Quantum Technology and Quantum Phenomena in Macroscopic Systems
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Lecture –1
Introduction and Basic Quantum Mechanics.

A very warm welcome to the first lecture of the course on quantum technology and quantum phenomena in macroscopic systems. As the title of the course suggests that this is a course on quantum mechanics applied to the macroscopic systems and the manipulation of these macroscopic systems for various applications. The outline of this introductory lecture is as follows.

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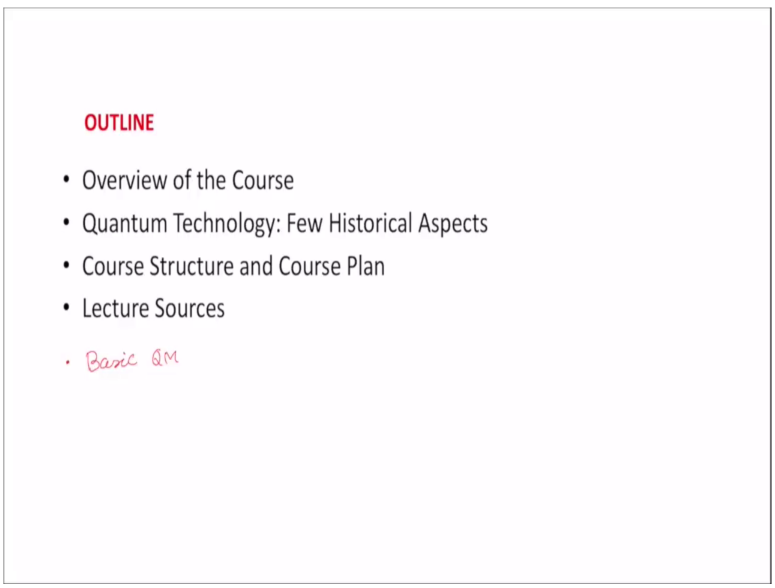

Firstly, I will give you a brief historical background of quantum technology. Then I will discuss about the course structure and course plan. It will be followed by informing you about various source materials based on which this course is structured. Finally, I will briefly remind you about some important postulates and mathematical tools of quantum mechanics. This will be helpful for those who are not in touch with quantum mechanics for a long time.

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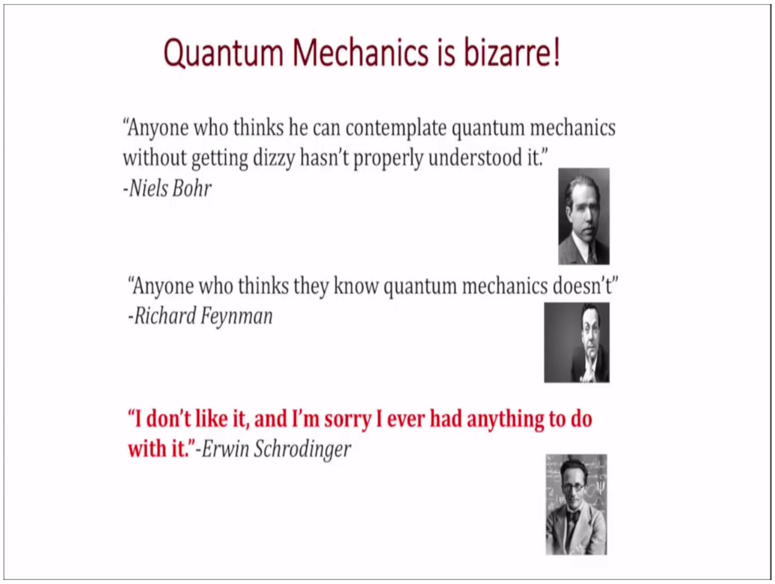

You know that quantum mechanics as a conceptual subject is very strange. In fact, Niels Bohr one of the founding fathers of quantum mechanics said that without getting dizzy one cannot properly understand quantum mechanics. Irwin Schrodinger who contributed so immensely for the development of quantum mechanics even said later in his life that "I do not like it and I am sorry I ever had anything to do with it". And you know Feynman famously said anyone who thinks they know quantum mechanics does not.

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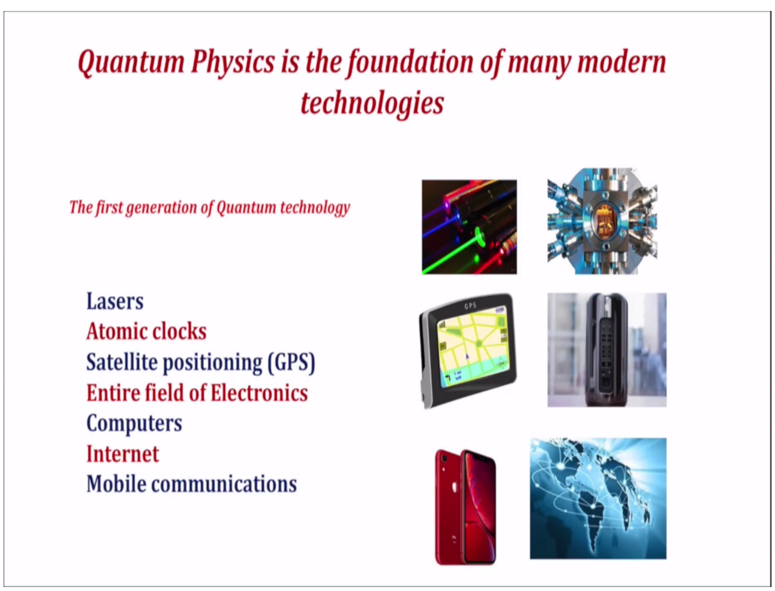

In spite of conceptual bizarreness not a single experiment so far has proved quantum mechanics to be wrong rather quantum mechanics turns out to be quite useful for various technological applications in a way it has brought revolution to technology. Quantum mechanics principles are behind the development of lasers, atomic clocks, GPS, the entire field of quantum electronics and computers that we see all around us, internet and even mobile communications.

In all these technologies the role of quantum mechanics is bit indirect as the system under consideration are not manipulated in the quantum domain.

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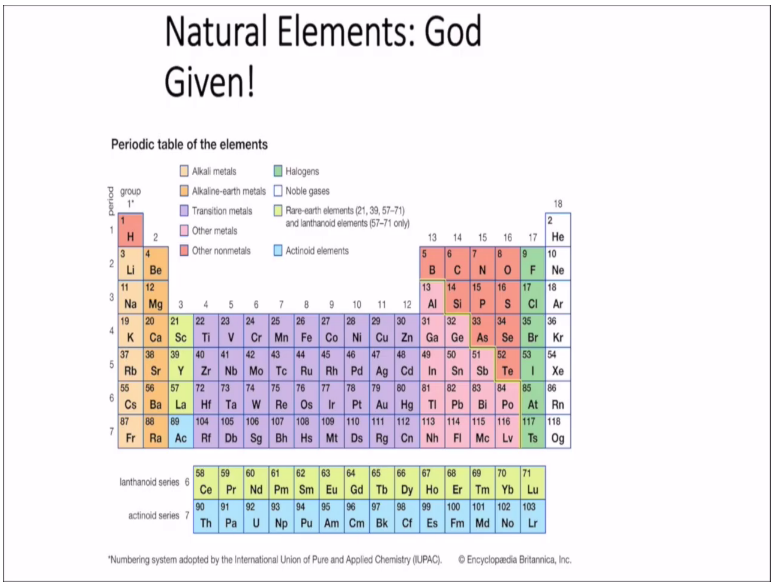

Entering as regards the manipulation of energy levels of atoms scientists were restricted by the god given elements which we know are enlisted in the periodic table. So, they are limited by these elements only and one does not have too much of freedom as energy levels of this neutral atoms can be altered or shifted by very tiny amount by application of electric field. You may recall about the start effect in this case or in the magnetic field case, you may recall about Zeeman effect, and artificial atoms were still to be discovered in the 70s or 60s.

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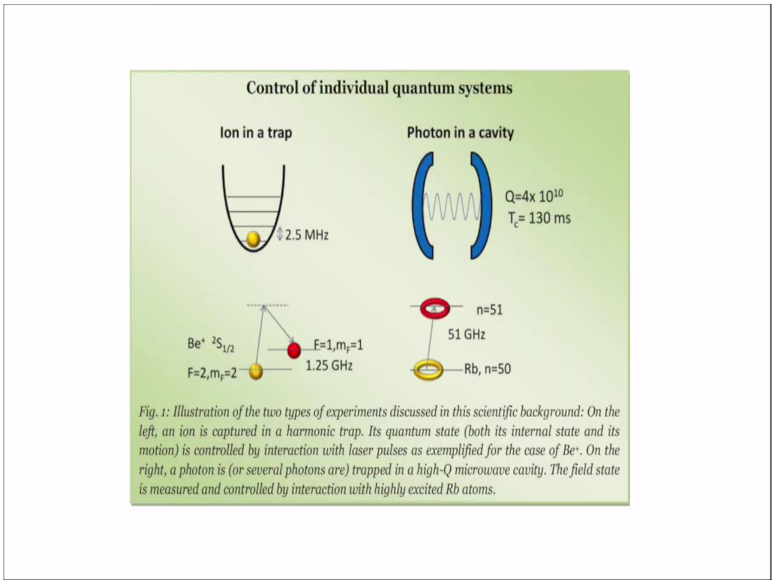

However, since 1980s due to enormous progress in technology and thanks to persistent research by some research groups it was possible to trap single ion or even trap photons in a cavity which led to the control of individual quantum systems.

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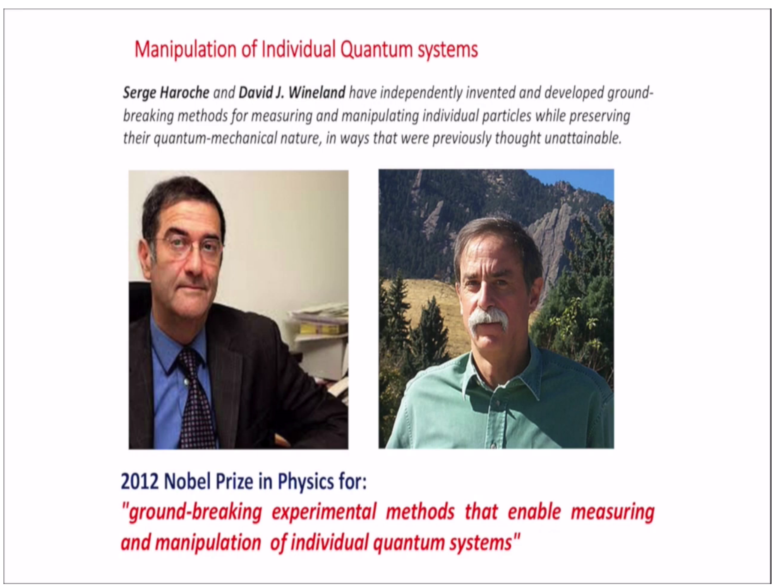

Two research groups, one lead by Serge Haroche and other by David J Wineland, contributed hugely towards manipulation of individual quantum systems. Their work in fact revolutionized the technological application aspects of quantum physics and very deservingly they were awarded the 2012 Physics Nobel Prize.

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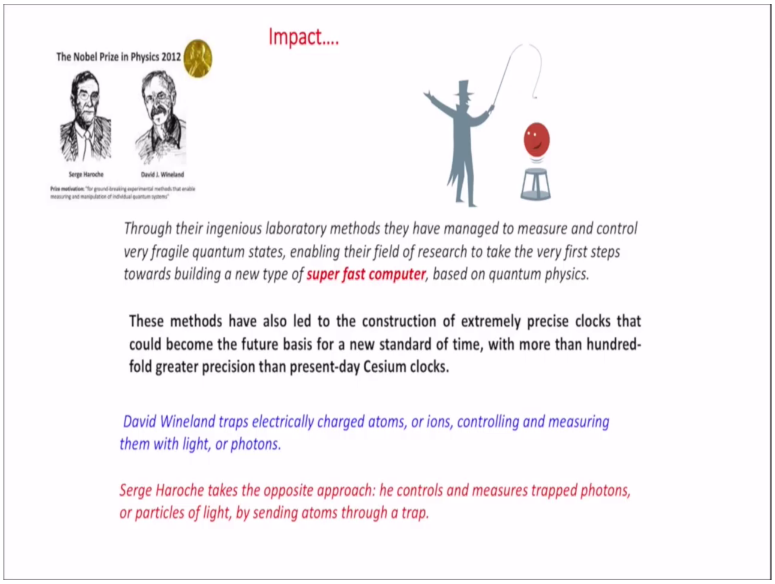

In fact, their works were partly responsible for huge resurgence of research development and towards development of quantum computers.

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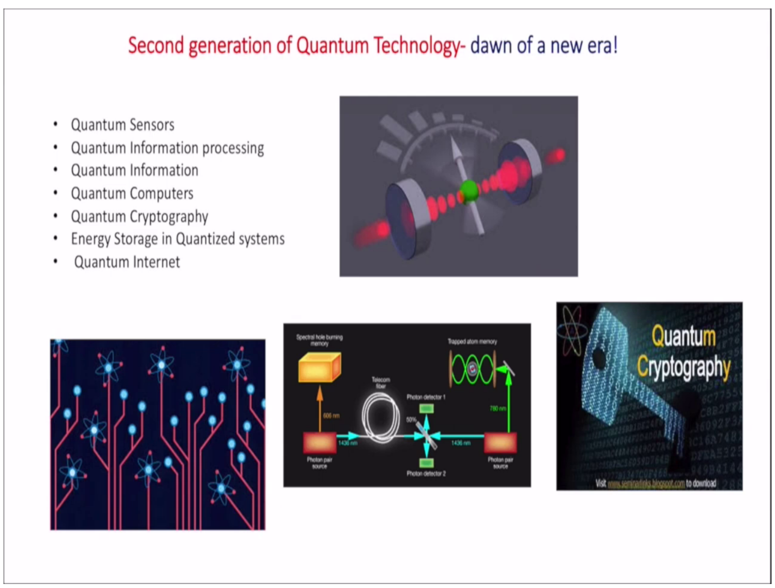

Today we are said to be in the domain of second generation of quantum technology where quantum mechanics is going to play direct role in various applications such as quantum sensors, quantum information processing, normal quantum materials, quantum cryptography and even in the so-called quantum internet.

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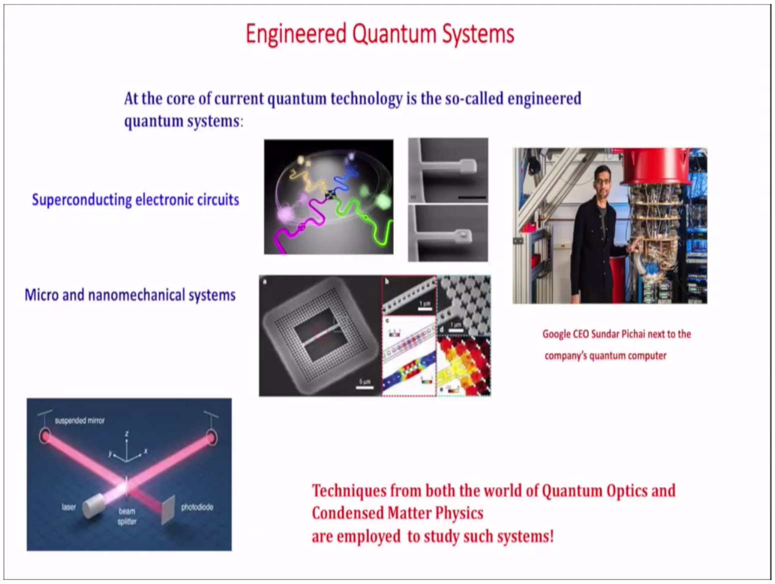

At the core of the quantum technology are the so-called engineered quantum systems such as superconductors, electronic circuits, micro and nanomechanical systems quantum dots and so on. In this course we will focus on superconducting circuits which are discussed as a topic under circuit quantum electrodynamics. Some of you may be aware that Google's latest quantum computer is based on superconducting circuits.

Apart from superconducting circuits we will study micro and nanomechanical systems they are discussed under the umbrella of cavity optomechanics. These systems has numerous applications. In fact, the LIGO which is basically the abbreviation for the laser interferometer gravitational wave observatory in reality is a large cavity optomechanical system. To understand these systems, one needs to know the tools of both quantum optics and condensed matter physics. And this is the major goal of this course; to familiarize you with the essential tools.

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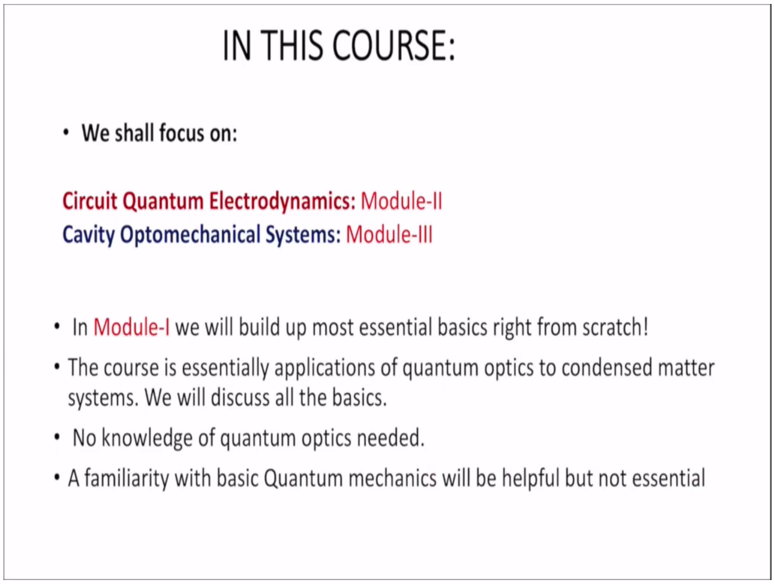

As I said in this course you will focus on circuit QED and Cavity optomechanics. Circuit QED will be covered in module-2 and cavity optomechanical system will be discussed in module-3. In module-1 we will build up most essential basics right from scratch. This course is essentially application of quantum optics to condense matter system and we will discuss all the basics. No knowledge of quantum optics is needed. However, a familiarity with basic quantum mechanics will be helpful but may not be essential.

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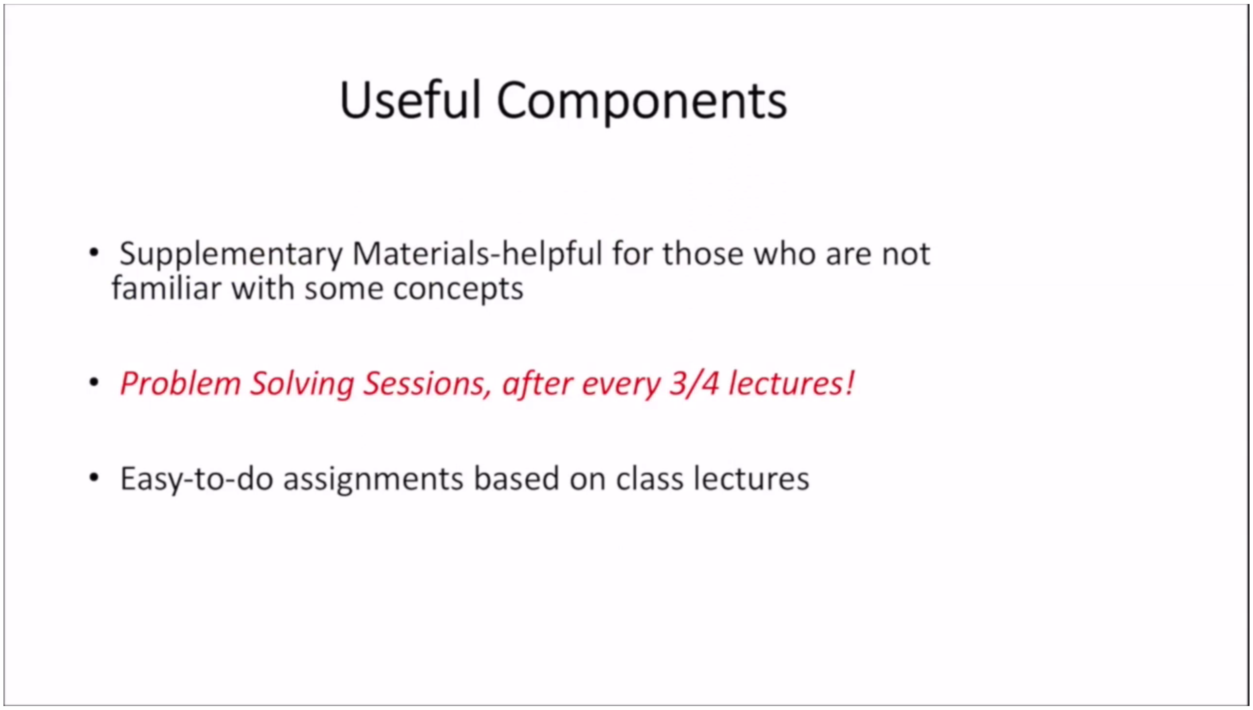

The useful components of the course are that we are going to have supplementary materials time to time which will help some of you to familiarize with some concept maybe you have forgotten or maybe you would like to learn it a little bit in more depth. So, supplementary materials will hopefully compensate those things. And all the lecture materials should be supplemented by problem solving sessions. After every three four lectures will have problem solving session.

In these problem-solving sessions some derivation steps for example which may skip in the lecture maybe, we will complete that in the problem-solving sessions. And these sessions are going to be useful because you will have confidence as you solve more and more problems. And will have easy to do assignments based on class lectures. No need for remembering the derivations. You just need to understand how things are worked out.

So, steps are necessary to do but for examination purposes you need not have to border about long derivations.

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The source materials on which this course is relied on are taken from research papers, books and even from internet. For quantum mechanics, you may look at the elementary book by David Griffiths. Fourth edition has come up. Apart from that there is a little bit on the higher side and that book is by Sakurai's Modern Quantum Mechanics, and for quantum optics you may look at Introductory Quantum Optics by Gerry and Knight, and there is another nice book by Grynberg and Aspect. Introduction to Quantum Optics is written actually by three authors Gilberg Grynberg, Alain Aspect and Claude Fabre.

And this book is bit advanced that is Quantum Optics by Girish S Agarwal. And this book is used to discuss some materials in module-3 where we are going to discuss cavity quantum optomechanics.

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Also, I will rely on materials from Florian Marquardt's lectures, Marcus Aspelmeyer, and I will use some notes by Mishkatul Bhattacharya who is my friend and he has kindly shared some notes with me. I will use some of them. And I will also use for circuit QED. I will use lectures by Stephen M Girvin and also the Qiskit Global Summer School lectures by Zlatko Minev. And there is a very good book on quantum optomechanics. This book is on advanced side but once you complete the module-3 you will be able to understand lots of the stuffs in this book. This book is called Quantum Optomechanics written by Warwick P Bowen and Gerard J.

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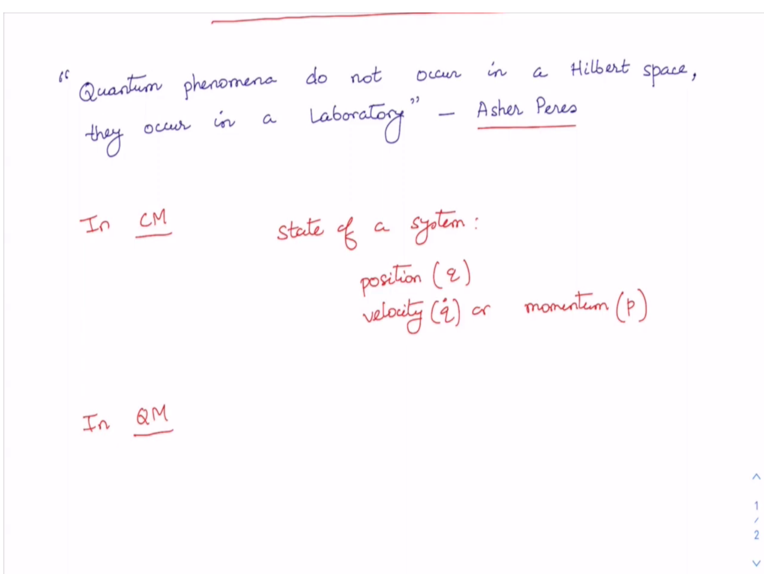

Let me now remind you about some basic quantum mechanics quickly. While in reality quantum mechanics happens in laboratory or actual life but, as put by the quantum physicist Asher Peres, to discuss quantum mechanics we generally rely on the abstract Hilbert space. As you know that in classical mechanics which I am going to write shortly as CM the state of a system or a particle is defined by the position and velocity or momentum at a given instant of time.

So, position let me denote it by q, velocity by q dot that is the time derivative of q and momentum is generally denoted by the symbol p. So, if we know the position and momentum of a particle at a given instant of time then in principle we can predict the past and future of the system by using Newton's laws of motion. However, in quantum mechanics this is not possible because we cannot know the position and momentum of a system or a particle with absolute accuracy and there is always a restriction imposed by the so-called Heisenberg uncertainty principle.

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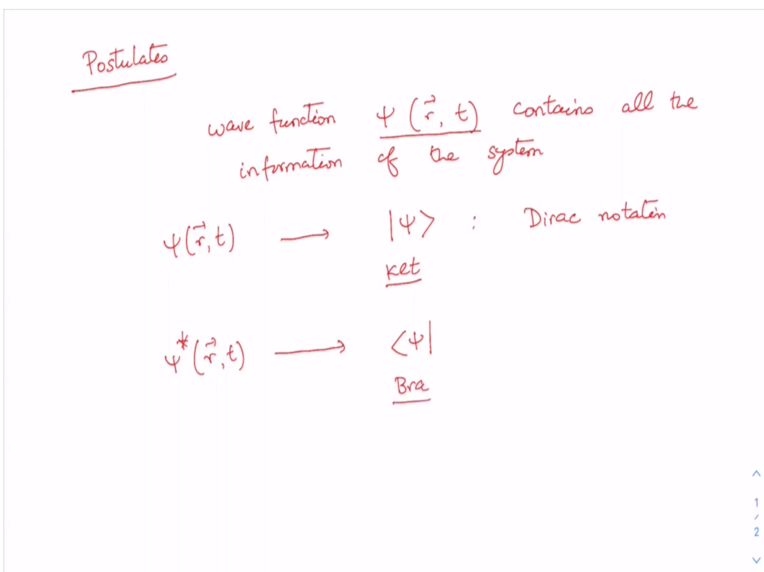

If the uncertainty in measuring the position is a del q and the momentum is del p then their product, uncertainty product, is greater than or equal to h cross by 2 where h cross is the reduced Planck’s constant and it is defined as h divided by 2 pi where h is the Planck’s constant. So, we need a completely different mathematical apparatus to discuss quantum mechanics here we will briefly talk about the bare essentials only. To gain some confidence we shall solve some problems in the next problem-solving session. Moreover, I am not being very rigorously perfect while discussing certain concepts. So, first of all let me discuss some important postulates of quantum mechanics based on which the whole quantum mechanics is best.

So, one of the basic postulates of quantum mechanics says that all you want to know about the system is contained in a complex wave function and this complex wave function is called wave function and the wave function contains all the information of the system. So, the state of a quantum system is defined by this wave function which is a complex quantity. And in Hilbert space this wave function can be represented by the so-called state vector and this is called Ket psi.

And this notation is called Dirac notation and this is called Ket and because psi of r,t wave function is complex. So, its complex conjugate would be denoted by psi star of r,t and the corresponding quantity in the Hilbert space or in Dirac notation it would be denoted by this symbol here and this is called bra.

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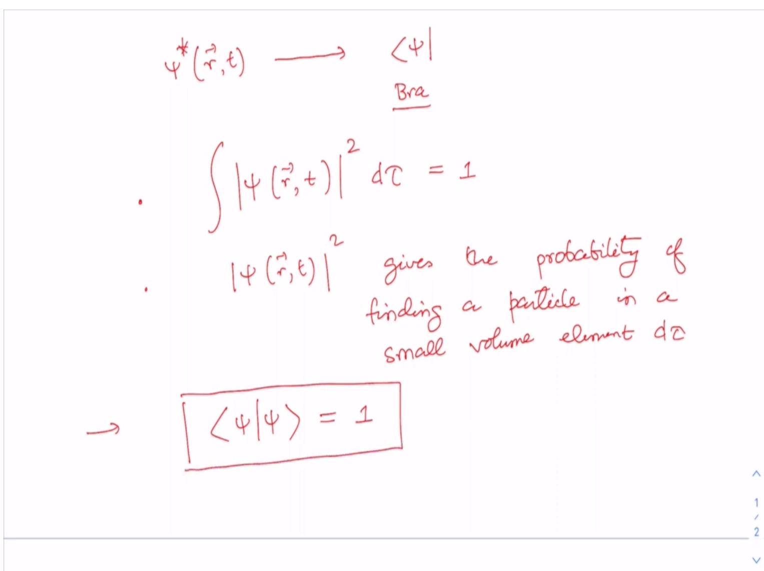

Also, it is known that this wave function if it has to be a valid representation of the system then it has to be normalized and that means this relation has to be satisfied that integral of modulus of psi of r,t d tau, d tau is the volume, that has to be equal to 1. And physically it means that the probability of finding the particle in the entire volume must be equal to unity and this quantity psi of r,t gives the probability of finding a particle in a small volume element d tau.

The corresponding normalization condition in Dirac notation would be simply this. This is called the inner product or scalar product and that is equal to scalar product of the state vector with itself and this is the normalization condition in the Dirac notation.

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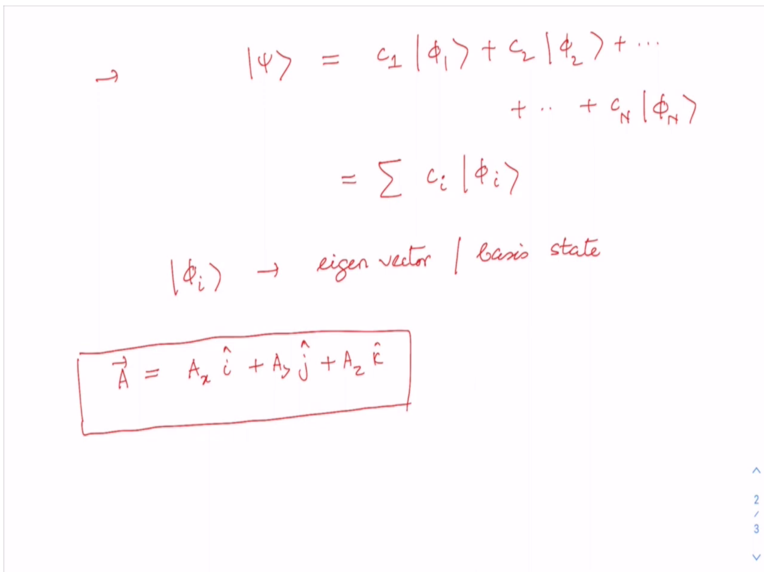

Now suppose before making a measurement a quantum system could be considered to be in any arbitrary state say ket psi. This state psi can be expressed as a superposition of eigenstates accessible to the system. So, we can write any arbitrary ket as a superposition of eigenstates like say phi 1 phi 2 and so on. If we are talking about a n dimensional system that would be cN phi N. For infinite dimensional system it will go on and will tends to infinity and this we can write it as a shorthand notation as Ci phi i.

Here phi 1 is called eigenvector or it is called basis state or basis vector. In fact to give you an analogy, as you know in classical mechanics or in vector analysis you know that if suppose we have a vector in three dimension this vector can be written in its component like this A x i cap, i cap is the unit vector along x direction, A y j cap, j cap is the unit vector along y direction and A z k cap and k cap is the unit vector along the z direction.

So, in three-dimensional space you need three unit vectors to define the vector in the similar spirit you can consider that in n-dimensional Hilbert space you need n number of eigenstates or the basis vectors to define a arbitrary state vector say psi.

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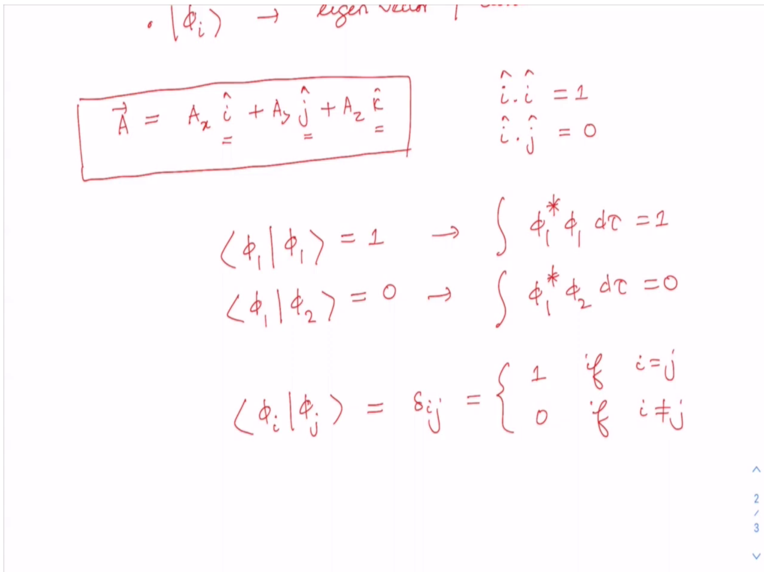

Now it is interesting that all these basis vectors or the eigenvectors for example again giving the analysis here if you take i dot i, i cap dot i cap is equal to 1. So, and similarly if you take the product of scalar product of 2 vectors i cap j cap then you will get zero. In the similar spirit again this vector phi 1 its scalar product with itself is equal to 1. So, that means the phi 1 is normalized but 2 difference states eigenvectors say phi 1 and phi 2 their scalar product will give 0 and in integral notation this would be simply phi 1 star phi 1 d tau is equal to 1.

And here it would be phi 1 star phi 2 d tau is equal to 0 and in shorthand notation I can combine both this as say phi i phi j the scalar product of 2 eigenstates would be simply delta ij and delta ij is equal to 1 if i is equal to j and it is equal to 0 if i is not equal to j. If a measurement is made the system self-collapse into one of the eigenstates. So, the probability would depend on this coefficient say c 1, c 2.

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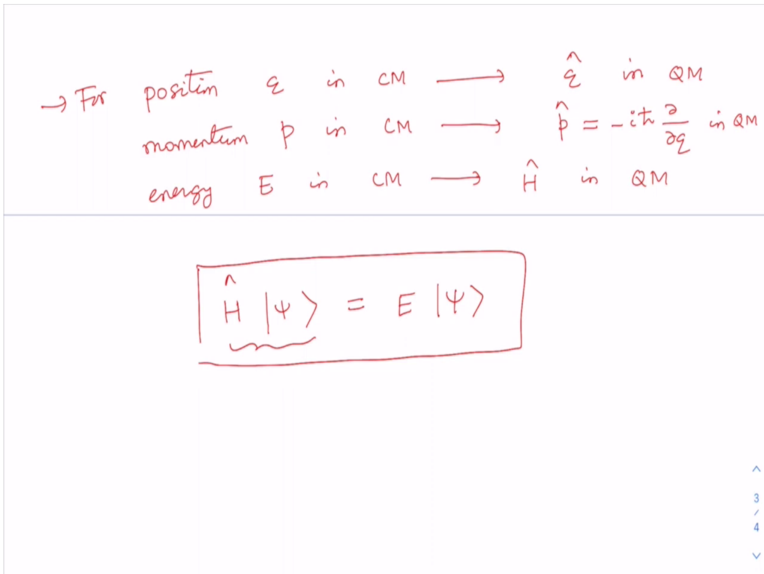

For example, say we are in an arbitrary state psi and if we make a measurement then the probability that it will collapse into the eigenstate say phi 4 it would be given by the probability c 4 mod square. So, in general the probability that, this is a very important aspect of the postulates, , system is collapsing into an eigenstate say phi i from psi due to measurement is expressed as say Pi is the probability that you are going from the arbitrary state psi to the eigenstate phi i and its probability would be mods of this square.

This scalar product mod square and if you calculate it you can show that this would be nothing but c i mod square. Another important postulate of quantum mechanics states that for every physically observable quantity in classical mechanics there is a corresponding operator in quantum mechanics. For example, if we have this position variable q in classical mechanics say for position q in classical mechanics we have the position operator q cap in quantum mechanics.

For momentum p in classical mechanics we have the momentum operator p cap in quantum mechanics and in coordinate representation we have it as minus I h cross delta q. And for energy E in classical mechanics we have the Hamiltonian operator in quantum mechanics and so on. As all the information are contained in the wave function or the state vector psi one needs to act on it by appropriate operator to get the relevant information.

For example, if I want to know the information about the energy of the system and when the system is represented by the state vector say psi then I have to operate on it by the Hamiltonian operator then I will be able to know the information about the energy. And generally, I end up with an equation of this type and then this equation is known as the eigenvalue equation. Physically speaking, what happens is that by this equation actually mean that when I am operating on the step vector psi by the Hamiltonian operator it amounts to making and measurement on the system to get information about the energy. So, it would be basically an energy measurement.

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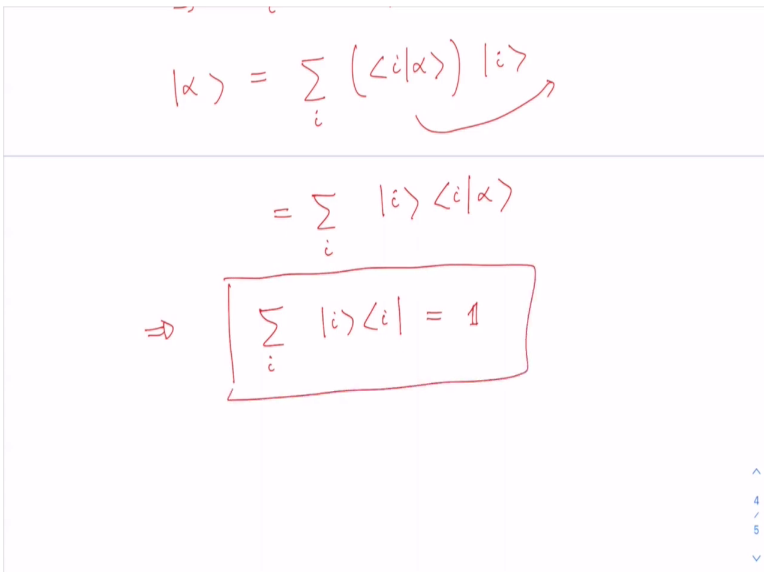

Whenever we have equation of this type say for a physically observable quantity A we have this operator A cap it operates on the state vector psi and if it pops out a value lambda which is a real quantity because the observable has to be measurable. So, lambda has to be real and the system remains or the state vector remains unaffected. Whenever this equation is there this kind of equation is called the eigenvalue problem or eigenvalue equation which I am sure all of you know this is the eigenvalue equation.

So, in the case of this Hamiltonian here you see lambda is here the energy. So, it is a well-known textbook problem to show that for the eigenvalue lambda to be real A has to be a Hermitian. A operator must be a Hermitian. Now another important postulate says that the time evolution of the state vector psi is given by the so-called Schrodinger equation and you know this Schrodinger equation is this one.

Time evolution of the state vector is given by the Schrodinger equation. And from this Schrodinger equation one can immediately write that the state vector at an arbitrary time t is equal to e to the power -i by h cross H of t psi of 0 and this particular operator, this is an operator, is known as the time evolution operator. One very useful relation in quantum mechanics is the so-called completeness condition.

Let me briefly remind you about it and it is very important relation, completeness relation or sometimes it is called closer relation also. As you know any arbitrary state vector state vector say Ket alpha in the Hilbert space can be expanded in terms of the eigenkets of a Hermitian operator say A then I can write it as a superposition of this eigenstates say i, ket i and these are the coefficients. Now if I multiply both sides of this equation by bra j. Let me do that. If I apply both sides then this is a number c i.

And I have here j i. Because these are now eigenvector or basis vectors. So, I can impose the condition that this is equal to delta ji and this is Kronecker delta and from here I get it as c j. So, this implies that this coefficient c i, I can write it as i alpha, inner product of i alpha. So, I can now write ket alpha as summation in the place of c i. Let me write this quantity, this is a number and I have it here because it is a number, I can take it to the other side.

So, if I do that I can write it as i alpha. So, this means it is very clear that this summation, this outer product of this eigenvectors, this is actually an identity operator, and this is known as the completeness condition.

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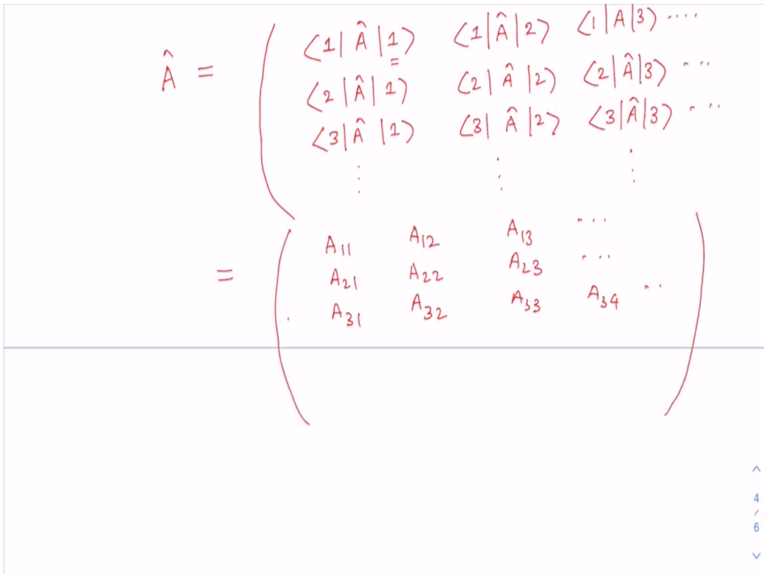

You can see the utility of the completeness condition when we try to represent an operator in the matrix form. Let us do that say. We have this operator A then I can multiply both sides of this operator by an identity. So, let me write the first identity as summation of this outer product. Sum of the outer products of ket i. Then I have this A and here it will be sum of the outer products of ket j. So, I can do that. Now I can write it as, if I just write only one summation but actually it is double summation.

So, in shorter notation I can write it in this way. So, then I have i A j j. Now you see this is just a number. So, I can write it here also i A j ij. There are all together N square of form i A j where N is the dimensionality of the ket space. So, we may arrange them into an N by N square matrix such that the column and the row indices appear as follows.

So, I have this form. Here, this would be row and this would be column. Let me illustrate it a little bit more clearly. So, we will have a N by N matrix for this operator A. So, we will have 1 A 1. So, this I can go on writing the first this brass side here represents the row first row and this ket psi represents the column. So, I have here 1 A 2 and then I have 1 A 3 and so on. And here I will have the second row A, this is the first column, this is the second row, second column second row, third column and so on.

And then here I will have third row, first column, third row, second column, and then you will have third row, third column and so on. So, we will have a total N by N, if it is N dimensional then you will have N square elements. This can be written in shortened rotation as A11, A12, A13 then you will have A21, A22, A23 you will have A31, A32, A33, A34 and so on. I hope you are getting the idea you will have a big matrix depending on the dimension.

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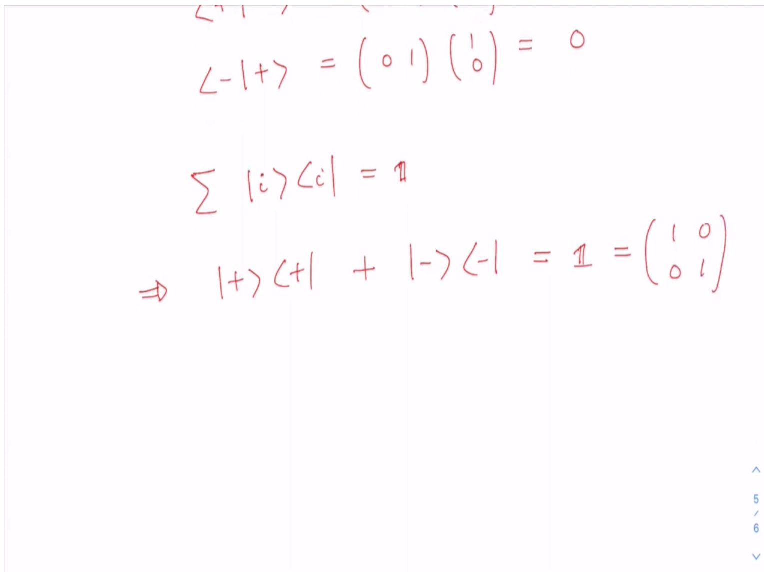

We will give you some example. Let me give you an example right away. Consider a spin-half system as an example consider a spin half system and you know a spin-half system is a 2-dimensional system because it has 2 eigenstates, one is the spin up state which is denoted by say in this notation ket up state or sometime people write it as plus ket. This sign represents that the spin is in the up state and because it is a two-state system,

so, this can be represented by this column vector 1, 0 and the other state is the spin down. Spin down state is represented in this notation and obviously by this minus ket minus here and the column vector would be 0, 1. It has to be like this because as you will see that these are the ket states. So, this has to be normalized. This is the eigenstate and this is also an another eigenstate. So, they have to be normalized and if you write it as 1, 0, this is the bra and it will be 1, 0 here and here it would be 1, 0.

This is the column and if you take the matrix multiplication immediately you will see you will get 1 and you will have minus minus then here you will have 0, 1 and here you have 0, 1 and this is also going to give you 1. On the other hand, if you take the scalar product of these 2 different eigenstates you can immediately see that it would be 1, 0 and this would be 0, 1 and if you take the matrix multiplication it will be zero.

So, this clearly shows that ket plus and ket minus are orthogonal. Similarly, you will have minus plus is going to give you here to be 0, 1 and here to be 1, 0 and you will again get 0. So, what about the completeness condition? Recall that the completeness condition was this. So, in this case we have it is 2-dimensional. So, the ket vectors are plus plus and you have minus minus and this would be identity matrix and you can check it that you will get actually 1, 0, 0, 1 identity matrix. 2 by 2 matrix you will get.

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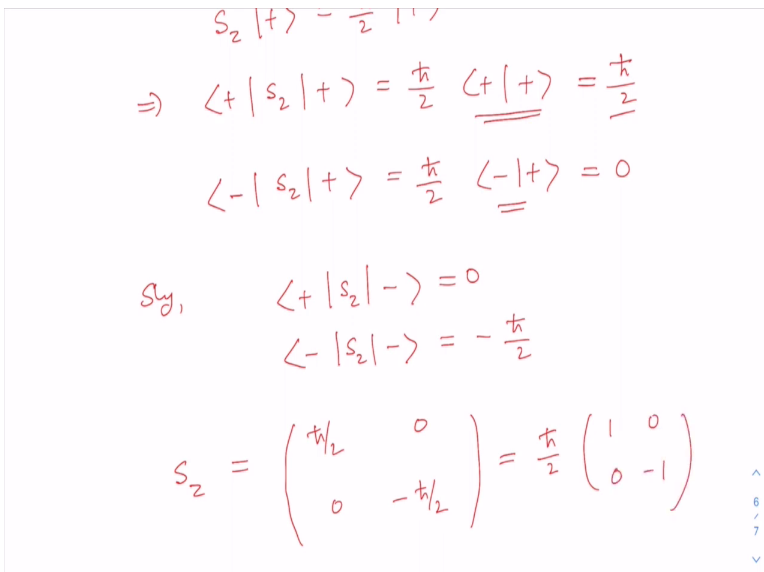

Suppose we have an eigenvalue equation for the spin half system such that if I have an operator say S z it if it operates on the upstate then it gives us the eigenvalue h cross by 2 plus and if it operates on the eigenstate minus that is the down state, we get the eigenvalue minus h cross by 2 Then, we can very easily express S z. So, now we can express S z in the matrix form. So, it would very easy because we have it as a 2 by 2 matrix.

So, elements would be you have plus S z plus and here you will have, this is your row, this is your row, this is column. So, this is the plus S z minus and here you will have minus S z plus and you will have minus S z minus. And now you can work it out because we have S z plus equal, when it operates on it, we are having h cross by 2 plus. If I multiply both sides by the bra plus here then I will get S z plus and here it would be h crossed by 2. This is what I will get and this is normalized.

So, you will have simply h cross by 2. Again, if I do minus S z, if I multiply it by the minus of this bra, then what I will get? You will get h cross by 2 minus plus and this is orthogonal. So, therefore you will ket 0. Similarly, you can show that you will have say plus S z minus that would be equal to zero and you will have minus S z minus this will give you minus h cross by 2. So, therefore the matrix representation of this operator would be then, here you will have h cross by 2, here it will be 0, 0, minus h cross by 2 which also you can write it as h cross by 2, 1, 0, 0, minus 1.

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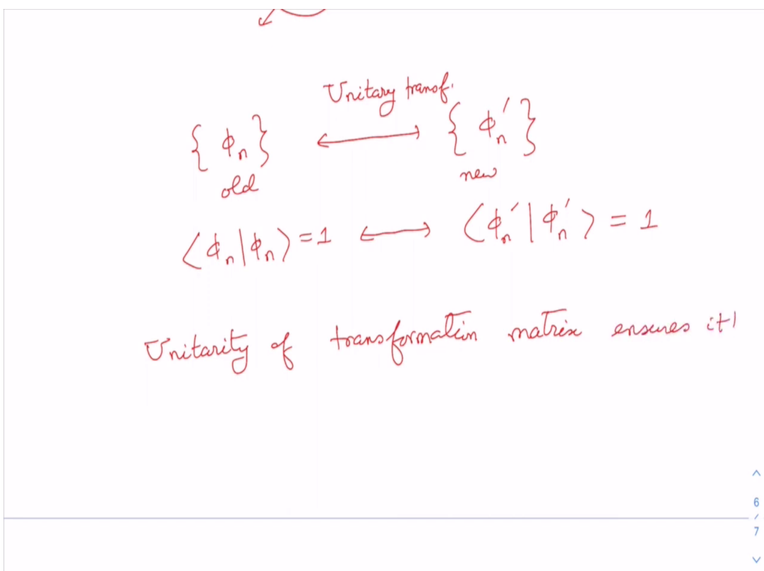

Many a times we would prefer to work in a particular basis states over that of the other ones. Just like in classical mechanics sometimes using spherical polar coordinates is more useful or easy to work on than the cartesian coordinate system. For example, in classical mechanics whenever in a particular problem there is spherical symmetry in the problem then we prefer to use the spherical polar coordinates say r theta phi rather than Cartesian coordinate x y z.

And we can go from cartesian coordinate system to the spherical polar coordinate system by the so-called coordinate transformation. In the similar spirit in quantum mechanics we can go from the basis state say phi n to a new basis states phi n dash or vice versa and we can do that by using a transformation called unitary transformation. By unitary transformation we ensure that the probability is conserved that means that we must have these conditions that scalar product has to be equal to 1 which is, you know, the normalization condition.

And while we go from this say old coordinate to new coordinate, this has to be maintained. The scalar product has to be equal to 1. So, this is what we mean by unitarity and this is ensured by unitarity of the transformation matrix.

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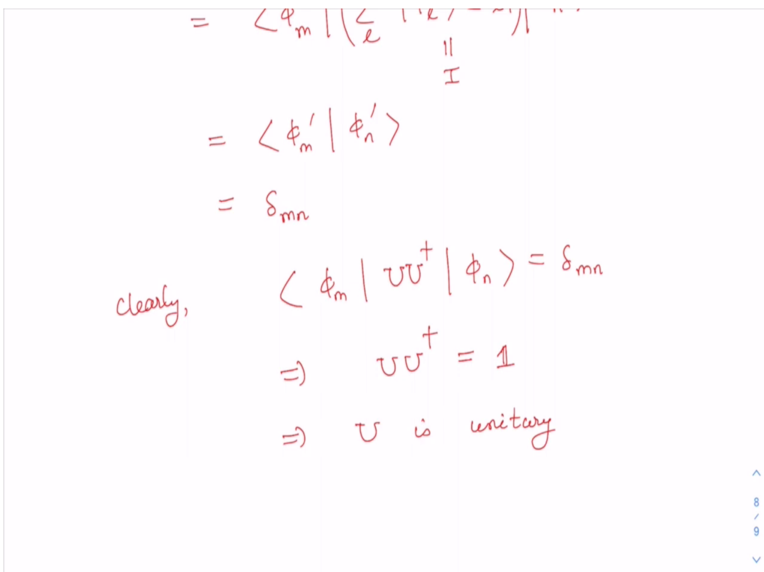

Let me elaborate it. Let me write say, we have this eigenstate phi n which I can write because of the completeness condition. Now I can write it as phi m dash phi m dash these are basis states and this is what I have. I can always do that because I know that this is the identity operator. This I can write slightly differently. I can write it because you see this is just a number. So, I can write it as phi m dash phi n phi m dash.

And if I write it as summation over m this is if I write it as Umn phi m dash where Umn is the matrix element of the unitary operator where I go from the old basis state phi n to a new basis state say phi m dash. Let me show that U is unitary. To prove it I just need to show because U is a unitary matrix, so, you know that a matrix is unitary U is unitary provided U U dagger is equal to identity operator. U dagger is the Hermitian operator corresponding to the operator U.

So, now let me prove it. I have U U dagger. If I multiply, if I take this relation let me work it out phi m phi n if I can actually prove that this is equal to delta m n then it will ensure that U is unitary. I will show this later. I can write it as phi m and utilizing the completeness condition let me put here another basis state say phi l like this and we have U dagger phi n. This then I can write it as summation over l. I can write it as U ml U star of nl where I defined U ml is equal to this is the matrix element of this unitary operator U phi m phi l and U nl star is equal to phi l U dagger phi n which is actually phi n U phi l if I take the complex conjugate.

Now we have U ml. This is going from the old basis states say phi l to going to the new basis state phi m dash as per our definition which is here, this one you note, utilizing that I can write U ml like this and U nl star I can write it as phi l phi n dash. So, therefore I can write sum over this quantity, I can write it as U ml U nl star is equal to this. I have phi m dash phi l phi l phi n dash and as you can see, let me write here I can write it as phi m dash summation over l phi l phi l and here I have phi n dash and this is identity operator.

So, I will have phi m dash phi n dash. So, this is the scalar product in the new basis and and we know that this is nothing but Kronecker delta mn. So, clearly, we now have therefore proved that phi m U U dagger phi n is equal to delta mn. What it means that that this matrix U U dagger is equal to identity operator and which means that U is a unitary matrix. U is unitary. If some of you are finding this particular portion difficult please do not worry we will work out a set of example problems in the problem-solving session which should surely boost your confidence. Let me stop here for today. In the next lecture we will start the course please note that you will get some supplementary materials time to time. These are there to help those students who have forgot the basics or those who want to understand certain topics more clearly. Also, please note that if I make some mistakes in the lecture materials they would be rectified in the problem-solving sessions. I hope you will enjoy the course see you in the next lecture, thank you.