Physics of Biological Systems

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Lecture - 62
Schnakenberg Kinetics

Just to a we can look at the, so last class I showed a simulation of this what I called is the Schnakenberg kinetics. So, just as an example we can take a look at it. So, for this particular Schnakenberg kinetics on this f and u is this clear. So, let me see.

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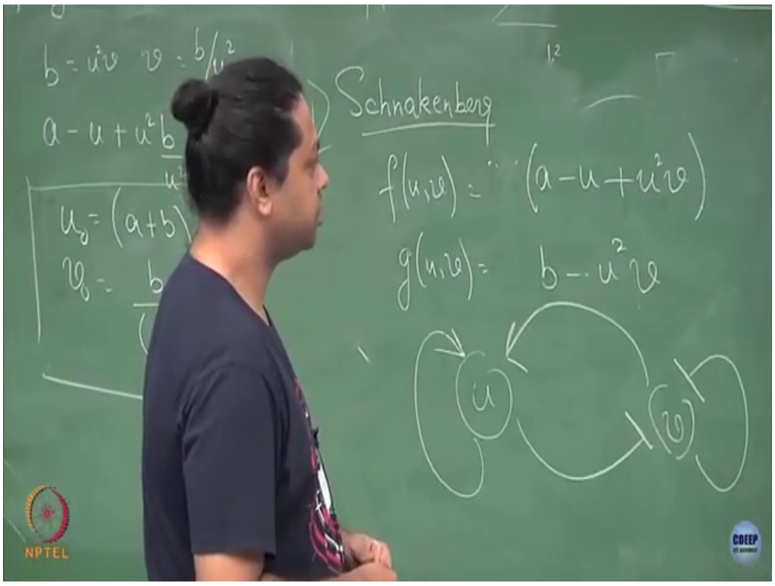

So, this is Schnakenberg; Schnakenberg. So, for Schnakenberg f u, v is gamma or whatever, I am anyway writing gamma outside. So, this is a minus u plus u square v and g of u comma v is sum b minus u square of v ok.

So, if I have so, this is the f and g, so once I specified the f and g I know, what is the chemical reaction, and I can try to find out whether or not there will be a pattern. But just to so if I look at the species u and v, the more of v I have, the more of u I produce right because, this comes with a plus sign. So, the more of v I have the more of u I produce, the more of v I have the less of v itself I produce right because, this comes with the minus sign. So, which means this is an inhibition terms to itself, the more of u I have again the less of v I have.

So, again this is an innovation term right and the more of u I have the more of itself of u I produce so, this is an activation term ok. So, this is what the Schnakenberg kinetics looks like at the network level that u up regulates itself it down regulates v; v down regulates itself, but it up regulates u ok.

So, you can look at the; you can look at this f and g terms and say what sort of a chemical reaction network you have basically underlying this. So, now, you can sort of do whatever this 4 can look at, what these 4 conditions mean for the Schnakenberg kinetics. For example; trivially let us just say that what is the steady state.

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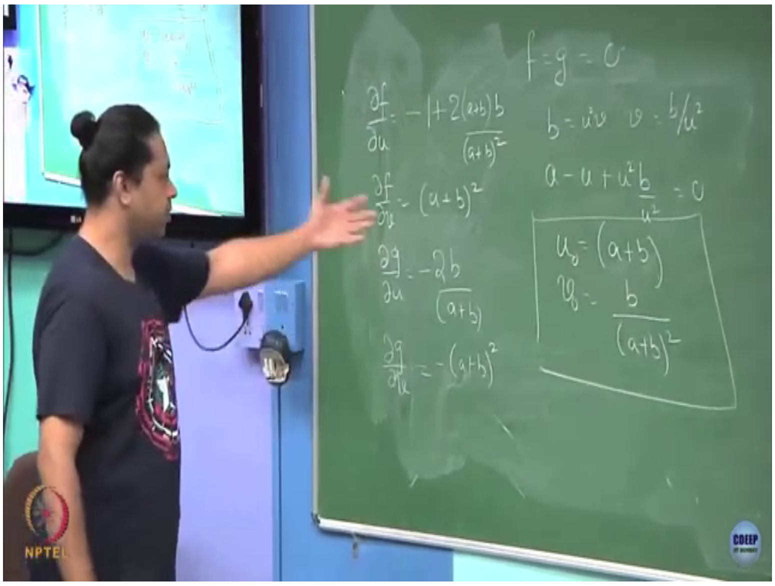

So, what is the in the absence of diffusion, what is this homogeneous steady state that we would have? Remember the homogeneous steady state is given by f equal to g equal to 0 right, because in the absence of diffusion del u del t 0 del v del t 0, Which means what? Which means, b is equal to u square v so, v is equal to b by u square and a minus u plus u square b so, b by u square is equal to 0, which means u is equal to a plus b, let me say u naught and v naught, then is b by u square, so b by a plus b whole square. So, given the Schnakenberg kinetics this is my homogeneous steady state in the absence of diffusion. I can now look at whether when will this steady state be stable.

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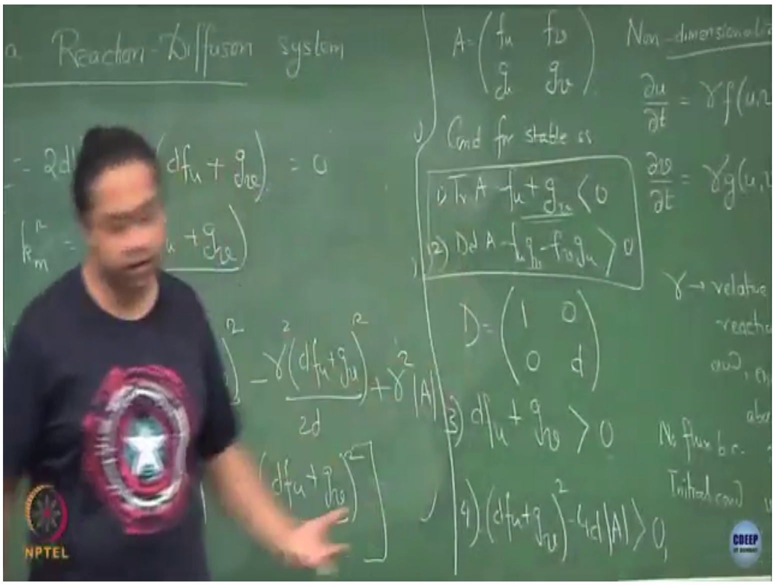

So, I can look at these two conditions 1 and 2. You just do so, what is del f del u let me construct del f del u is minus 1 plus 2 u v, at this so 2 u naught v naught a plus b, b by a plus b whole square, del f del v is u square so u naught square so, a plus b whole square del g del u is minus 2 u naught v naught minus 2 b by a plus b and del g del v is minus u square, so minus u naught square, minus a plus b squared.

So, this once I know if v of course, and I know what is the steady state. I can construct my stability matrix these are the four elements of my stability matrix. So, what this means is that fu plus. So, what that condition means is that f u plus g v less than 0; f u is minus 1 plus 2 b by a plus b g v is minus a plus b whole square is less than 0 and the determinant is whatever (Refer Time: 06:27) writing it down.

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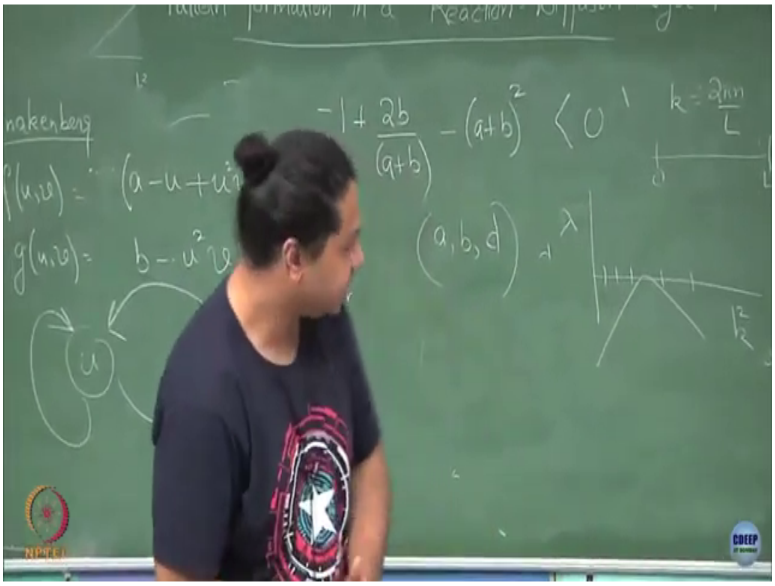

So, the condition is that so the point is that; given this sort of an equation which is two parameters a and b. And you have this third parameter d by applying these four conditions sequentially you can find out for what value for what ranges of a b and this ratios of diffusion coefficients a, b, d sorry d you will have patterns that will destabilize the steady state u naught v naught in the Schnakenberg kinetics.

So, you can take I forget the upload the code, but I will do that sometime this you can take that MATLAB code. And try to see that whether you know given these conditions what happens if you choose a d inside this, allowed by these conditions what happens if you choose a d outside these conditions and see whether the steady state actually gets stable destabilize or stabilized ok.

You can do it with the first Schnakenberg is sort of one of the canonical kinetics, there are a bunch of canonical kinetics that people use to study pattern formation in these systems, but for any of this spirit is the same that you calculate this to find out the stable steady state in the absence of diffusion you find out the stability matrix and then you apply these conditions to see that whether there exists any wave vector where, you will get actually you get a pattern ok.

Another thing that I should also mention is that it is good to be careful that for example, let us say I was plotting this lambda as a function of k square right. And let us say I got something like this ok. So, there are some k is which are allowed, but still you may not see a pattern simply depending on what you are allowed k vectors are right. So, if you are thinking of for example, a one d domain between 0 to L you are allowed wave vectors would look something like 2 pi n by L , which means you have discrete wave vectors right.

And it might so maybe this wave vector is allowed simply because of the boundary conditions ok. So, it might be that this wave vector is allowed this way vector there is no wave allowed wave vector here at all. Even though this lambda k square is greater than 0, the length of your domain says that that wave vector itself is not allowed. So, even that is something to keep in mind there whatever allowed wave vectors you find that must be physically possible given the domain that you are trying to solve this pattern formation problem ok.

So, I think I will stop here. You can play around with the code either with this code or with any of the other standard kinetics where is observe (Refer Time: 09:06) or any of the others and yeah that is it. That is all I have to tell you for the whole course it is done.