Quantum Technology and Quantum Phenomena in Macroscopic Systems
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Lecture – 14
Quantum States of Radiation Fields-II Squeezed States

Hello, welcome to lecture 10 of this Course. This is the final lecture in module one. In the last lecture we have learned about coherent states of harmonic oscillator. In this lecture we are going to learn and discuss about another important quantum states of harmonic oscillator called squeezed states. These states have very important applications particularly in precision measurements and in particular one can obtain precision better than the one set by the so-called Heisenberg uncertainty relation. Finally, we are going to summarize what we have learned in the module 1 of this course.

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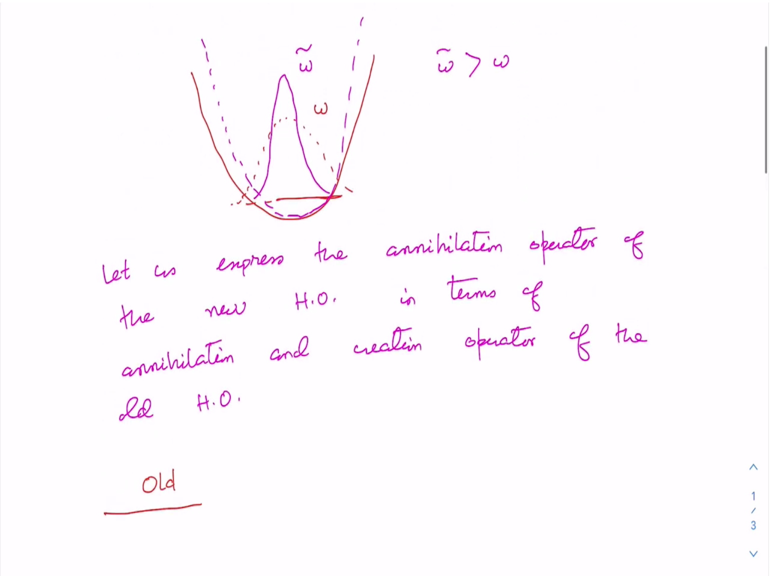

Let us begin our discussion on squeezed states. It is an important class of quantum states of harmonic oscillator or quantized electromagnetic radiation. As we learned in the last class about coherent state by considering a ground state and then displacing it. So, we have taken a ground state and if we displace it then we get the coherent state. Similarly, we can understand squeeze state by taking the ground state of a harmonic oscillator and then squeezing the ground state and then we are going to get the so-called squeeze stage.

So, squeeze state is going to be represented by this ket Xi. Let me explain it a little bit more clearly. Say let us take the usual ground state of a harmonic oscillator with say frequency omega. You know that the ground state of a harmonic oscillator is Gaussian. So, it is what we have. Suppose this is what we have and this harmonic oscillator has the natural frequency to be say omega.

Now increase the curvature of the harmonic oscillator. If we increase the curvature of the harmonic oscillator potential that implies squeezing of the harmonic oscillator. So, this is what we are going to get, because of that you see the ground state is getting squeezed and the frequency of the new harmonic oscillator is actually, now we increase the curvature and then as a result of this ground state is getting squeezed and its frequency is say omega tilde is the new frequency of the harmonic oscillator when after increasing the curvature and omega tilde is greater than omega. Let us now do one thing, let us express the annihilation operator of the, let me just write here, let us express the annihilation operator of the new harmonic oscillator in terms of annihilation and creation operator of the old harmonic oscillator.

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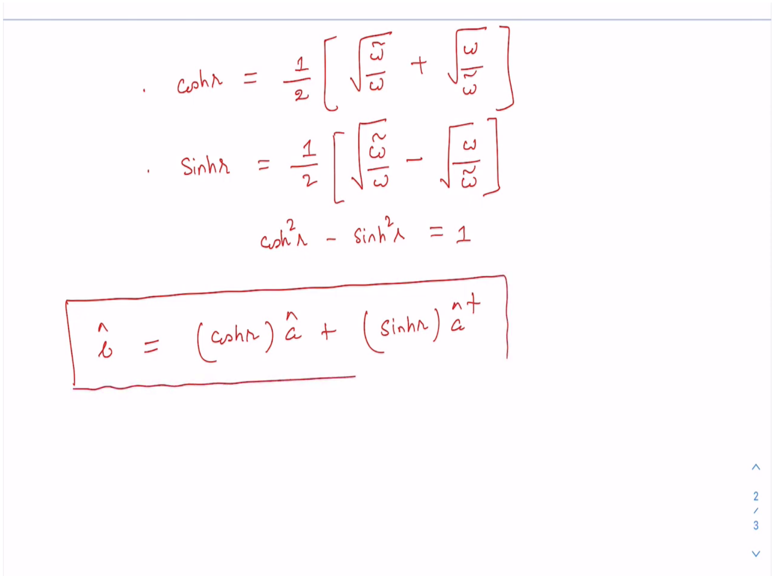

Let me write here, for example what I mean by old harmonic oscillator say in old harmonic oscillator, the frequency of the harmonic oscillator was omega and new harmonic oscillator the frequency is omega tilde. So, the position operator would be represented by x cap. You know that that would be say q 0 + a + a dagger, a is the annihilation operator, a dagger is the creation operator and q 0 is equal to h cross divided by twice m omega under root that is for the old.

Then I will also talk about the new a little bit later and then the corresponding momentum operator would be i m omega q 0 a dagger - a and therefore you can immediately write a annihilation operator as 1 by twice q 0 x + i p by m omega all right. So, this is what I have and now the new harmonic oscillator, let us its position operator is x cap say zero-point fluctuation is q 0 tilde and now annihilation operator let me write it as b and then creation operator is b dagger for the new harmonic oscillator.

You see that q 0 tilde is now, it is (say) frequency. It is the same harmonic oscillator only we have just changed the frequency or increased the frequency. So, it would be the only the frequency is now getting changed and momentum operator would be i m omega tilde q 0 tilde b dagger - b and therefore b this operator I can write it as 1 by twice q 0 tilde, you have here x + i p by m omega tilde all right.

So, now the next thing is to express this annihilation operator b in terms of a and a dagger that is you can trivially do that. So, you have 1 by twice q 0 tilde. So, in x let me write it this one okay. So, x would be q 0. So, I am just using this one a + a dagger and you have here i 1 by m omega tilde and p operator you can write it as i m omega q 0 a dagger - a. So, this is what you can do and if you actually do the calculation, it is very trivial.

So, you should be finally, I encourage you to do it. You should be able to show that this would turn out to be one half square root of omega tilde by omega and you will have a 1 + omega by omega tilde + a dagger 1 - omega by omega tilde all right. So, this is what you will have. Now clearly the new annihilation operator is a combination of old annihilation and creation operator.

So, if I Now write say cos hyperbolic r is equal to half square root of omega tilde by omega + omega by omega tilde and sine hyperbolic r, I can write it as one-half omega tilde by omega - omega by omega tilde under root. So, you will immediately see that if
I write it like this cos hyperbolic r - sine hyperbolic square r is equal to 1 you can verify it very easily. Then in terms of this cos hyperbolic r and sine of particular I can express my new annihilation operator b as cos hyperbolic r a + sine hyperbolic r a dagger.

So, you see the annihilation operator of the new harmonic oscillator after we squeezed it, how it is related to the annihilation and creation operator of the old harmonic oscillator.

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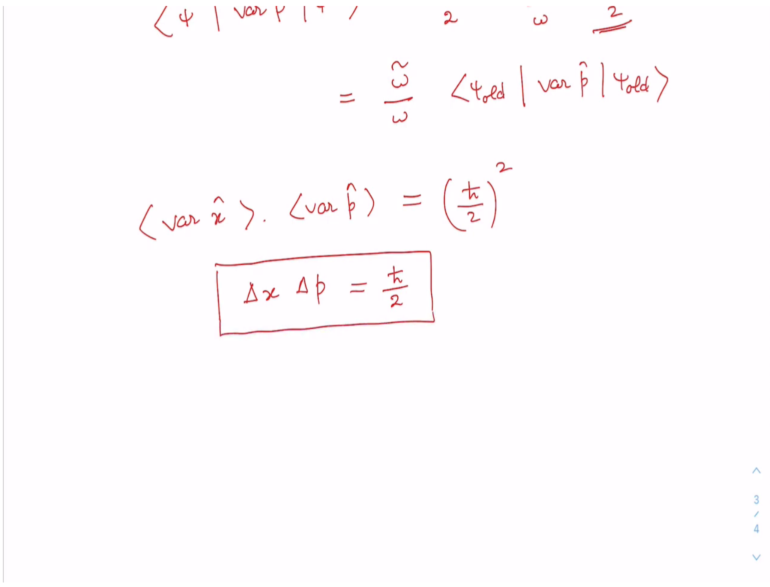

This transformation is actually known as Bogoliubov transformation. So, I am going to build up everything around it. So, that is why I have introduced it this way and you will see the meaning and another thing let me just comment here that because cos hyperbolic square - sine hyperbolic square r is equal to 1 and since omega tilde is greater than omega so, this implies that this parameter r is greater than 0.

Now let us look at the variance of the new harmonic oscillator. So, variance of position or x operator of the new harmonic oscillator I can write it as h cross divided by twice m omega tilde, where psi tilde is the wave function of the new harmonic oscillator and because we are considering the ground state of the harmonic oscillator, so the variance is nothing but the square of the zero-point fluctuation which is simply h cross by twice m omega tilde.

This I can further write as h cross by twice m omega and omega by omega tilde and you can easily recognize that this is again nothing but the variance of the position operator for the old harmonic oscillator. It will be multiplied by, omega by omega tilde. Now as you can see that omega tilde that is the harmonic frequency of the new harmonic oscillator which is greater than the old harmonic oscillator frequency.

So clearly, we can see that the variance of x for the new harmonic oscillator is less than the variance of x for the old harmonic oscillator okay and it is a clear indication that the uncertainty in position is decreasing. On the other hand, if you look at the variance of the momentum operator for the new harmonic oscillator, that you can simply write it as h cross m omega tilde by 2 because we are in the ground state which further, I can express it as omega tilde by omega h cross m omega by 2.

This guy is nothing but the variance of momentum of the old harmonic oscillator. So, this is I can write it as say variance of this position operator for the old harmonic oscillator. So, here again you can see that the uncertainty in momentum is now getting increased. So, what you actually see that the so-called position, uncertainty in position is decreasing at the cost of increase in the uncertainty in the momentum.

However, you should know that the product of the variance of the position and the variance of the momentum, that would be a constant and that would be h cross by 2 whole square okay and this is basically same with the original unsqueezed harmonic oscillator and that means that the uncertainty product that is delta x into delta p is equal to h cross by 2 and this is still preserved.

Actually, because of that reason both coherent states and the squeeze states are called Gaussian states because for Gaussian states only the uncertainty product that is uncertainty in position and uncertainty in momentum that product is minimum that is h cross by 2.

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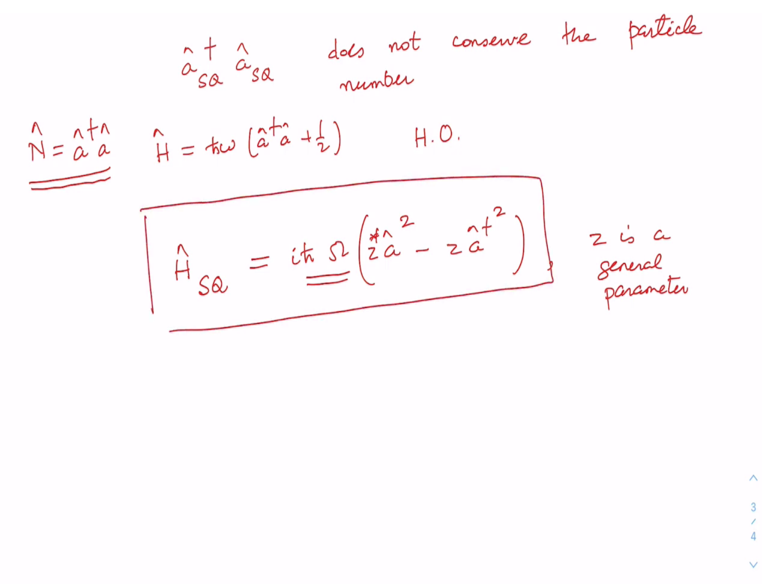

Now what about the squeezing operator? So, let us now discuss that what about the squeezing operator that is S Xi, but before I go into discussion of squeezing operator, first let us guess the squeezing Hamiltonian. Now you see that we got the squeezing annihilation operator from our preliminary analysis like this, cos hyperbolic r a + sin hyperbolic r a dagger.

Now you can actually prove that if you take a dagger for the squeezing annihilation operator this is a SQ, if this is the number operator for the squeeze state this does not conserve the particle number unlike a and a dagger, a dagger a which is the number operator originally, this conserved the particle number and that is the reason we write down the Hamiltonian for the normal harmonic oscillator as h cross omega a dagger a + half okay for harmonic oscillator.

But now the thing is that what about the Hamiltonian for the this is squeezing okay for the squeeze states of the harmonic oscillator. So, let us guess let us say our squeezing Hamiltonian for the squeeze state is of this type. So, it should be quadratic in the annihilation and creation operator. Let us say it is a dagger, let me first write and I will explain i h cross omega z star a square - z a dagger square because it has to be Hermitian where z is a general parameter z is a general parameter all right.

So, this is what I have. So, h cross omega has the dimension of you know energy. So, this is our assumption about the Hamiltonian for the squeeze state harmonic oscillator. So, if whether it is correct or wrong, we will get to know if we look at the time evolution of the annihilation operator.

(Refer Slide Time: 16:29)

$$\dot{\hat{a}}^{\dagger} = -2\Omega z^{*} \hat{a} \rightarrow (2)$$

$$\ddot{\hat{a}} = -2\Omega z\left(-2\Omega z^{*} \hat{a}\right)$$

$$\boxed{\ddot{\hat{a}} = 4\Omega^{2} |z|^{2} \hat{a}} = \lambda^{2} \hat{a} \; ; \; \lambda^{2} = 4\Omega^{2} |z|^{2}$$

$$\rightarrow (3)$$

Because the time evolution of this annihilation operator. If it coincides with what we have got as the annihilation operator for the squeeze state, then we will get to know that our Hamiltonian is correct because you remember that this one, we worked out as annihilation operator we denoted it by b. So, now from Heisenberg representation which we discussed in an earlier class, we can write this time evolution of this annihilation operator as e to the power i by h cross.

Now our squeeze Hamiltonian is this and then this would be a e to the power - i by h cross H SQ t all right. So, this is what I have. Now to work out the time evolution let us begin by writing down the Hamilton's equation of motion. So, I have a dagger of t that would be Heisenberg equation of motion will give me 1 by i h cross a Hamiltonian H SQ.

So, if you already I guessed what is my if I just use this equation then you can immediately work out this as let me just write it down, you please verify you should be able to get it - omega z a a dagger square this is what you will get and this can further you can simplify and then you will get a dagger dot time derivative of a would be equal to - twice omega z a dagger.

For the creation operator it would be a dagger dot that would be equal to - twice omega z star a. So, let us say this is my equation number 1 and this is my equation number 2. Now from these 2 equations I can get an uncoupled equation if I take the time derivative of equation 1 then I can write it as a dagger double dot, a double dot annihilation operator I am taking the time derivative.

So, I will have twice omega z and then this would be - twice omega z star a which you can easily see. So, therefore what I will get here would be I will get it as 4 omega square mod z square a. So, therefore I get an equation for the annihilation operator and I can write it as say lambda square a, where lambda square is equal to 4 omega square mod z square. So, let us say this is my equation number 3.

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$$\lambda t = r$$

$$\hat{H}_{SQ} = i\hbar\Omega\left(z^{*}\hat{a}^{2} - z\hat{a}^{\dagger 2}\right)$$

$$= i\hbar\Omega\,|z|\left[e^{-i\theta}\hat{a}^{2} - e^{i\theta}\hat{a}^{\dagger 2}\right]$$

$$\boxed{\hat{H}_{SQ} = i\hbar\Omega\,|z|\left[e^{-i\theta}\hat{a}^{2} - e^{i\theta}\hat{a}^{\dagger 2}\right]}$$

So, this equation could be easily solved and we know the general solution of this type say a of t is equal to some constant say c 1 cos hyperbolic lambda t + c 2 sine hyperbolic lambda t all right and let us say this is my equation number 4. Now we know that at t is equal to 0 we have a at t is equal to 0 is simply let us write it as a of 0 and if I put this initial condition in equation 4, immediately you see that c 1 is a of 0 on the other hand if you take all right.

So, let me just work it out if you take the time derivative of this equation 4 then you will get it as c 1 lambda sine hyperbolic lambda t + c 2 lambda cos hyperbolic lambda t and then because of the fact that lambda square is equal to 4 omega square mod z square. So, let us take lambda is equal to - twice omega into mod z and this minus sign is taken rather than the plus sign so that we can reproduce equation number one okay that is the reason I have taken it like this and therefore now at t is equal to 0 I have a dot at t is equal to 0 would be simply from this equation you see that would be c 2 of lambda and then I can easily work out from equation 1. Let us go to equation 1 here you will see I have a dot that is from here I can get these are very simple analyzes I am doing it in details but I could have left it for you.

But, let me do it twice omega z a dagger t is equal to 0 is equal to c 2 lambda and from here what I can do I can write it further twice omega say mod z this is z. So, mod z e to the power i theta a dagger at t is equal to 0 and you have c 2 and lambda I have taken it to be like this. So, I have twice omega mod z. So, from here I can find out my other constant that is c 2 is equal to a dagger at t is equal to 0 e to the power i theta.

So, therefore what I get finally is that i get a of t is equal to a of 0 cos hyperbolic lambda t and + a dagger 0 e to the power i theta sine hyperbolic lambda t or I can write it as a of t is equal to a of 0 cos hyperbolic cos hyperbolic r + e to the power i theta a dagger 0 sine hyperbolic r okay where i have taken lambda t is equal to r. So, this is what I got and in fact if you look at our earlier treatment which was very elementary treatment, we have taken we got it exactly the same equation apart from here what the new thing we got is that.

Now we got e to the power i theta which is actually in our earlier treatment that was not the case. So, let me write it again this is not what we got from our earlier treatment because our squeezing was only along the x direction but here, I am now talking about a general squeezing can be done in general anyway I will come to that. So, you see that means that the Hamiltonian that we have considered for squeezing is perfect.

So, our guess is correct. So, the squeezing Hamiltonian we can write it as i h cross omega z star a square that was our guess and by this guess we got the correct expression. So, I can now write it as i h cross omega mod z e to the power - i theta a square - e to the power i theta a dagger square or I can finally write it as the squeezing Hamiltonian as i h cross omega mod z e to the power - i theta a square - e to the power i theta a dagger square okay let me box it. So, this is what I got.

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$$S(\xi) = e^{-i/\hbar H_{SQ} t} = \exp\left[-\frac{r}{2}\left(e\ a - e\ a\right)\right]$$

$$\xi = r e^{i\theta}$$

$$S(\xi) = e^{-i/\hbar \hat{H}_{SQ} t} = \exp\left[\frac{1}{2}\xi \hat{a}^{\dagger 2} - \frac{1}{2}\xi^{*} \hat{a}^{2}\right]$$

$$\boxed{S(\xi) = \exp\left[\frac{1}{2}\xi \hat{a}^{\dagger 2} - \frac{1}{2}\xi^{*} \hat{a}^{2}\right]}$$

Now what about the squeezing operator? So, squeezing operator would be simply e to the power - i by h cross H SQ t all right. So, this actually you can verify but let me just simplify it. It is exponential omega mod z t e to the power - i theta a square - e to the power i theta a dagger square. So, this is my squeezing operator and because we have already taken that r is equal to lambda t which is - twice omega mod z that is what our lambda.

So, therefore I can finally write this expression i by h cross H SQ t is equal to exponential r by 2 e to the power - i theta a square - e to the power + i theta a dagger square or this is basically my squeezing operator and therefore if I now define a quantity say Xi is equal to r e to the power i theta then we can write S Xi is equal to e to the power - i by h cross this I am writing again and again just you should remember it.

Then I can write it in the very familiar form exponential half Xi a dagger square - half Xi star a square. This is what we have or finally let me just write down the squeezing operator separately like this, exponential half Xi a dagger square - half Xi star a square. So, this is a very important result and it is an important operator and this is we are going to exploit in this course.

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$$= \langle 0|0\rangle \qquad \mathbb{1}$$

$$= 1$$

- $S^{\dagger}(\xi) = S^{-1}(\xi) = S(-\xi)$
- $S^{\dagger}(\xi)\,\hat{a}\,S(\xi) = \hat{a}\cosh r + \hat{a}\,e^{+i\theta}\sinh r$
- $S^{\dagger}(\xi)\,\hat{a}^{\dagger}\,S(\xi) = \hat{a}^{\dagger}\cosh r + \hat{a}\,e^{-i\theta}\sinh r$

This squeezing operator is unitary squeezing operator is unitary and the squeezed state Xi squeezed state Xi can be created from vacuum just by applying this squeezing operator on the vacuum state. Also, you can see that the squeeze state is normalized because of the fact that S Xi is unitary. So, because of this you can immediately see because vacuum state is this is anyway identity operator.

So, vacuum state is normalized. So, this squeeze state is also normalized. Let me reiterate once again that when we originally obtained the expression for the annihilation operator earlier let me go to that expression once again where we got it from our elementary treatment here. You see here we did not have this term e to the power e to the power i theta there and because originally, we assumed that the direction of squeezing to be along a particular direction and in this case, it was along the x direction.

But the new expression which we are having e to the power i theta this factor is also there. So, that is more general and actually now we can talk about squeezing along any direction so, this is much more general expression we have and generally in literature this is the expression that you are going to get. Finally let me quickly list some of the useful relation that you can immediately derive just the way we have done it for the displacement operator in the context of coherent state.

For squeezed state also squeezing operator case you can have as these properties. So, S dagger Xi is equal to S - S inverse Xi and that is equal to you can show that, that would be nothing but because of the structure of the operator this relation you can immediately prove and I leave it as a exercise to you can show that S daggers Xi a S Xi you should be able to get a cos hyperbolic r + a dagger e to the power - i theta sine hyperbolic r and you will for the creation operator you will get this kind of expression.

So, that would be a dagger cos hyperbolic r + a e to the power I think this is + i theta here it will be e to the power - i theta sine hyperbolic r. These are very useful relations and I encourage you to work it out. Okay there is a small correction this should be a dagger.

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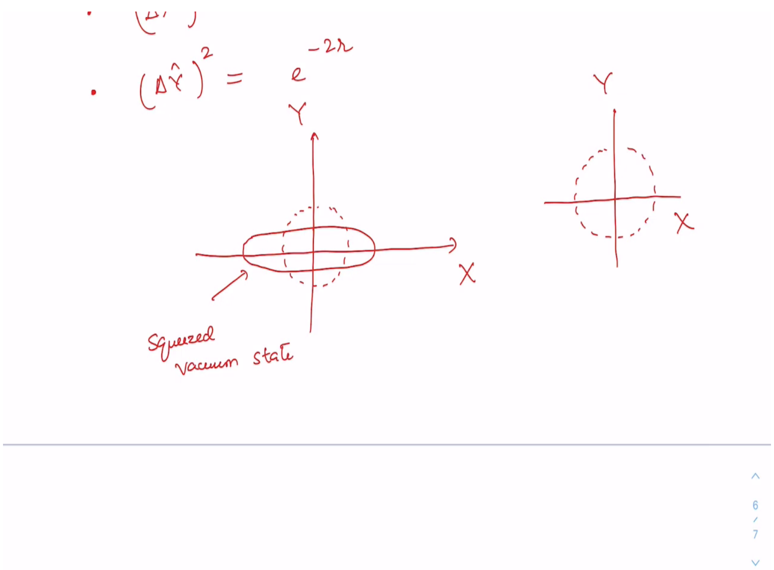

Now finally let me give a quick graphical representation of squeezed state in free space. Here let me make one point clear that the squeezed state I am considering here is the so-called squeezed vacuum because I have obtained it by squeezing the ground state or the vacuum state. So, this is called squeezed vacuum state. To represent it graphically, let me define 2 quadrature operators say X cap as say a + a dagger and Y cap as i into a dagger – a.

As you can see from the construction that X cap is analogous to the position operator and Y cap analogous to the momentum operator. Now if we calculate the variance of this position operator in the squeezed state, the variance is defined as expectation value of X square - expectation value of X whole square. If we calculate it in the squeezed state and maybe we will do it in our problem-solving session or you can try it yourself you will get it as e to the power 2 r in the squeeze state we are calculating or in on the other hand the variance of this momentum operator or Y quadrature would turn out to be e to the power - twice r.

So, you see this the variance is variance in Y is decreasing while the variance is X is getting increased. So, this we can represent it graphically in this phase space by this is my X axis and this is my Y axis. Initially I had my vacuum state like this now after squeezing what is happening that the quadrature in X is getting increased. So, this is what I get here and Y is decreasing.

So, you see the Y quadrature is getting squeezed and the Y quadrature is getting amplified. So, this is what I mean by squeezed vacuum state this is what we mean by squeeze vacuum state. Similar analysis you can do for coherent state for coherent state you will see that the quadrature actually does not change but what happens is that as you know that the coherence state is basically displaced vacuum. So, quadrature remains maintained but this vacuum state is getting displaced in the phase plot. So, this is what is our coherent state all right.

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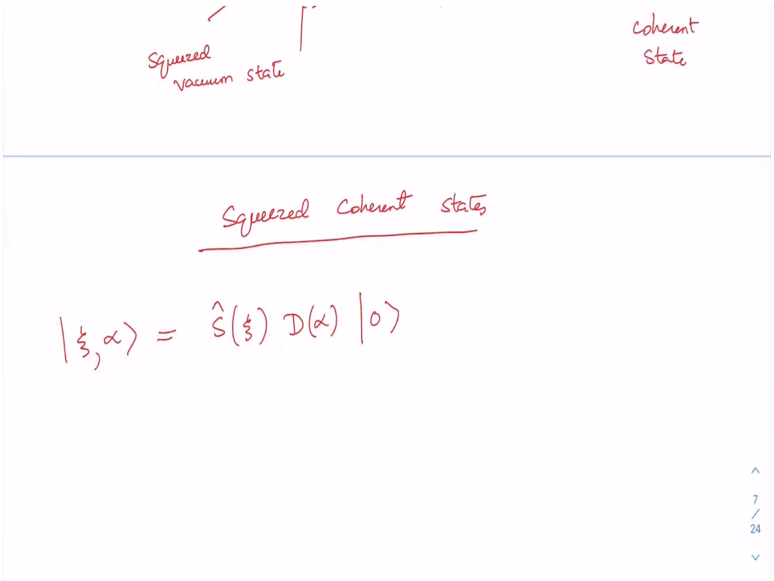

Another point let me make that apart from this squeezed vacuum state we can have also squeezed coherent states as well. This can be obtained first by creating a coherent state and then you squeeze the whole thing then you will able to get the so-called squeezed coherent states. So, with this we have completed the first module of this course.

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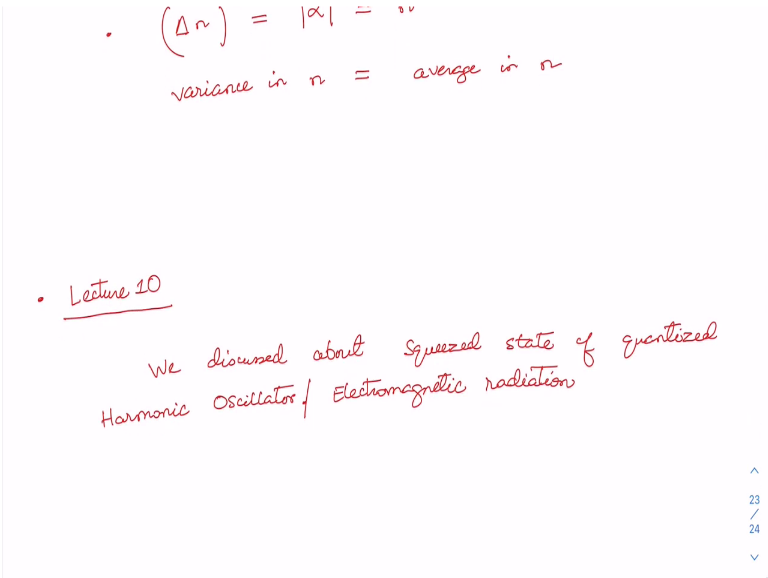

Now let us have a quick recap of what we have learned in the first module of this course in lecture one I have given a general introduction to the course followed by motivation behind the course then in lecture 2 and 3 we learned about 2 level systems or 2 level atoms there we saw how one can write the Hamiltonian in a very compact form in terms of the Pauli matrices and we also learned about the so-called Heisenberg representation or Heisenberg Picture.

In this picture, we learned that the operator any operator quantum mechanical operator (say) a become time dependent and the wave function is time independent. On the other hand, in the so-called Schrodinger picture which we are generally used to, there the time dependency is associated with the wave function and the operator there is time independent. In an analogous way to the Schrodinger equation which gives the time evolution of the wave function, in the Heisenberg picture the operator evolves with time as per the so-called Heisenberg equation of motion here I have written down the Heisenberg equation of motion provided the operator a is not exclusively dependent on time. Then we saw you applying this Heisenberg equation of motion that the time evolution of the Pauli operator and we also saw that the expectation value of the Pauli operator it basically precesses around an arbitrary vector say epsilon.

It precesses with some frequency say twice e by h cross that is the precession frequency with which the Pauli, expectation value of the Pauli vector rotates and this expectation value of Pauli vector or it has a name, it is also called the so-called Bloch vector all right. Then after that we worked out the eigenvalues and eigenvectors of this Hamiltonian.

Also, we discussed about the so-called avoided crossing and we saw that in when there is a term, a non-zero term in the off diagonal of the Hamiltonian then it basically because of that this energy level no longer cross and that is what is called the avoided crossing. After that we discussed the 2-level system under a time dependent driving and there also we learned that under the so-called rotating wave approximation the Hamiltonian can be again written in a compact form using the Pauli matrices and then we went on to discuss the so-called Rabi oscillations.

In lecture 4 we studied about a 2-level atom interacting with a classical field. The classical electric field was taken at the position of the atom because we considered that the atom is, size of the atom is much smaller than the wavelength of the electric field electromagnetic radiation. The ground state of the atom was considered to have an energy 0 and the excited state has energy E equal to h cross omega 0, omega 0 is the resonance frequency.

In this case the atom is considered as a quantized entity and the electric field is considered as a classical entity and that is why this particular treatment is known as the semi classical treatment. Using the so-called rotating wave approximation and in the rotating frame of reference the Hamiltonian of this 2-level system is of the whole system is described by this Hamiltonian where delta is the detuning parameter that is omega - omega 0, omega is the laser field frequency, omega 0 as I told is the resonance frequency of the atom and capital omega is the so-called Rabi frequency. Then we can actually write it in this basis state as well, here sigma for example represents the atomic lowering operator and sigma dagger represents the atomic raising operator. Then we went on to calculate the probability of getting the system in the excited state and then this is given by this general expression here, omega tilde is the generalizable Rabi frequency.

We see that when the resonance condition is there that means delta is equal to 0 the atom can go from the ground state to the excited state fully when omega t is equal to pi. But that is not the case if this resonance condition is not there if delta is not equal to 0. So, these things we discussed. Then in lecture 5 we discussed the dressed state picture of the 2-level atom.

The idea was that, when the laser light is impedes on the 2-level atomic systems. The ground state and the excited state are no longer the eigen state of the 2-level system and then still we can find out the eigenvalues and eigen state of this Hamiltonian dressed Hamiltonian and we found it to be it has 2 eigenvalues E + and E- and the dressed states and the corresponding eigen states is represented by this plus ket and this minus ket here.

This angle theta is known as the Stuckelberg angle and this is associated with the Rabi frequency and the detuning parameter by this expression here and we went on to discuss the so-called avoided crossing here as well. In lecture 6 we learned the density matrix formalism in the context of 2-level system, we saw that if we consider a single 2-level system or a pure system which is simply represented by this wave function then the density operator is written in this form.

In the ground state basis z e basis basically the density operator can be written by this 2 by 2 matrix and the expectation value of an operator a can be worked out just by taking the trace of the product of the density operator and the density operator. And we also learned about the properties of the density operator for example the density operator is Hermitian trace of rho is equal to 1.

Eigenvalues of rho are real and non-negative but rho does not obey Heisenberg equation and this is basically due to the fact that though rho is Hermitian it does not correspond to any physically measurable quantity. Then the mixed state if we consider a mixed state the density operator is given by this expression and how to find out whether a system is mixed at or pure state you just have to take the rho square of the density matrix and then take the trace.

If the trace is rho square is found to be less than 1 then it corresponds to a mixed state. On the other hand, if trace rho square is equal to 1 then that corresponds to the so-called pure state. So, here we are if it is less than 1 then the density operator is basically representing a mixed state. Then we discussed the reduced density matrix if we are interested say only in the system not in the surrounding then we can always trace out the system by taking the partial trace of the whole density matrix of the system plus the surrounding.

We can now then focus only on the system only just by writing the density operator for the system. Then in lecture 7 we refresh our knowledge on quantum harmonic oscillator. Basically, we learnt the canonical quantization procedure here. The idea is to first, though the example was for classical harmonic oscillator but this is valid for any classical system the idea is to write down the Lagrangian here.

We are talking about classical harmonic oscillator Lagrangian of the harmonic oscillator we just find out, then we find out the conjugate momentum of the system and then we can write down the Hamiltonian of the system using this formula or expression and then we can express the whole Hamiltonian of the classical system in terms of this coordinate and momentum q and p.

Then the next step is to find out the canonically conjugate variable. To do that we just have to find out those variables which satisfy the so-called Hamilton's canonical equation of motion. If we find that out then that will give us the canonically conjugate variable and they satisfy the so-called Poisson bracket equation and then quantization is straightforward, this canonically conjugate variable q and p are then replaced by the operators, corresponding operators.

This Poisson bracket is now replaced by this commutation relation and then we can just write down the Hamiltonian of the system. So, this is basically the way and in the context of harmonic oscillator we find out we define the so-called annihilation and the creation operator and the so-called the number state and we also discussed couple harmonic oscillator how to write down the 2 harmonic oscillator interacting with each other by some coupling coefficients say kappa.

Suppose this is harmonic oscillator. So, this is what we have. So, harmonic oscillator 1 and harmonic oscillator 2 having say spring constant say k 0 k 0 and mass m 1 m 2 and then we learn how to write down the Hamiltonian for this coupled harmonic oscillator system under the so-called rotating wave approximation. Then we went on to discuss how to quantize a travelling electromagnetic wave the idea was to again here to find out the canonically conjugate variables.

We started with this travelling with electric field and using the Maxwell equations we worked out this equation where this E 0 characterizes the state of the electric field in a given mode we focus only one single mode and the real part of the electric field E 0 and imaginary part of E 0 corresponds to the canonically conjugate variable within a multiplying factor but we prefer to do in the normalized units.

So, therefore we wrote E 0 t in this form where alpha is a normalizable variable A is a constant to be worked out and this is straight away we get this equation from the Maxwells equations and then defining the variables and ultimately, we did the calculation and we were able to get the Hamiltonian in the form of the well-known form of the so-called harmonic oscillator.

Then we went to find out that if we just write this alpha as alpha in terms of q and p if we defined it like this and then we will be able to write it as yeah here we have written it harmonic oscillator. So, and then to decide whether this q and p satisfy the are they really the canonically conjugate variable then we found out this well-known equation using the Hamilton's canonical equation of motion that confirms that this q and p are canonically conjugate variables.

Then quantization is straightforward and thereby we were able to show that this electromagnetic wave, a travelling electromagnetic wave, the mode of a travelling electromagnetic wave behaves like a harmonic oscillator quantized harmonic oscillator. The electric field operator also we can write in terms of this annihilation and creation operator. Then in lecture nine we went to on to discuss the quantize the standing electromagnetic wave.

There the procedure was analogous but with some difference because now we have to satisfy some boundary condition because if it is a standing wave the electric field is 0 at the boundaries at say z is equal to 0 and at z is equal to L. So, this boundary condition has to be satisfied. So, thereby immediately we find that this wave vector is discretized and because the boundary conditions have to be satisfied the vector potential and the electric field has to be taken in this form.

The magnetic field is taken like this and we work in the so-called Coulomb gauge. So, divergence of A is equal to 0 the background electrostatic potential was taken to be equal to 0 and then electric field is simply a we can express it in terms of the vector potential. Then we using the Maxwell equation we arrived in this equation and then defining a normalized variable alpha of t in terms of the vector potential and the electric field then we got this equation from Maxwells equation.

Then we write down the Hamiltonian like this and ultimately, we were able to show that this again even a standing electromagnetic wave behaves like a harmonic oscillator and the quantization was straightforward. Again, we got the canonically conjugate variables as our q and p in physical unit they are basically the vector potential and the electric field with multiplying by appropriate factor they behave like canonically conjugate variables.

Then we were able to finally show that this also this standing electromagnetic wave when quantize it behaves like a harmonic oscillator. Now we can express the vector potential and these are physically measurable quantity, the electric field in the operator form and after that we discussed the so-called coherent states it is one of the important quantum states of harmonic oscillator or electromagnetic radiation.

Because we now know that quantized electromagnetic radiation behaves like harmonic oscillator. We show that this coherence state is nothing but displaced ground state and D alpha is the displacement operator and this coherence state can be expressed in terms of the number state as well by this popular expression and this coherent state is normalized. Then we found out that this average number of quanta in the coherent state is basically mod alpha square and it satisfied the so-called Poisson statistics or Poisson distribution.

We found that the variance in the number of quanta or number of photons is equal to average in the number of photons. Of-course this is basically one of the characteristics of Poisson distribution. Then today in this class we discussed about squeezed states of quantized harmonic oscillator or electromagnetic radiation. Now as we are equipped with required fundamentals of quantum optics, we are ready to go to the next phase.

In module 2 of this course, we are going to encounter our first artificial quantum system in the form of a cooper pair box which is the building block of circuit quantum electrodynamics that we are going to discuss in our course. I will first give an overview of circuit quantum electrodynamics then I will build up the fundamentals of circuit cavity from the first principles. So, see you in the next class, thank you.