Semiconductor Optoelectronics
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Lecture - 20
Amplification by Stimulated Emission

Okay, so we start today's topic is Amplification by Stimulated Emission.

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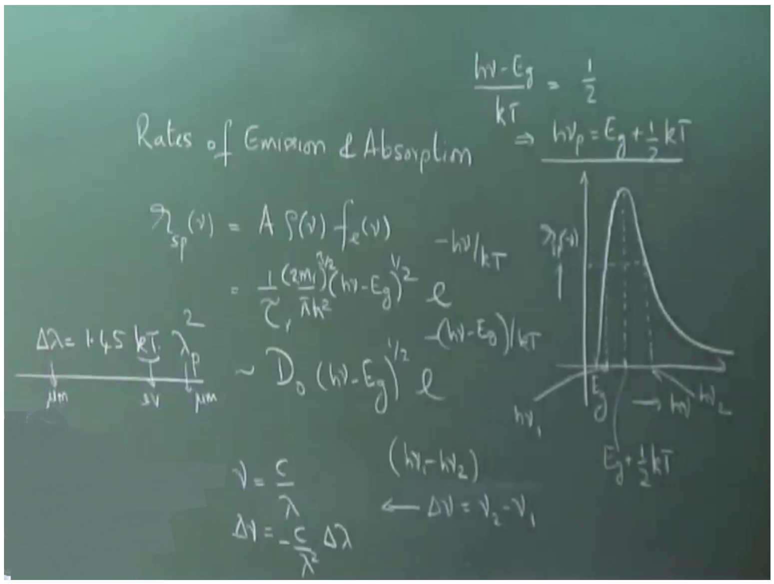

So recall that in the last class we discussed about rates of emission and absorption, and in particular we discussed about the rate of spontaneous emission. And we have got an expression for spontaneous emission that RSP of nu= A*rho of nu density of states*fe of nu, where fe of nu is the emission probability, probability of emission. We substituted the various parameters that is for rho of nu we substituted the density.

So we have 1/tau r I am just recording this is recapping, so 1/pi h cross square*h nu-Eg to the power 1/2 * we had shown that this is approximately fe is e to the power-h nu/kT, and then we wrote this in the form of twice mr there is an expression here twice mr to the power 3/2, and we wrote this in the form of D0*h nu-Eg to the power 1/2*e to the power-h nu-Eg/kT. And then I had shown that the variation looks something like this.

So what I have plotted is RSP of nu, it is spontaneous emission spectrum as a function of h nu and it starts at Eg, and then we could find out the peak value the peak value corresponds to Eg+1/2Kt the peak of spontaneous emission occurs at energy value Eg+1/2Kt. I had also asked you to find out the full width at half maximum, so you can find out the full width at half maximum, so if this is the maximum then half of it there you drop down and find out what are the corresponding values of h nu1 and h nu 2.

And therefore you can find out the bandwidth corresponding to this, so corresponding to the frequency here so we have let us say this is this point here is h nu1 and this end here is h nu2, then we have h nu1-h nu2/kT, so this is h nu so h nu1-, I have written that just S h nu so h nu1-h nu2, and therefore from this we can find out what is delta nu=nu2-nu1, and once you know delta you can find out delta lambda that is the line width delta lambda=lambda square/C just a minute.

So nu=c/lambda and therefore delta nu=c/lambda square*delta lambda with the negative sign, so delta lambda=lambda square*delta nu/c, so we have I had given an exercise to find the delta lambda. So I had written an expression that show that delta lambda is approximately =1.45*kT*lambda p square approximately equal to, so here kT is an electron volt and lambda and delta lambda here are the micrometers micron in this expression.

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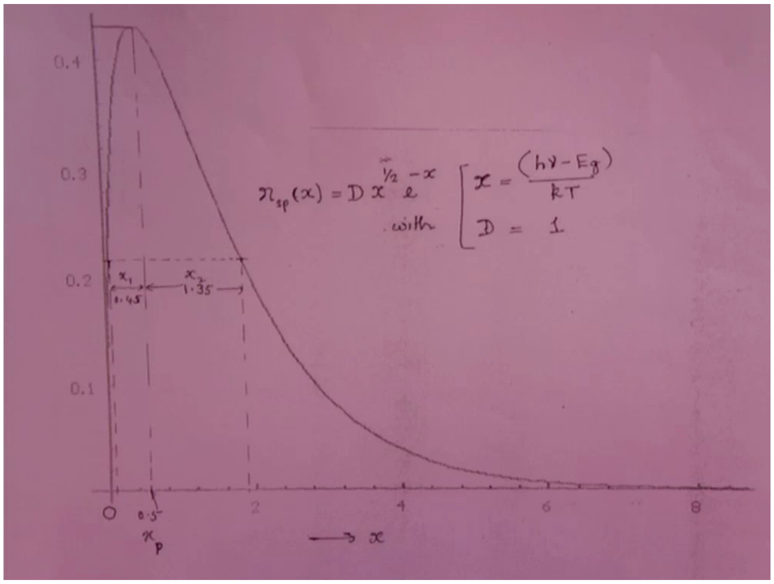

Here, I have a graph I have actually plotted this function, I hope you can see this you may not be able to read it, I have a slightly bigger one let me put a slightly bigger one this is too small let us see whether this comes out. So what I would like you to see is this is actual plot I have been qualitatively showing you here the variation like this, so what I have plotted is RSP of nu I have defined x as h nu-Eg/Kt, then the expression for RSP becomes RSP=D*x power 1/2.

So you can divide by kT to this and there is already a kT so e to the power of-x, and if I divide by square root kT this is x power 1/2, and kT to the power 1/2 has been absorbed in this D0 so RSP can be written in this form with x=this. So this is a plot of RSP versus x, so you can see where the peak occurs, so the peak occurs these are the actual numbers calculated plotted by the computer, so you can see that xp the peak occurs at 0.5, xp=0.5.

So h nu-Eg/kT=1/2 this implies h nu=h nu p=Eg+1/2 kT as I had written earlier here, so you the point is actually this shape is really you get a shape like this, and how you can find out at half the maximum value you find out what is the value of x here, and what is the value of x here and find out the difference between them to find out the delta lambda line width of a given source. The equation cannot be solved analytically you have to numerically resolved or graphically you can obtain the solutions and get the expression.

So today I will start with the rates of stimulated emission and absorption, please try to do this exercise and put some numbers and see what is the line width that you get, because I had plotted I had drawn qualitatively the spontaneous emission spectrum, but you can see that if you plot that function you get indeed a shape which is like that.

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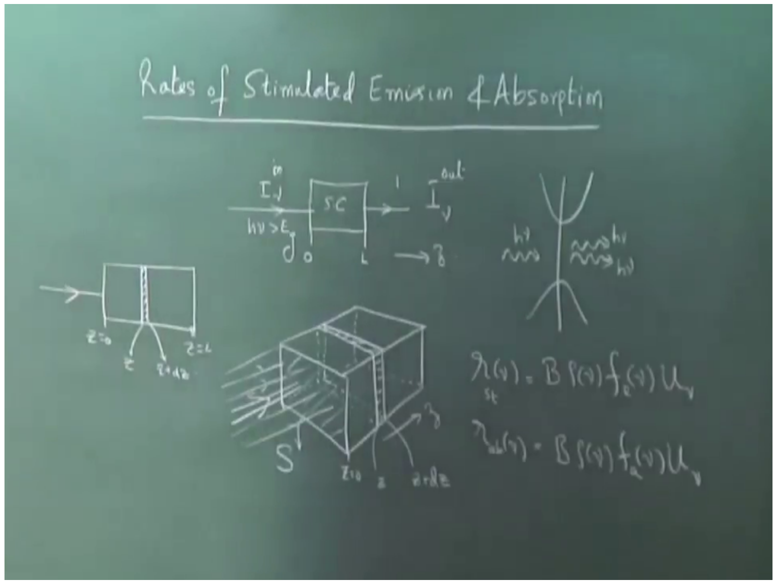

So today we will discuss the rate of stimulated emission and absorption. Our objective would be given a piece of semiconductor, a beam of irradiance I nu of frequency h nu of energy h nu>Eg, beam of h nu here a light beam of irradiance I nu entering this semiconductor here, what is the material property effect of the material property on the beam? That is the output here if I call this as l=0 let us say this is a z direction of propagation this is l=0 and this is L length of the semiconductor.

What is if this is the input I nu of input, then how is I out of nu related to I in, whether there is attenuation, amplification or no change under what conditions our specific objective will be determine under what conditions we will get amplification, and what happens in thermal equilibrium how does the semiconductor behave. So amplification if we are looking at amplification, amplification can take place only by stimulated emission.

Because that is the basic process of amplification basic process of stimulated emission, where you have one photon which is incident here leaves to generation of 2 photon in simple terms so h nu here and gives out, so this is the process of amplification. Amplification refers to coherent amplification, it is coherent amplification so stimulated emission is the one which is responsible for amplification, you can have if by the process of absorption, the photon can get absorbed and the output can get diminished or attenuated.

So we want to find out what is the coefficient of gain or coefficient of loss in this semiconductor that will be our objective. So let us consider so let me draw this diagram again, so this is a piece of semiconductor, so a light beam is incident at z=0 so this is the direction of propagation is the z direction, so this is z=0. If I consider a small volume element I consider a small slice of this here infinitesimal a slice of infinitesimal thickness, so this is z here and this is z +dz.

In the 2-D picture here, let me draw the 2D picture the radiation is entering from here, I have considered a small slice here so this is z=0 and this is z=l, and we have considered z at some position z and z+dz, let S be the area of cross this area of cross section of, so we want to get an analytical expression so let S be the area of cross section of this surface, and the beam is entering through the surface everywhere the beam is present everywhere in the cross section, so S is the area of cross section.

If I consider this small volume element and we know that the rate of stimulated emission at a frequency nu=B*rho of nu*fe of nu*the energy density u nu, rate of stimulated emission is=the Einstein coefficient, joint density of states, probability of emission and density at nu energy density u nu. And rate of stimulated absorption are simply absorption is=B*rho nu*absorption so probability of absorption *u nu, both stimulated emission and absorption depend on the energy density of the radiation u nu.

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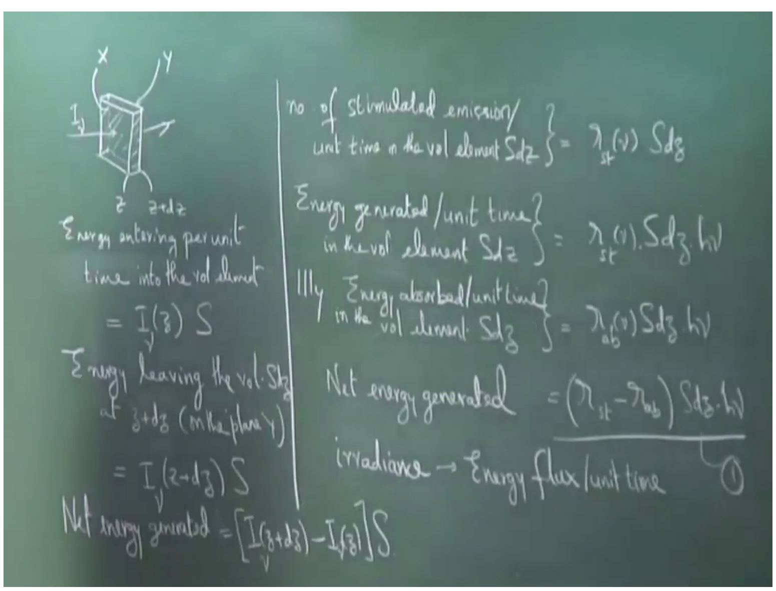

If I look at that small volume element I am now focusing on the small volume element, so let me draw the volume element here that small slice, so this is of thickness dz so z and z +dz. The number of stimulated transitions stimulated emissions let me write it stimulated emissions per unit time in the volume element Sdz= rate of stimulated emission is this one, what does this mean this tells you the number of stimulated emission per unit time per unit volume.

Why unit volume because density here rho nu is per unit volume, so this is the number of emissions per unit time per unit volume. Therefore, number of emissions per unit time in the volume Sdz will be=RSP we are considering at the frequency nu*Sdz, so every stimulated emission every emission generates a photon of energy h nu, and therefore energy generated we see logic energy generated per unit time in the volume element Sdz=what will that be this is the number of emissions therefore, energy will be =multiplied by h nu.

Because every emission brings down a photon of energy h nu therefore, multiplied by this is the energy generated in this volume element, there is a radiation of energy density u nu present. And therefore, similarly energy absorbed I can find out number of absorptions per unit time in the volume element Sdz will be r ab* nu*Sdz, Sdz is the volume, and therefore, energy generated here because of emission because of absorption energy which will be absorbed or energy lost.

So energy absorbed per unit time in the volume element Sdz=r ab*Sdz*h nu. Therefore, net energy generated per unit time in the volume element Sdz=r st-r ab this is generated and this is generated this is absorbed therefore, the net generated is=Sdz*h nu okay, so this is the net energy generated. I want to relate this energy generated to the intensity, so I nu is the intensity or irradiance of irradiance is energy flux per unit time.

This is energy flux means energy per unit area per unit time, it has a units of intensity because energy flux is energy per unit area per unit time, energy per unit time is power and power per unit area is intensity that is why it is I, energy flux energy per unit area per unit time energy per unit time is power and power per unit area is intensity. So if I nu of z is the irradiance on this plane, so let me show this end plane as plane X, and other plane at the other end as Y.

So energy entering per unit time into the volume element is =I nu of z that is the irradiance on the plane z plane x I nu of z*the cross-sectional area, energy entering per unit time is intensity*cross sectional area S is the cross-sectional area of this surface, so I nu*S. What is the energy leaving from the other side? Energy leaving the volume element Sdz at Y that is at the plane at X at +Sdz on the plane Y at z+ dz from the plane Y at that plane is =I nu of z+ dz*S.

I is the intensity or irradiance and multiplied by the surface area tells you the energy entering, energy leaving. Therefore, the net energy generated in the volume element per unit time is=what is leaving-what is entering so I nu of z+ dz-I nu of z*S, please see this. Net energy generated in the volume is this much r st-r ab*Sdz*h nu and net energy generated in terms of intensity is this much, so they are the same.

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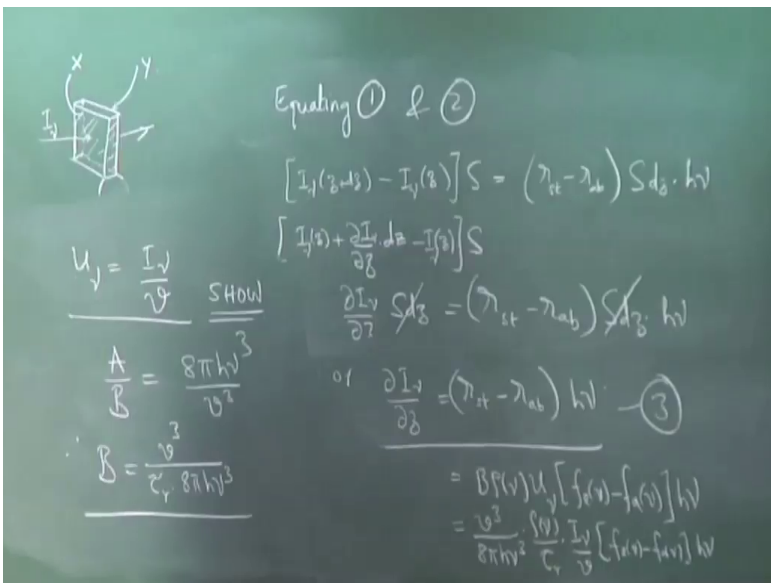

So equating 1 and 2, I nu of z+ dz-I nu of z*S= r st-r ab*Sdz*h nu, we have considered an infinitesimal volume element therefore, I can write this I nu of this z+ dz as I nu of z+ del I nu/del z*dz, where dz is the thickness of this, this -I nu of z*S=this. So this is=del I nu/del z*Sdz= r st-r ab*Sdz*h nu or so this goes off we have the del I nu/del z=r st-r ab*h nu. Substitute for r st and r ab, and we should get the expression.

So r st so this is=B*rho nu*u nu*fe of nu-fa of nu this portion*h nu, how is I nu irradiance related to energy density u nu=I nu/velocity, this is energy density you can see energy per unit volume, and what is this? This is energy per unit area per unit time so you can dimensionally immediately you can see that there yes they match dimension. If I nu is the you can show this let me write this to show that u nu=I nu/v.

Just use the for fundamental definitions of u nu and I nu and show that u nu=I nu/v, so this is =, and what else do we have we know that A/B the relation between the Einstein coefficient is 8 pi h nu cube/v cube, see if it is in free space but if it is in the medium it is v that is c/n, where A= therefore, B=A A is 1/tau r therefore, we have v cube/tau r*8 pi h nu cube, so this is the coefficient B Einstein coefficient B is we have used A/b=8 pi h nu cube/v cube, therefore v=this.

So we substitute there here in this expression, so let me substitute here so v cube/8 pi h nu cube*rho of nu*tau r radiative recombination time*u nu which is I nu/v velocity*fe of nu-fa of nu, it is a very simple derivation but for the sake of completeness let me not skip any step and show you. So 1 h nu goes here with this h nu 1 h and this becomes nu square and 1 v goes here with this.

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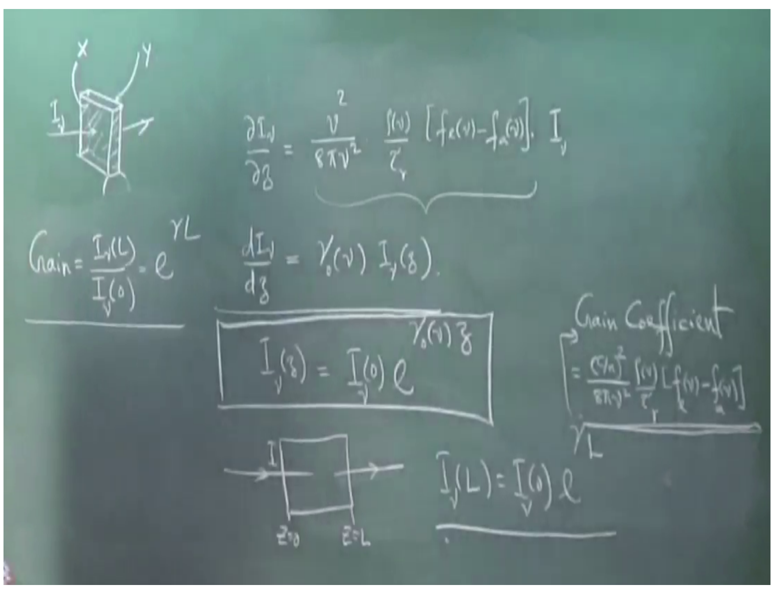

And therefore we have del I nu/del z=v square/8 pi nu square this v or I can write c/n*rho of nu/tau r*fe of nu-fa of nu*I nu intensity is a function of z, please see this here does not depend on z independent of z, intensity I enters the medium and depending on the position z it may be getting attenuated or amplified whatever but I is a function of z. And therefore, since I is a function of z alone I can write this as d I nu/dz= this whole coefficient here it depends on the frequency therefore I called this as gamma 0 of nu*I nu of z.

So what is the solution this gives I nu of z=I nu of 0 that is the entrance at the entrance point*e to the power gamma 0of nu*z. We have got an expression for any value for the intensity at any value of z, if you are looking at the output so this is z=0 if I nu of 0 is the intensity here at the output at z=L this will be I nu of L=I nu of 0*e to the power gamma*L, so I nu of L or I nu of z will be > I nu of 0 if gamma is positive, if gamma is positive then I nu of z>, when will gamma be positive.

So this tells that there is amplification so what is output by input therefore, the gain=I nu of L output/input I nu of L/I nu of 0=e to the power gamma*L, this is the gain, not gain going coefficient this is gain and gamma is called the gain coefficient, so gamma is the gain coefficient. So what is the gain coefficient? Gain coefficient gamma=B square or c/n just because otherwise sometimes nu and v may get confused, so 8 pi nu square*rho of nu density of states/tau r*fe of nu-fa of nu.

I have already called it as gain, but when will it be gain? It will be gain if the gamma is positive, and gamma is positive when everything here are positive constant, this is the density of states so it is the positive number, this is a number it is a time positive number everything is positive, only this can be positive or negative, and when will this be positive? This will be positive when probability of emission is > probability of absorption, and when will that happen?

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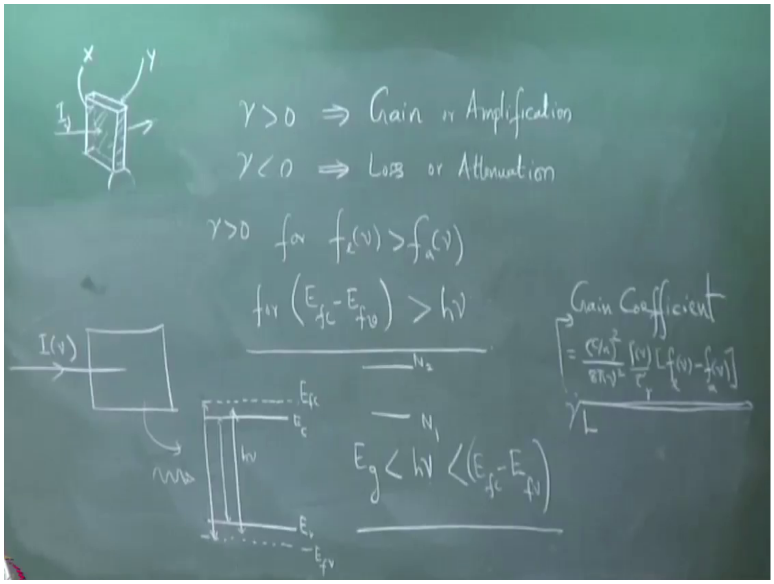

When will probability of emission be > probability of absorption? This is the condition that we have already derived and so gamma is > 0 implies gain or amplification, and gamma < 0 that is negative implies net loss or attenuation. And therefore, it is the sign of gamma which is important and which will be determine that? It will be determined by this factor fe of nu-fa of nu, and so gamma is > 0 or fe of nu > fa of nu, and this happens when?

When Efc-Efv the difference between the quasi fermi levels is > h nu. So the separation between the quasi fermi levels if this is >h nu, then for all those frequencies we have gained in the medium, so if you have a semiconductor here and light of radiation I of nu, now I put I of nu if that radiation contains a spectrum of frequencies I of nu, then and if the material here is characterized by a band gap like this, energy band diagram I am showing.

Now let me show this is Ec, this is Ev and assume that if this semiconductor we have maintained Efc here and Efv here, so these are valence band conduction band and this is the Eg, for h nu > Eg for all energy between for all values of h nu between this that is from Eg here so Eg < h nu < Efc-Efv for all frequencies nu such that Eg < h nu < Efc-Efv, we have this condition satisfied, it means if I have radiation which is coming such that the energy difference corresponds to some values here, let us say h nu.

Then for this h nu which is >Eg but < Efc-Efv we will have gain in the medium, so if you pass a spectrum then those frequencies for which energy corresponds to energy satisfies this equation we will have gain in the medium so Efc-Efv, so this is the condition for gain we have analytically showed that this is the condition for gain in the medium, this is equivalent therefore, this condition.

Those of you have studied atomic laser physics you know that there you have to have N2>N1 or a state of population inversion. So a state of population inversion means is N2>N1, this is the necessary condition for amplification by stimulated emission, and in the case of semiconductors the necessary condition for amplification by stimulated emission is this one. So there is an equivalent it is incorrect to say that number of electrons in the conduction band should be >number of electrons in the valence band that is not the condition the condition is this.

If you can maintain this condition then all frequencies nu, which satisfy this condition will get amplified. What happens to other frequencies? So if we go here for example a radiation which corresponds to this frequency difference, so this condition is not satisfied and immediately that means will be fe of nu will be < fa of nu and therefore, this will be negative number which means gamma is a negative number, which corresponds to loss or attenuation.

This will determine the amplification bandwidth we will discuss this in the next class, the amplification bandwidth of the amplifier. So our next topic will be laser amplifier and an amplifier is characterized by gain and bandwidth and of course noise also.

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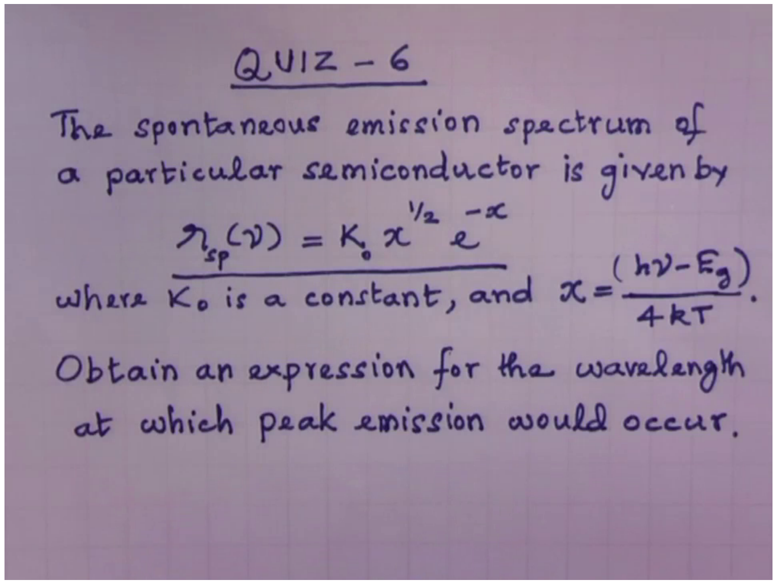

So we will take a quick quiz, a simple quiz this is a very simple quiz, the spontaneous emission spectrum of a particular semiconductor is given by an expression is given to RSP of nu=K0*x to the power 1/2*e to the power-x, where K0 is a constant, and x=h nu-Eg/4kT. Obtain an expression for the wavelength at which peak emission would occur.