly.

So, distribution would be something like this,so near the surface. So, it would be say, positively this is positively charged and this is negatively charged. So, overall we know that the charge is quantized. So, say Q is equal to twice that of the electronic charge into the number of cooper pairs, here n is the number of copper pairs that have say, tunnel from the lower island to the lower island from the upper island and we can also assume immediately it should strike you that may be you can model this whole thing by this simple picture here.

So, you have a charge minus Q minus Q because Q is equal to twice Q e right Q e is the this is the electron charge ,this is electron charge and electron charge is negative. So, that is why I am writing here minus Q in the upper island and plus Q in the in the lower island. So, this whole thing if you look you can model it as a kind of a capacitor. This is I can just say that this is kind of look like a capacitor and having capacitance C.

And this capacitance actually contains all the details about the distribution charge and the geometry of these 2 superconducting islands and the electrostatic energy or the energy that is stored in this capacitor is equal to Q square by twice C right. So, we can actually turn this into a quantum mechanical problem, then the first question is what is the Hamiltonian H?

So, what is H here? So, first of all we have to write down the Hamiltonian. Now to write it naturally ,we have to select a basis so we can obviously select ket N as our basis where N is,so, let us take N as basis where n refers to the number of cooper pairs number, of cooper pairs okay, and in fact this ket N refers to, it actually refers to many particle many particle ground state ,many particle ground state as you know these 2 superconductors when they are cooled they are in the ground state.

So, N refers to many ,ket N refers to many particle ground state and we can now define a number operator .We can define a number operator N cap that this basically counts the number of cooper pairs, number of cooper pairs in the cooper pair box and it is defined such that when this operates on ket N then you get this equation and this is familiar to you. And also let us define the, another operator called the charge operator Q cap. And this is obviously that would be twice of q e n N cap, right.

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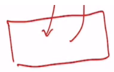

$$\hat{H}_{tunnel} = -\frac{E_J}{2} \sum_{N=-\infty}^{\infty} \left\{ |N+1\rangle\langle N| + |N\rangle\langle N+1| \right\}$$

$E_J \equiv$ Josephson energy

So, this is the charge operator .This is our charge operator and in terms of the charge operator we have one part of the Hamiltonian of the system that is H charge is equal to Q square by twice c and basically this refers to the charging energy, charging energy of the cooper pair box ,of this cooper pair box. So, this is only one part of the Hamiltonian the other part is related to the tunneling or exchange of cooper pair from ,okay, from one island to the another island.

So, as you have seen the cooper pair can tunnel from this island upper to the lower or lower to the upper and there is no limit to the number actually, any number of cooper pair can tunnel from one island to the other islands. So, therefore the other part of the Hamiltonian has to deal with the tunneling or exchange of copper pair from one island to the another island and this is actually very easily we can write it.

So, let me explain. So, suppose we are having n number of cooper ,we start from n cooper pair in the lower island say, and then cooper pair is now say getting tunnel from the upper island to the lower island then we are going from N to, N state to n + 1 state and of course we have to sum over all the possible N and we can do that we have to sum up all possible.

So, we can take N is equal to minus infinity to plus infinity. It may look odd that we are summing it up from minus infinity to plus infinity but that is not the at all a problem because ultimately this total number of cooper pairs is going to be taken care of by the charging energy and also you should know that this Hamiltonian has to be Hermitian. So, the reverse process has also to be allowed.

So, what may happen is that rather than tunneling from the upper to the lower ,tunneling may happen from the lower maybe you start at N + 1 state and then you will have the in that case the cooper pair is now getting tunnel from the lower island to the upper island. And this Hamiltonian is done ,Hermitian this is still not Hamiltonian because you see it it is dimensionless.

So, it has to has the dimension of energy and generally this conventionally this energy is written as minus E J by 2 where E J is called a Josephson energy ,this is called Josephson energy and this depends on the geometry and the material properties of the cooper pair box. This actually you can consider as the kind of amplitude of the cooper pair tunneling or the coupling strength.

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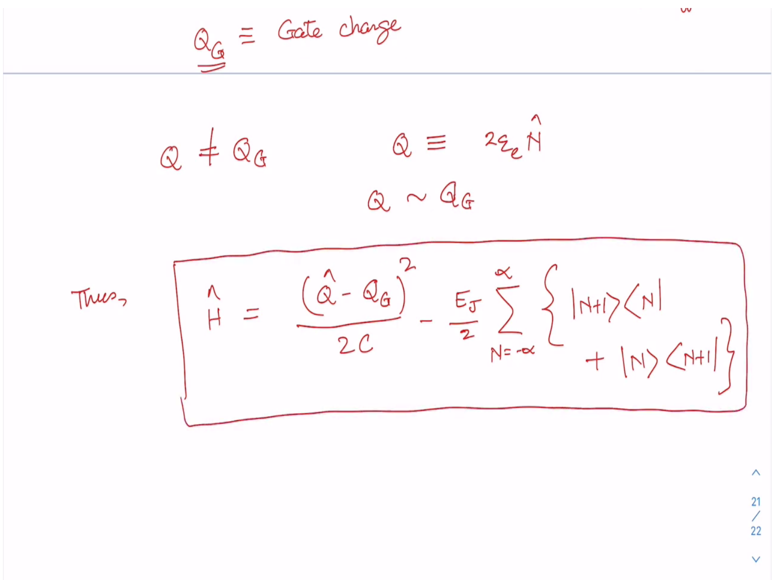

So, another so now then we what we obtained is that the total Hamiltonian, therefore H total has 2 parts one is the charging part charging part of the Hamiltonian and then we have this tunneling part H tunnel very simple. However you should know that in real devices or real system we would like to manipulate this cooper pair box or tune the energies. So, to do that let us say we have this cooper pair box and a cooper pair is getting tunnel from the upper to the lower.

So, we have this situation here and we want to manipulate it. So, let us apply an electric field external electric field let us apply. So, to do that generally what is some metallic electrodes or pieces are put here. So, that we have this we can have an external electric field we can apply from outside. So, this is E external and how to model it .Now you know that we have initially a kind of a let me just we have a dipole kind of a situation here right when the cooper pair got tunnelled.

So, we when we put it in this dipole in the external field so, the energy due to the dipole. So, dipole energy we can write it as minus p dot E external and again this dipole you know that that is actually proportional to the charge so this is Q. So, I can write it at Q external. So, therefore this external energy is directly proportional to the charge Q. Now the question is that how we can incorporate this into our model.

Well the overall energy as you see is, we can write it as Q - Q G square by twice C and then we, if we just open it up then we will get Q square by 2C which is our original charging energy in the Hamiltonian then we have a term like say C G by C and then this Q and then this is Q G square by twice C ,anyway this is a constant term then this is kind of energy offset we can neglect it ,we can neglect it and you we have this required term we needed a term which would be proportional to the charge as from this and this external part.

So, this external thing I think this is Q right. So, this Q G is known as gate charge, Q G is referred to as gate charge and it is called gate charge and this is associated with the external field in other words, we can model the take the external electric field into account by incorporating this term in our Hamiltonian that is Q G and Q G is actually a preferred charge of the system because the system always would like to have low energy.

And by the way in general it is not possible to have Q is equal to Q G it is not possible in general because of the fact that this total ,this charge Q that is belonging to the cooper pair is cooper pair box is quantized as you know and it is a it is an operator and but we can almost have Q is equal to Q G is a possibility and therefore finally if we have this tuning is also taken into account the total Hamiltonian therefore we can write it as H is equal to Q minus Q G square by 2 divided by 2c ,alright, and then I have minus E J by 2 that is the tunneling part of the Hamiltonian.

So, here and goes from say minus infinity to plus infinity and then I have this tunneling term and to say N + 1 and then I have other terms say N + 1 to N okay. So, this is what I have finally, we have finally this Hamiltonian for the cooper pair box. Now our next job is to find out a way how to solve it and not only that how I can actually convert this whole thing into a 2 level system. So, let us do that.

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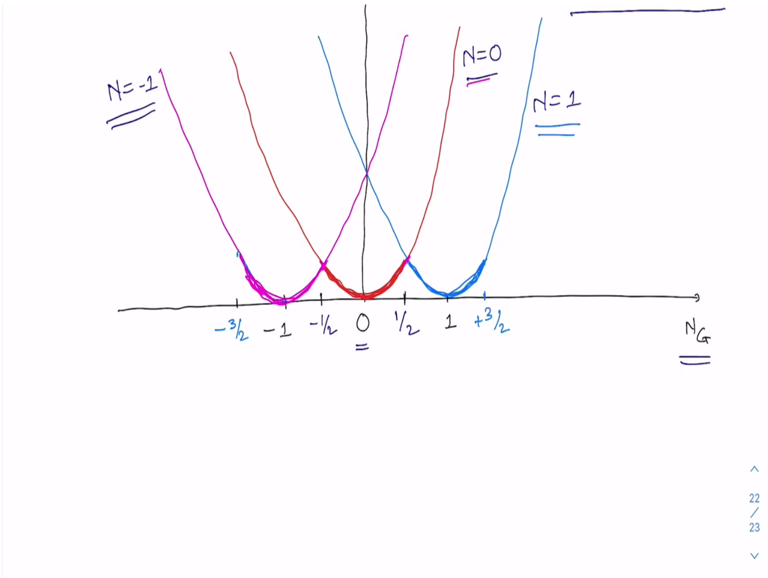

To solve the Hamiltonian ,as a first step let us neglect the second term in the Hamiltonian which is which refers to the tunneling part and we are going to consider it as a very very small perturbation. So, in the first step at first we will consider the charging energy part of the Hamiltonian only. So, in step one we will just consider this term the charging Hamiltonian charging part of the Hamiltonian which is Q minus Q G square divided by twice C.

These actually we can write in a more convenient form because we know that Q is equal to twice Q G twice the elementary charge into the cooper pair okay, and if we define Q G is equal to twice Q in the similar line is N G ,you recall that this Q G is the gate charge and it we can write it in terms of gate voltage and a gate capacitance like this and this gate voltage is related to the external electric field by which this cooper pair can be cooper pair box can be tuned and that is how we can say this N G is related to the gate voltage and therefore N G this parameter is a controllable parameter.

So, in terms of this definition we can write this Hamiltonian as 4q e square divided by twice C N – N G square alright. And by convention this constant this is written as 4E C where E c is the charging energy constant. So, now another thing you can notice is that for every n there is a finite value of the charging term okay. So, this charging operator is diagonal ,this is diagonal in the N gate basis this basis this N basis this is simply diagonal.

Now what we are going to do ,we are going to plot this charging energy the charging energy is 4E C N - N G as a function of this controllable parameter capital N G and we will keep this cooper pair as a fixed quantity ,this is fixed but this N G is a controllable or tuning parameter. So, if we do that we are going to get a plot like this. Here this red one refers to the case where n is equal to zero and as you can see from this expression here then this is a parabola.

And this parabola would be centered around N G is equal to 0 N G is equal to 0 and if we now take N is equal to 1 that is the parabola referring to the this blue color and this would be centered around N G is equal to + 1. And also you see that this parabola this blue parabola is going to intersect the parabola n is equal to 0 at N G is equal to plus half and similarly this one this pink one refers to the parabola for n is equal to - 1 and again this is going to intersect the parabola n is equal to 0 at minus half, alright.

In fact N is equal to minus up physically means that there are more number of cooper pairs more number of cooper pairs in the upper island. This way in fact we can go on and on for various values of a capital N and we will get a, get an array of parabolas. Now if the tunneling term is 0 the ground state energy as a function of N G simply refers to the lower branch for example if say, N G lies between say, N G lies between minus half and plus half then the ground state preferred start state would be capital N is equal to 0 and this ground state would correspond to this lower branch.

On the other hand ,say, N G is between let me pick up another colour here. If N G is lies between minus half and say plus half say this would be say plus 3 by 2 and then the ground state would correspond to this lower branch and the preferred charge state would be N is equal to – 1. Similarly if N G lies between say minus 3 by 2 to minus half then this preferred you know energy state okay this the ground state will correspond to this lower branch . I hope you get the idea here.

And now you may notice that that this is the clear case of energy crossing here in the absence of tunneling. So, this energy levels for example n is equal to 0 charge state N is equal to 0 and charged at N is equal to 1 is you know intersecting here. Similarly n is equal to 0 and N is equal to this 1 N is equal to 0 charge state and N is equal to minus charge state is intersecting or that means their energy levels are actually crossing.

Now this kind of energy level crossing already we have studied in the context of 2 level system earlier as you can recall in module one. Now the next thing that we are going to do is, we are going to turn on the perturbation then obviously we will see that these energy levels are no longer going to cross and we are going to get the situation of avoided crossing this we are going to discuss in the next class.

So, let me stop for today, hopefully in this lecture you had a brief idea about circuit quantum electrodynamics which we are anyway going to do in great details in the whole module and we learned about some elementary concept of superconductivity and also we saw how to write down the Hamiltonian for a cooper pair box. In the next lecture we will see how this cooper pair box could be turned into a two-level system.

And then it would be followed by our discussion on transmission line concepts and that is going to be extremely important for circuit quantum electrodynamics. So, see you in the next class, thank you.