Solar Energy Engineering and Technology
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Lecture No. 05
Geometry, angles and measurement - I

Dear students. Today, we will discuss solar radiation geometry. So, before we start solar radiation geometry let us summarize what we have discussed in the last lecture. So, we are discussing about different instrument used for measurement of solar radiation and duration of sunshine hours.

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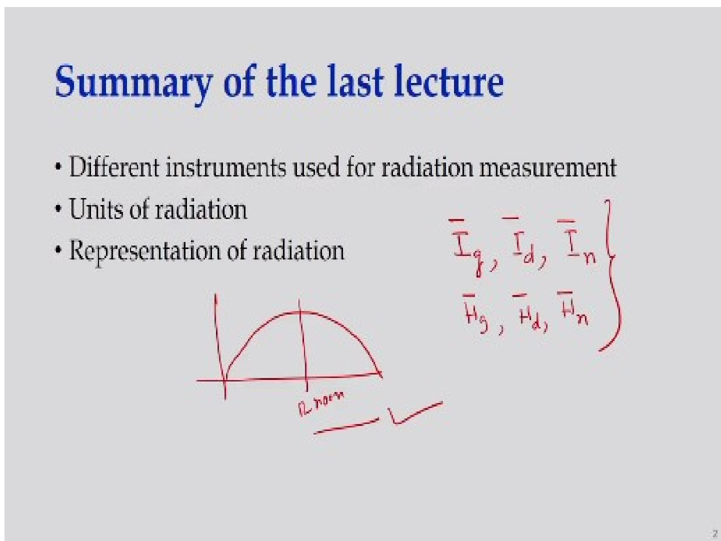

So, we have learned how these different instruments work. So, for measurement of normal radiation we have to use pyrheliometer and for measurement of global and diffuse radiation we have to use pyranometer.

And for measurement of duration of brightness of the sun we have to use sunshine recorder. So, these 3 instruments are very, very important for measurement of radiation parameters. Also we have studied the different units like langleys, what is W/m^2. So, how these parameters are represented if we are dealing with instantaneous values? We will use only I_g if we are interested for global radiations.

Then I_d in case of diffused radiations, then I_n is for normal radiation. And if we are interested about hourly average then we will put a bar on the top of these parameters. And sometimes we are also interested to know the variation of radiations in daily basis. So, under that conditions what we are going to use is H_g, H_d then H_n. So, again if we are interested to know the average values say for example monthly average value if we are interested.

Then monthly average of daily radiation if we would like to know then we will use a bar on the top of it. So, these parameters we have learned in the last class. Also we have studied the radiation patterns on a clear sky and on a cloudy sky. So, in cloudy sky, lot of peaks and valleys are observed, but in case of clear sky or cloudless sky, so these variations is something like like this. So, normally maximum radiation is observed at the noon. So this is about 12 noon.

So, these parameters were studied and also we have studied the radiation map. So, sometimes radiation maps are used to know the radiation spectrum of a particular region. So, we have studied for example for the country like India.

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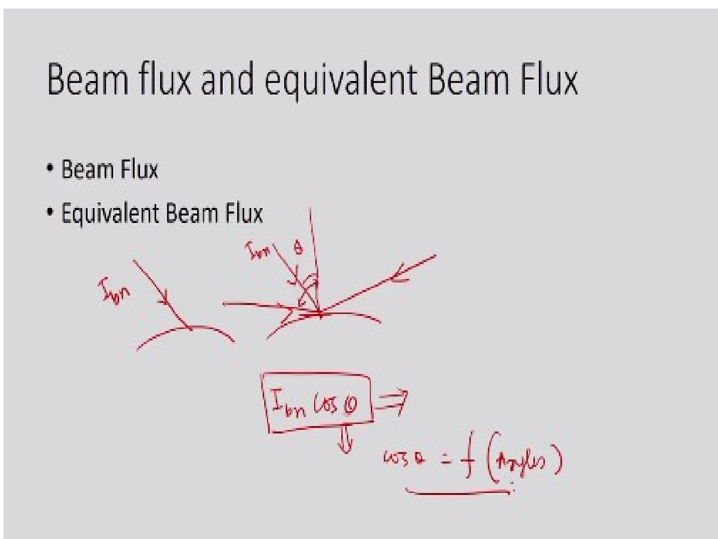

Now come to our today's lecture, so radiation geometry we need to understand very clearly, then only we can do lot of calculations which is required for estimation of global radiation, diffused radiation or normal radiations. So, first thing we must know what is beam flux and equivalent beam flux? So, if we have to know this beam flux, suppose this is the Earth surface and solar radiation is coming.

So, solar radiation may come from any direction, so this may be I_n. This is the beam flux or I should write I_{bn} I_{bn} is a beam flux; b stands for beam radiation. So, this is the beam radiation falling on a particular place. So, if I am interested to convert the equivalent beam flux then what we need to do? Then this is the orientation of the radiation and then we have to draw a line which is normal to the surface.

So, let this angle be θ and this is I_{bn}. So, equivalent value will be equivalent beam flux will be I_{bn} cos θ. So, for all the calculations we need to do this. So, this is nothing but this is normal to the surface. So, solar radiation may come from this side also, maybe from this side also. So we have to convert it with respect to the normal to the surface. So that's why it is called equivalent beam flux.

So, this theta is a functions of many angles like declination, tilt, solar azimuth angle, altitude angle and so on. So, we need to understand how this cos θ is varies with respect to different angles.

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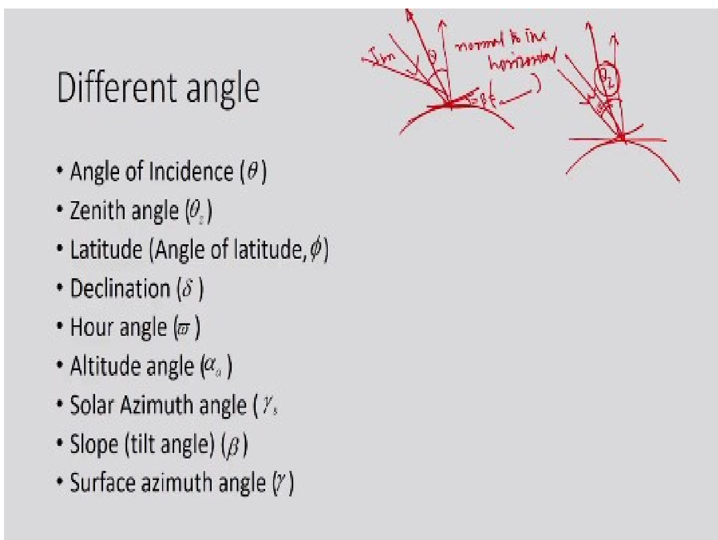

So, let us see what are different angles are present. Angle of incidence this θ what we have studied in the last slides. So, this angle is something like this is the I_{bn}, this is the surface, this is the solar radiations and this angle is θ. So, for example here what we are considering is the horizontal surface. So, this is the beam radiation falling from the Sun and is striking on the surface which is horizontal and this is your normal to the horizontal. This is normal normal to the horizontal normal to the horizontal.

So, there are some cases where we have to study when it is inclined. So inclined at an angle say β and β may vary, β may vary. So, we will learn how it varies. So, this angle is β, this is known as slope or tilt angle. So, first case what we have studied that is horizontal surface and second case, it is inclined surface. So, if it is inclined then our things will be something like this. This is the normal and solar radiation maybe here.

So, this is your normal to the horizontal surface. And already we have studied what is zenith angle. If it is horizontal surface and its normal to the horizontal is this and solar radiation is falling here, so this angle is θ_z. So, if we consider a inclined surface and if solar radiation is falling here and normal to the inclined surface is something like this.

So, this angle will be θ, but when we are concerned about normal to the horizontal and when this Sun beam makes an angle with the normal to the horizontal, that is called zenith angle, so this is θ_z. These things should be clear when we draw many angles and with time you will understand the importance of those angles and how these angles are different and latitude.

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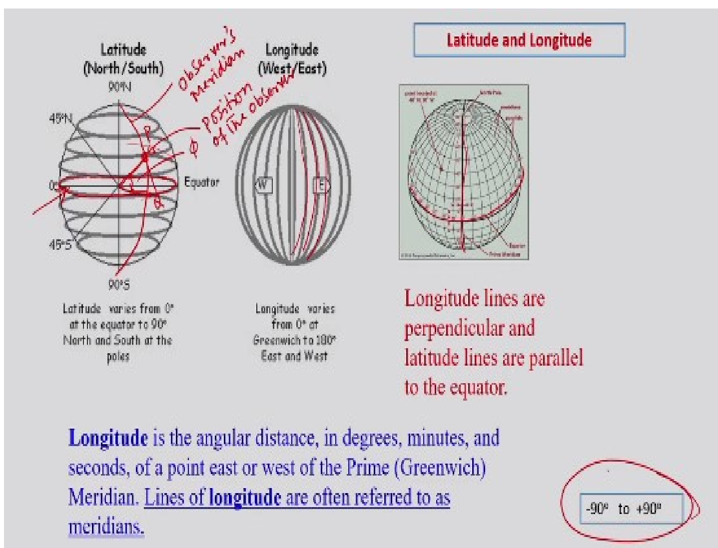

Let us learn what is latitude? So, this slide shows different pictures to represent latitude and longitude. So, this parallel planes these are the parallel planes, parallel to the equator are latitude and if it is perpendicular to the equator are longitude. So, this latitude and longitude are used for coordinate system. Of course we need elevation as well to specify the coordinate completely.

So, here if we have to define latitude say for example, let us consider a place here. I am interested to know the latitude of that location then what I will do? I will draw a line from this location to the center of the Earth and we will project it project this line on the equatorial plane. This maybe our equatorial plane now. So this will be something like this and this angle will be phi. Of course we can write this and we can have this observer's meridian.

This maybe observer observer's meridian or meridian and this is the position of the observer maybe position of the observer. So, I hope you understand this. So, this is the position of the observer and we will connect the center of the Earth to the position of the observer and we project it on a equatorial plane. This maybe P I can say, this maybe Q and this angle is ϕ. So, this ϕ is nothing but latitude of the location.

So this ϕ we need to calculate in many of the occasions for calculations of many radiation parameters. And also we can see these lines, these lines are longitudinal lines. So, as you can see this, this is the prime meridian where our longitude is 0 and then we can see the coordinate system and this is the equatorial plane what we have explained here this is the equatorial plane and this ϕ varies from -90 to +90. This is the variation of ϕ.

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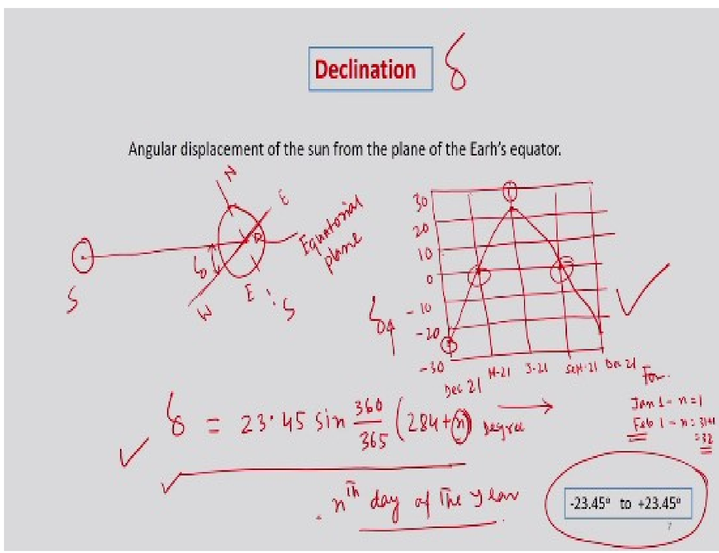

So, let us discuss the other angle like declination. So, what declination signifies or defines like the angular displacement of the Sun from the plane of the Earth's equator. So, if we have to present or we have to define, let us draw this sun here and maybe Earth here and we will connect the center and maybe we will draw this as north, south and of course we will have east and west here and this is this is equatorial plane.

This is equatorial plane and if here if we have to define declination δ, so what we do? This is Sun, this is Earth. We connect the center of the Sun and center of the Earth and this line will project on the equatorial plane. So, this angle is δ. So, when we connect center of the Sun and center of the Earth and that line will project on the equatorial plane and that angle is known as δ or declination.

So, declination is very, very important and how this declination varies we will discuss now. So, for example, if we have to show the variation of declination throughout the year that is variations in such declination we can study. So, maybe we can show it here 0 then we have this maybe -10, this maybe -20 and this maybe -30 and here we will have 1, 2 and 3 this is maybe 10, this is 20, this is 30.

And we will have this maybe June this is maybe December 21 and we need 1 then we have 2 and 3 we need 4. So, how this will vary? December 21 then we have March I will write M 21, this maybe June 21 then we have September 21 and we will have again December 21. So, declination will vary, so this is delta and this is the month and this will vary. So, this will be -23.45 here 23.45 will be somewhere here and in March 21 it will be 0.

And June 21 it will be +23.45, it will be here somewhere and in September 21 again it will be 0 and because these two are equinox; winter equinox and summer equinox. So this is winter equinox, this is summer equinox and here again it will be -23. So, if we plot these variations, this will come something like this. So variation of the Sun's declination is something like that.

So if we have to find out the declination of any day, we can calculate it by using δ is $23.45\sin\left[\frac{360}{365}(284+n)\right]$, n is the n^{th} day of the year. So, we can use this plot or we can calculate by using this δ if we know which date we are doing the calculation. So, n is the n^{th} day n^{th} day of the year of the year. So, as you can see from this plot, this variation of δ is varies from -23.45 degree which appears in December 21.

And maximum is +23.45 which appears in June 21. So, that's why this variation is from -23.45 to +23.45. So what we can summarize here, so declination is very, very important. It defines the angular displacement of Sun from the plane of the Earth equator. So, this is the Earth equator, plane of the Earth equator, this is the δ and this is the angular displacement. And also another way we can define, this is the angle between the line joining the center of the Sun and the center of the Earth and projection of that line on the equatorial plane.

That angle is nothing but δ or declination and declination varies from -23.45. So, -23.45 appears in December 21 and maximum is found to be +23.45 which appears on the day of June 21. And also we can calculate declination and this will be in degree or I should write in degree degree.

So, $\delta = 23.45 \sin\left[\frac{360}{365}(284+n)\right]$. So, n is the n^th^ day of the year. So, for example for example if we have to calculate the δ for say January 1. Say January January 1, so n will be 1. So under that condition, we can calculate what will be the value of δ or for example if we are interested for February, February 1; then what will be n? n will be (31+1), it will be 32. So, if we substitute the value of 32 here in place of n, then we will get the δ for February 1. So, that's how we can calculate the declination on a particular day.

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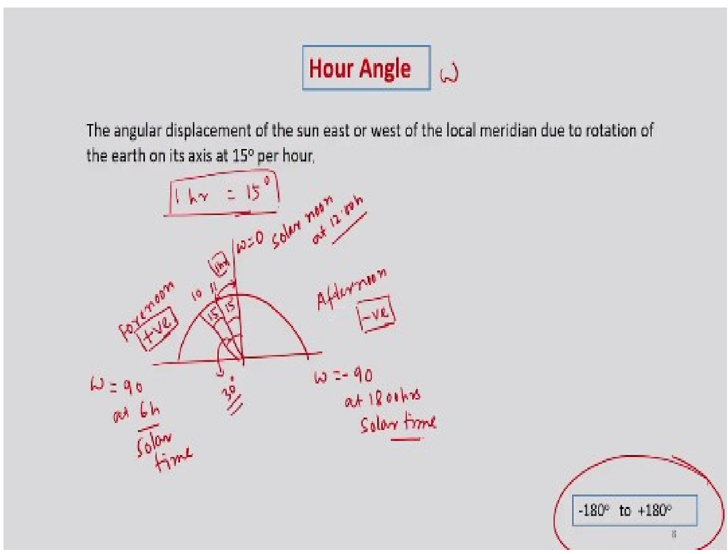

Now let us study hour angle. So, hour angle is very, very important. So we need to calculate local apparent time for calculation of hour angle. So hour angle we represent like ω. So, it represents the angular displacement of the Sun, east or west of the local meridian due to the rotation of Earth on its axis at 15° /hour. That means it signifies like 1 hour is 15° or 15° represents 1 hour.

So, if we plot it like this way and maybe we can write this and I can write a line here. So this is ω is 0 and this is called solar noon at 12:00 hour and if we divide maybe this is 11 or this maybe 10, so this angle is 15°. When this difference is one hour so one hour is 15° this is also

15°. Total hour angle will be here this angle will be 30°. So, this way we can measure and it measures the angular displacement of the Sun.

East or west of the local meridian due to rotation of Earth on its axis at 15° /hour. So one hour is 15°. We will do lot of calculations how to calculate ω and what happens in case of sunset and what happens in case of sunrise? So, all the things we need to calculate and here we can say ω is positive in the morning. This part is forenoon, this is forenoon and this is afternoon.

In case of forenoon ω is positive, for afternoon ω is negative. We normally this sign conventions normally considered and here ω is +90 at 6:00 hours solar time and this will be ω will be -90 at 18:00 hours and this is also solar time. So, what we need to learn here that is solar hour angle is very, very important for solar calculations and 1 hour is 15°.

And how we have defined this hour angle? Like it is a angular displacement of the Sun and it varies from -180 to +180. So, we will learn details when we solve some numerical problems related to this hour hour angle.

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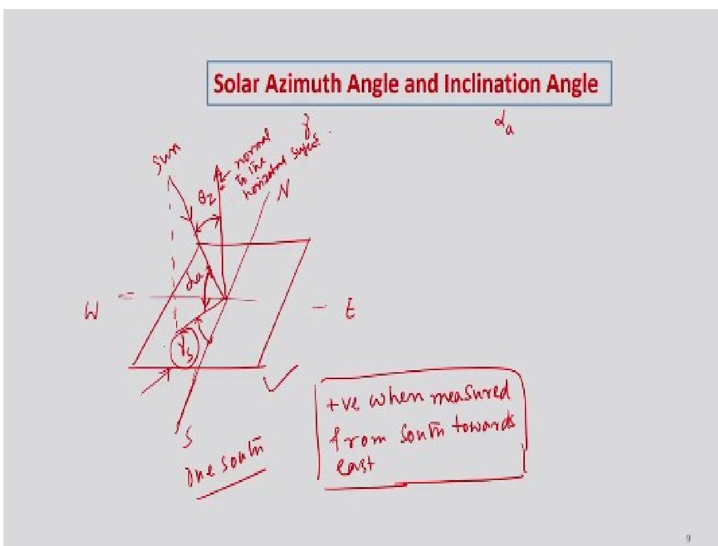

So, now let us learn something about solar azimuth angle and inclination angle. So, this solar azimuth angle is represented by γ_s and inclination is α_a and this is related to horizontal surface. Let us draw one horizontal surface something like this and maybe we will find out the center here. This is north, this is south, east and then we have west. So, normal to this horizontal surface is something like this and solar radiation is falling from here.

So, this angle is always known, this is zenith angle θ_z. Please remember that this is horizontal surface so we are not talking about inclined surface. This is a horizontal surface and normal to the horizontal is this, this is normal to the normal to the horizontal surface horizontal surface. And this is the incident radiation incident beam radiation. So, if this is projected here and of course we we need not to explain to this.

So, maybe we can extend this somehow and this angle this angle is γ_s, that is solar azimuth angle. So, this angle is angle made in horizontal surface, this is due South this is due South due South. So, this is the angle in the horizontal plane. So what happens this is Sun so for Sun beam radiation is falling on this horizontal surface and we have drawn this normal to the horizontal here, this is normal to the horizontal and this angle is always zenith angle.

And then if we project this beam radiation on the horizontal plane and this angle is this projected line and the line towards due South, this angle is γ_s and this angle is inclination angle or α_a. So, this is the angle on horizontal plane between the line due South and the projection of the Sun's ray on the horizontal plane. This is the horizontal plane and it is positive when it is measured from south or west.

So, it is positive when when measured from south towards east. So, please keep in mind that these things are happening in horizontal surface. So, we are not talking about the inclined surface. So, when we deal with inclined surface then we will talk about surface azimuth angle. So, this angle γ_s is the angle on a horizontal plane between the line due South and the projections of the Sun's ray on the horizontal plane.

So, this is important and this angle is inclination angle. So, these informations are also required while calculating these solar radiation parameters.

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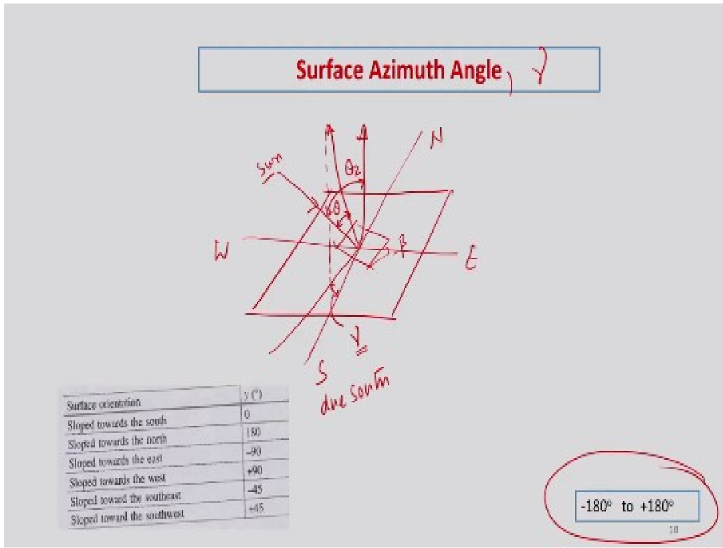

Now, let us learn about solar azimuth angle, so this azimuth angle is important because most of the time we will be dealing with this γ and this γ is nothing but surface azimuth angle. So, this is related with inclined surface. So, let us make an horizontal surface and maybe we will take some kind of this and maybe we can have this kind of inclined surface. So, this maybe this angle is β.

So, normal to the horizontal we can draw this way and this normal to this inclined surface will be something like this. So, solar radiation normal solar radiation is falling here, so this angle is θ, but this angle is θ_z. So, now I think you can understand what is the difference between θ and θ_z. Now, if we project this normal to the horizontal surface on this horizontal plane and this angle is γ.

So, how we can define this? So this is Sun, so it is the angle in the horizontal plane, this is the horizontal plane between the line due South, this is due South because this is N, this is S, this is east and this is west. So this is due South and the horizontal projection is a projection of the normal of the inclined surface. So, this is the normal of the inclined surface and this is the projection. So this angle is γ which is nothing, but solar azimuth angle.

So, this angle is very, very important because most of the time we are going to use this gamma for our calculations. And variation of γ can be seen, its -180 to +180. Also there are certain cases like slope towards the south if we say slope is towards the south, then value of γ will be 0 and slope towards the north will be 180, then slope towards the east is -90.

Slope towards west is +90, then slope towards the southeast is - 45°, slope towards the south west is +45°. So, these are different angles we need to understand and we can use it for reducing calculations.

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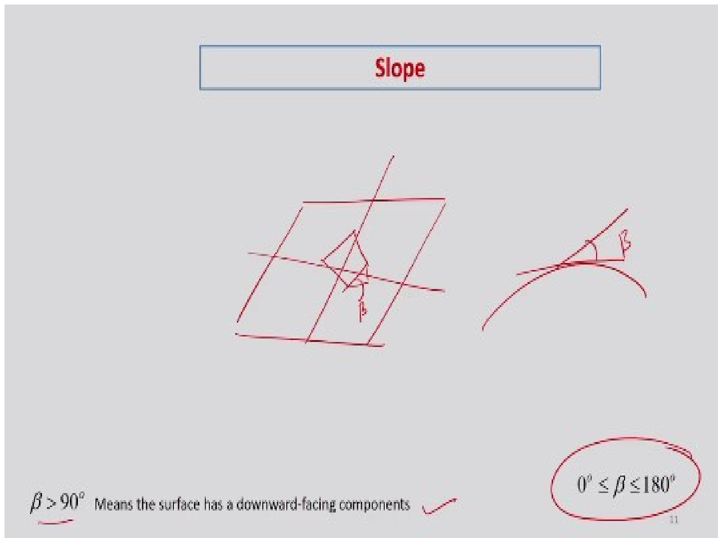

So, slope is already defined. So what is slope now? So, if we maybe we can draw again this surface and maybe horizontal surface and if we draw this kind of systems, so this angle this angle is β or maybe if we draw this kind of system if we consider horizontal surface and maybe inclined surface. So, this angle is β so this is β. So, it varies from 0 to 180 and sometimes its β > 90.

That means the surface has a downward facing components. So this slope also we will learn and let us combine all the angles and let's see how it look like.

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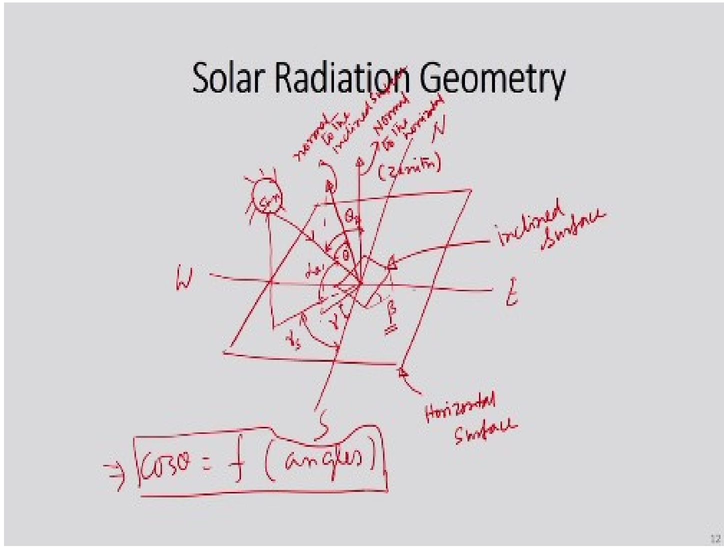

So, again I will draw one horizontal surface and we will write this is the north, this is south, east and west. And we will draw this draw this and that makes an angle this is an angle so this if we see this is the angle this angle is β and this is normal to the horizontal and we will have normal to this inclined surface. This is normal to the inclined surface and we will have solar radiation will fall here and already you know this angle is θ.

I will write here and this angle is θ_z, cos θ_z is zenith angle and θ is incidence angle. So, incidence angle already we have defined and if we project it on the horizontal plane and we will write here. So, this angle is azimuth surface azimuth angle γ and if we project it, this one here on this plane and the kind of angle what we will get this is solar azimuth angle and this is α_a.

This is your altitude or inclination angle α_a. This is Sun or we can write this way because Sun is radiating this beam energy and is falling on this inclined surface. So, now we can learn the different angles like in this surface we have introduced many angles like β then θ_z, θ, α_a, γ, γ_s. So, now what we will study, we will study the cos θ, how this cos θ is a function of many angles.

So, we are not going to derive the expression how this angles are related with cos θ, but finally we will use the result of this geometry. So, how this cos θ is a function of different angles? So, as you understand so briefly what we can discuss in this slides, so this is a horizontal surface. Let us let me write this is horizontal surface horizontal surface and this is inclined surface inclined, this is inclined surface. And this is normal to the horizontal this is normal to the horizontal horizontal surface and this is normal to the inclined surface normal to the inclined surface inclined surface.

So, also for your understanding and this is zenith. So, these are different angles involve and this is the inclination, β is slope and θ is zenith and θ_z is zenith angle and θ is incidence angle and α is altitude angle, then γ_s is solar azimuth angle and this γ is surface azimuth angle. Now, I will relate this θ with respect to all these angles.

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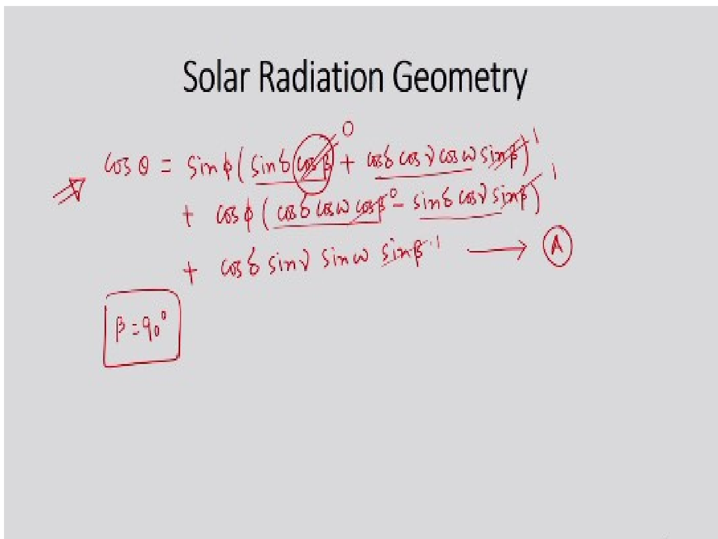

So, how this can be expressed? I will I will write

$$\cos\theta = \sin\phi(\sin\delta\cos\beta + \cos\delta\cos\gamma\cos\omega\sin\beta) + \cos\phi(\cos\delta\cos\omega\cos\beta - \sin\delta\cos\gamma\sin\beta) + \cos\delta\sin\gamma\sin\omega\sin\beta$$

So, this is an expression which relates cos θ with different angles. So sin ϕ then cos we have this and then we have. So this is maybe equation A. So, every time we need to do this kind of calculations if we know this δ, β, ϕ and ω, then we can substitute here and we can calculate what is cos θ. Because for all the cases, we need to find out what is cos θ, that is equivalent beam flux. This has to be calculated.

So, now we will study different cases like we can say these are special cases. What happens if β is 0, what happens if γ is 0, what happens if β is 90°? So all those cases we will study and then later on, we will solve numerical problems to strengthen your understanding. So, we will study some cases, maybe in case 1, for vertical surface. So, in case of vertical surface that is beta is 90°.

So, if we substitute β value 90° in equation A then what will happen? So we will go back to the slide and if we substitute β is 90°. So, if we substitute β 90° so here I can substitute here so cos β will be 0 and sin 90 will be 1 so this will be 1. So, this will retain and here and this will be retained, so this will be 1 and this will be 0, this part will be 0 means this expression will be 0, this will be 0 and then this will be 1.

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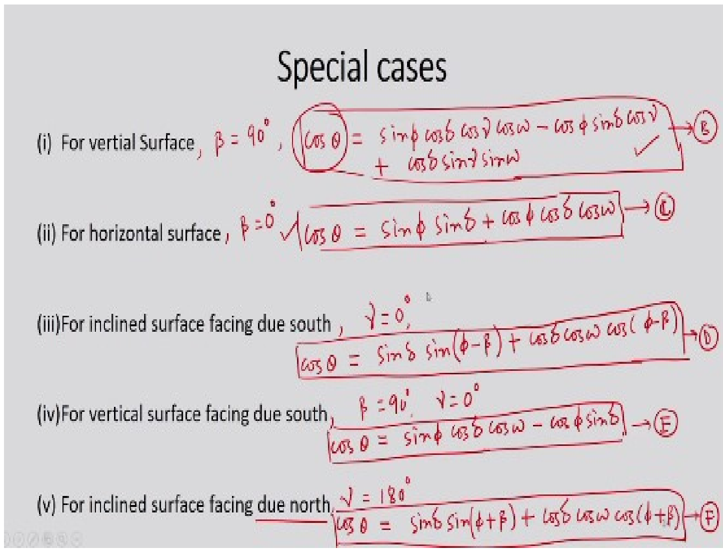

So, if we simplify then what expression we will get here; so I can write if β = 90 then cos θ will be it will simplify to a expression of something like this. $\cos\theta = \sin\phi\cos\delta\cos\gamma\cos\omega - \cos\phi\sin\delta\cos\gamma + \cos\delta\sin\gamma\sin\omega$. So, this will be the expression. So, for a vertical surface, β is 90^o. So we can use this expression to calculate cos θ.

This equation maybe we can name as B and for horizontal surface where β = 0. So, again we can substitute the value β = 0. So, β = 0 means here it will be 1 cos 0 one then this part will be 0, this part will be 0 again and this part will be 0, this part will be 0 and this part will be 1. So, if we simplify then what expression we will get, we can write here $\cos\theta = \sin\phi\sin\delta + \cos\phi\cos\delta\cos\omega$. So this is equation C so this is the equation C.

So, if the surface is horizontal then we can use this expression to calculate cos θ. For inclined surface facing due South, so due South means what we can do we have γ = 0 and inclined surface maybe at certain angle β, so only γ will be 0 here. So, if you substitute γ then we can simplify this and we can get an equation of something like this, sin of or get an expression of something like this. $\cos\theta = \sin\delta\sin(\phi-\beta) + \cos\delta\cos\omega\cos(\phi-\beta)$. So, this expression we can name as equation D. Now, come to the case 4, for vertical surface facing due South, so vertical surface means β = 90^o and due South is γ = 0 0^o 0^o and β degree here. So, if we substitute this value, then this equation or maybe equation B will simplify something like $\cos\theta = \sin\phi\cos\delta\cos\omega - \cos\phi\sin\delta$.

So, we can name this equation as E. So in the last case or there are many more cases so as per my formulation. So for inclined surface facing due South, so here what happens we have γ is 180°. So, this is facing due North, so our expression will be $\cos\theta = \sin\delta\sin(\phi+\beta) + \cos\delta\cos\omega\cos(\phi+\beta)$.

So, this equation will be something like this. So, this will be F. So, these are the special cases we have formulated. So at different conditions, we can use different expressions. But straight away we can use the original equations this equations which is good for all the cases just we need to simplify. We need to consider say for example vertical surface β is 90, for horizontal surface β is 0.

If it is due South, then γ is 0 degree so that way we need to consider and we can simplify this expression and we can get different equations. So, once we simplify it then it is very easy to calculate the value of cos θ.

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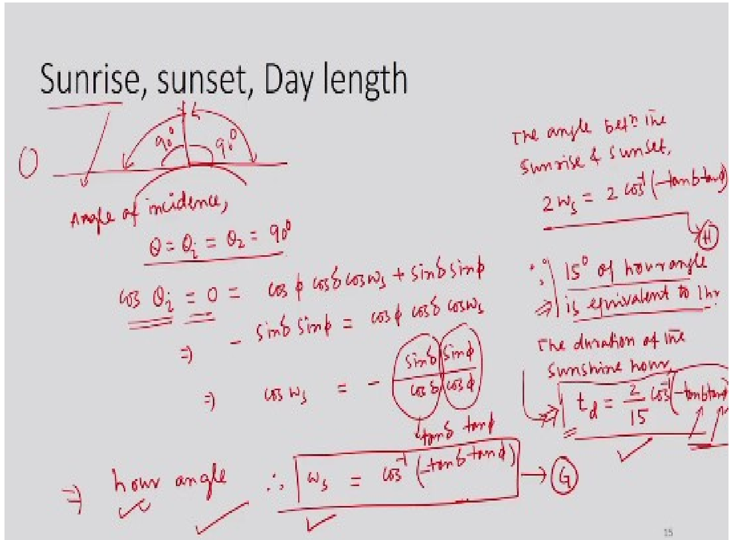

So, now let us discuss about sunrise, sunset and day length. So, at sunrise, the Sun's rays are parallel to the horizontal surface. So if this is the case if we take this Earth. So, when Sun is here, sunrise is taking place. This should be something like this. So, it is something like parallel to the horizontal surface. So, under that conditions, angle of incidence will be so for this sunrise, angle of incidence angle of incidence that is θ or θ_i.

I can write θ or θ_i or so maybe θ_z will be 90°; because this will be 90°. So, for sunrise, the sun rays are parallel to the horizontal surface and hence θ or θ_i or θ_z is equal to 90°. So, if we substitute this value in the original equation A, which is functions of all the angles, so what we will get, we will get something like $\cos\theta_i = 0 = \cos\phi\cos\delta\cos\omega_s + \sin\delta\sin\phi$.

So, how we can simplify now? So we can take sin δ, sin ϕ in this side and then we can keep cos of ϕ, cos of δ, cos of ω_s on other side and we want cos of ω_s which will be minus sin δ sin of ϕ and then we will have cos of δ. We can interchange multiplicative and cos of ϕ. So, this part sin δ by cos δ and sin ϕ by cos ϕ is nothing, but tan δ and tan ϕ.

So, now $\omega_s = \cos^{-1}(-\tan\delta\tan\phi)$. So, we can calculate what is ω_s which is nothing, but hour angle. So, what we have calculated this is hour angle hour angle. So, this expression maybe, we can write G. So, this angle between the sunrise and sunset will be how much? So, this angle is 90 and again sunset, it will be 90 it will be 90°. So, if it is ω_s for the sunrise, from here to here then it will be just $2\omega_s$.

So, the angle between the sunrise and sunset will be $2\times\omega_s$. This will be $2\times\cos^{-1}(-\tan\delta\tan\phi)$. So, this maybe we can write equation one equation number GH, I can write here. So, since we know this 15° of hour angle of hour angle is equivalent to equivalent to 1 hour, then what we can write, the duration of the duration of the sunshine hour will be, so if we represent t_d, this will be $t_d = \frac{2}{15}\times\cos^{-1}(-\tan\delta\tan\phi)$.

So, this is the duration of the sunshine hour. So, if we know δ as declination and latitude of a location, then we can calculate what is t_d which is nothing but duration of the sunshine hour. So, this is very, very important findings. So by using this equation, we can calculate the duration of the sunshine hour. So, what we have discussed in the slides, we tried to calculate the day length.

So, while doing so, what we have done we have done angle of inclinations is 90° for sunrise and sunset. So, if we substitute the value of 90° here in the cos θ, cos θ_i. So, we can simplify this and finally what we can calculate is the hour angle for sunrise and same expression hold good for sunset as well and also we know this 15° of hour angle is equivalent to 1 hr.

So, this knowledge we can apply and we can calculate what is the duration of the sunshine hour. So, as per calculation, it is found to be something like this. Like $t_d = \frac{2}{15}\times\cos^{-1}(-\tan\delta\tan\phi)$. So, let us summarize here about this and in the next class, we will discuss some of the numerical problems related to the content we have covered in this present video. So, thank you for watching this video.