Physics of Biological Systems
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Lecture – 15
Mean capture time

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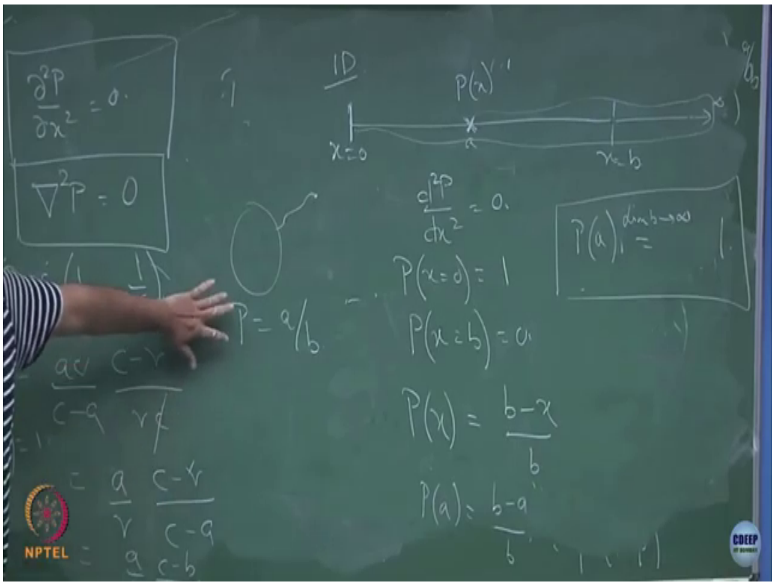

So, now, let us ask this related question that I have calculated this capture probability, what is the probability that the particle will get captured. Let me ask the related question that how long will it take to get captured.

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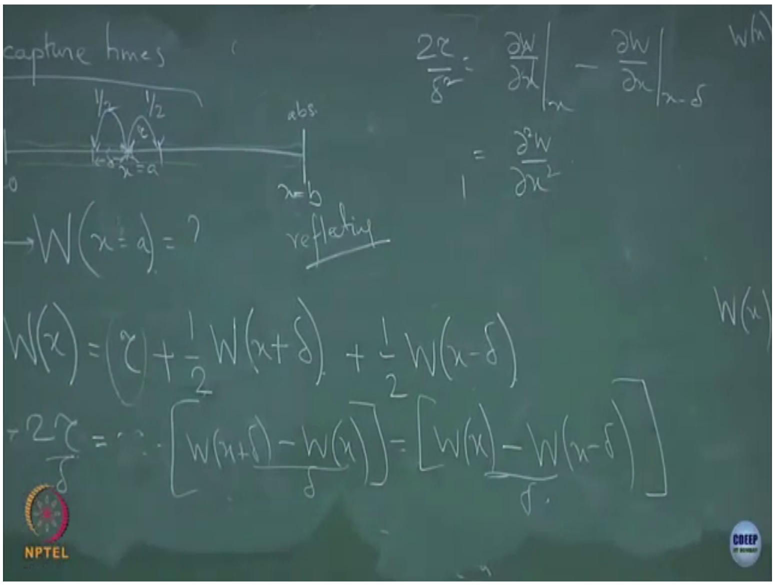

So, let me say that is the mean captured time, mean captured times. So, again let me derive it in 1D and we will generalize it to 3D, ok. So, let me take this domain, this is between x equal to 0 and let say x equal to b, and I release the particle somewhere over here at x equal to b.

And let say both these boundaries are absorbing, this is absorbing, this is absorbing, ok. I want to find out that what is that mean and time that it will take for a particle release that x equal to a to get captured at either of these boundaries, ok, it can get captured here and get captured there alright. So, before I calculate this maybe you can tell me what the answer will be.

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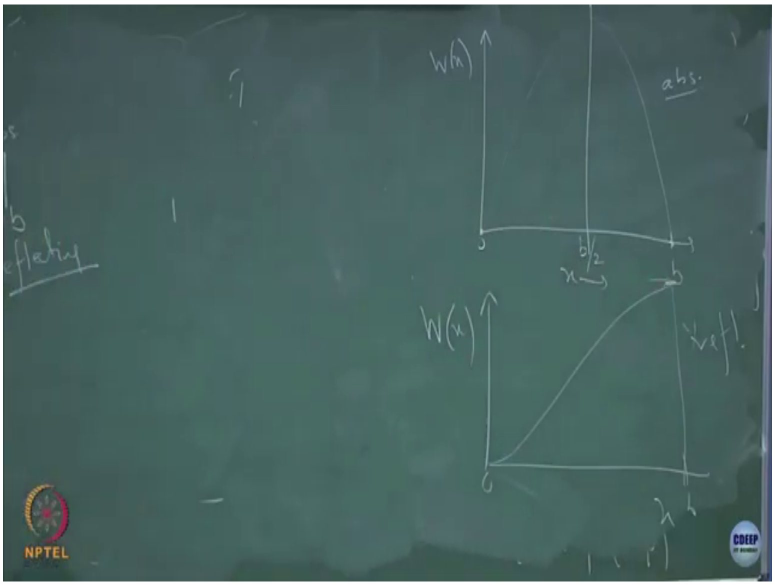

So, if I wanted to plot this W of x, W of x. Remember x is the starting position where I have release the particle. So, this and on the x axis is let us say x between 0 and b. So, if I release the particle at 0 how long will it take?

Student: (Refer Time: 02:15).

Student: 0 time.

0 time if I release the particle at b, how long will it take? Again 0 time. Where will it take the maximum time? Where will it take the maximum time?

Student: b by 2.

b by 2. So, I assume the curve to look something I do not know; that the exact how it looks we will work out, but I expect it to look something like this with a maximum at b by 2, right, naively. On the other hand, if I said that this x equal to b was a reflecting wall, so it could only get absorbed at x equal to 0. So, this is for the absorbing case, this is for the absorbing case. If I plotted the corresponding graph W of x versus x between 0 and b for the reflecting case, where would it take the maximum time?

Student: (Refer Time: 03:07).

Student: At b.

At b. So, therefore, maybe it would look something like this, you know, right. At b you would get the maximum amount of time.

So, any anything that you release here would have one path like this, but would also have another path like this all the way back and that would increase the times. So, whatever we calculate we will try to calculate this W of x, but that should at least naively conform to what expectation you have at the back of your mind, ok.

So, again I will try to calculate using this random walk language that I have this random walker, it can hop to the right or it can hop to the left with probability half. It covers a disk, each hop it covers a distance delta, so it is walking on a lattice in that sense and each hop takes a time tau, ok. So, each hop takes some time tau. So, it is a random walker that takes a step every tau units of time and each step it takes moves it delta distance to the left or delta distance to the right. With that I will try to write down an equation for this W of x, ok.

So, what I want to write is this W of x, ok. What is the mean time it takes to get absorbed at the boundaries if I start from x? So, if I start from x, what can I do? Again it can has two possibilities, it can hop to the left, it can hop to the right with equal probabilities. So, if it hops to the right, I now have the mean time from this new position, ok. If it hops to the left, I have the mean time from this new position x minus delta. Anything else? Anything else? It is hopped unit to the right or to the left. So, therefore, what do I need to complete this equation?

Student: Tau.

Tau, right. To make this hop it has taken a time tau, right. It has hopped either here or there that hop takes a time tau. Remember this is an W is a time, what is the mean time. So, it take that hop takes a time tau and then at the new position you have some mean time again, W of x plus b, W of x minus b, ok.

Once I have written this discrete evolution equation the rest is simply a matter of algebra again. So, again I will do what? Let me multiply by 2 by delta. So, 2 W x is equal to 2 tau plus W x plus d plus W x minus d. So, I bring it to this side. Let me bring the tau on this side minus 2 tau, this would be W x plus d minus W x that is 1 plus W x or sorry minus W x minus W x minus delta that is the other, right. I divide throughout by delta, delta, delta, delta. So, that is del. So, I have 2 tau by delta is equal to del W del x at x minus del W del x at x minus delta. I again divide by one more delta, so then this becomes del 2 W by del x 2, right. So, everything is the same as for the captured probability except this manipulation with these times, so that once you have put in the factors of delta correctly is 2 tau by delta square. What is 2 tau by delta square?

Student: (Refer Time: 07:25).

Student: (Refer Time: 07:26).

There is a minus sign somewhere, right, ok. So, minus. So, what is 2 tau by delta square? Yes.

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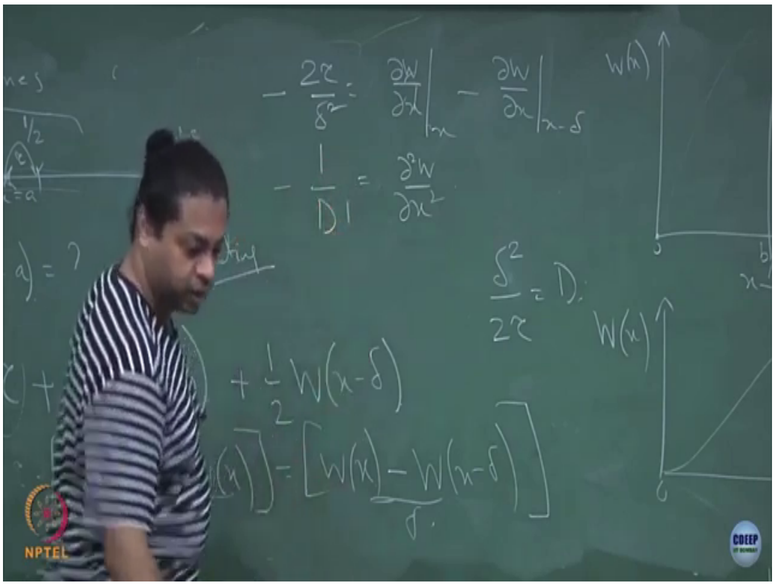

What is this unit or what is this unit? Length square over 2 times time D. This is nothing, but the diffusion coefficient, right. So, this is nothing but 1 over the diffusion coefficient. So, this is minus 1 by D, ok.

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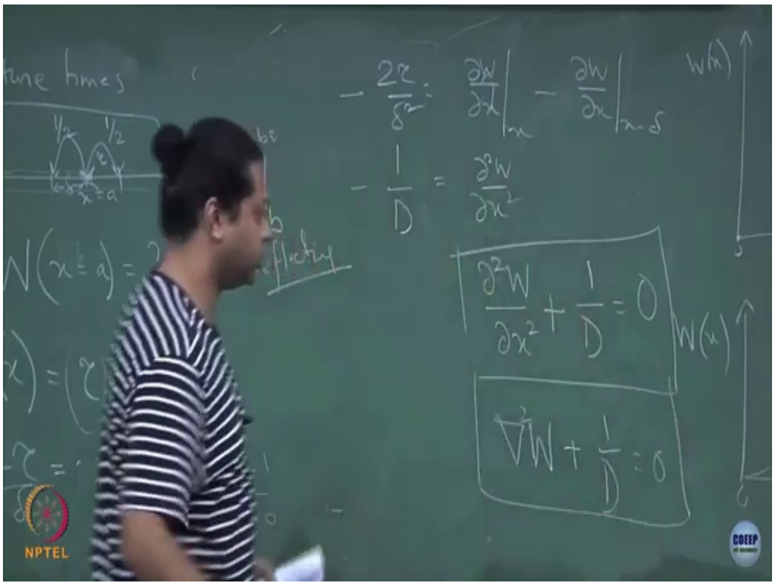

So, if I am looking for the mean captured time or the mean first technical term is the mean first passage time then that obeys the equation del 2 W del x 2, let me bring this over here only plus 1 over D is equal to 0. So, you write the recursion relation in the discrete set up, you take the continuum limits correctly, what emerges is this diffusion coefficient very nicely because of course, if something is diffusing faster it will take smaller time, if its diffusing slower it will take longer time.

So, as long as you are calculating times you expect it to depend on the diffusion coefficient, right. And this is what the equation comes. If you generalize this again this thing becomes the del square operator, so it becomes del square W plus 1 over D is equal to 0, in any general dimensions 2D, 3D whatever, ok, ok, fine, ok. So, now let us just then calculate and see whether what we calculated or what we gets make sense.

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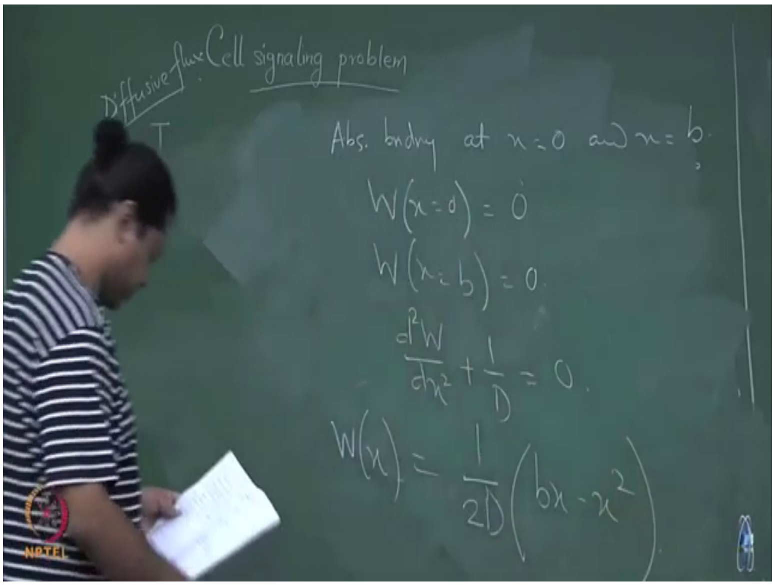

Let say I have do it this setup that I have absorbing boundaries, absorbing boundaries at x equal to 0 and x equal to b, ok. What does that mean? That means, that W of x equal to 0 is 0. If I started there it would take no time or if I started at b again it would take no time, right. That is the meaning of absorbing boundary conditions. And then you just solve this equation that d 2 W d x 2 plus 1 by D is equal to 0. And what would this W of x be equal to? Ok.

So, you can work this out and check. So, I will just write down the answer. So, this will come out to be 1 by 2D, bx minus x square. So, if you put x equal to 0 this become 0, if you put x equal to b then it becomes b square minus b square which is again 0, the maximum comes at x equal to b by 2 which is what we have here.

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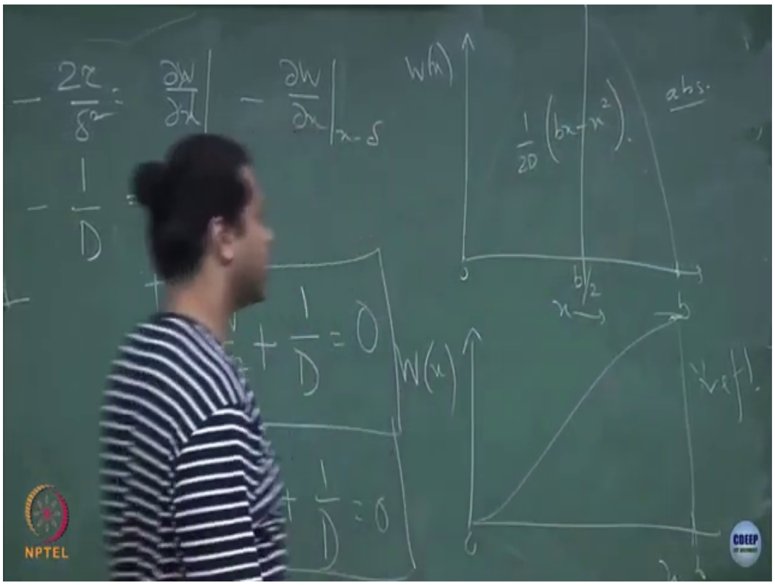

So, this curve that I drew then is 1 by 2D, bx minus x square. And the diffusion coefficient appears in the denominator which again make sense, if it is faster if you have a large diffusion coefficient your average times will be smaller, if you have a small diffusion coefficient your average times will be large, alright.

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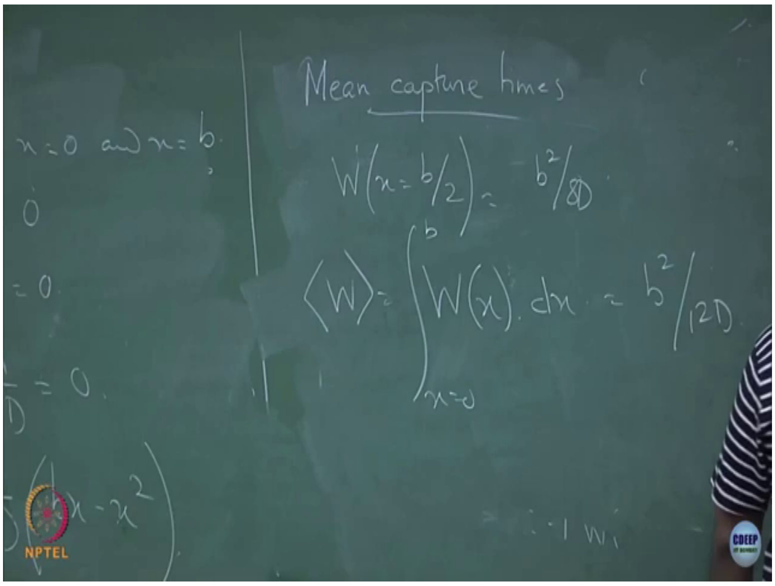

So, if I say that I release this particle, so that is the general solution for all W of x. If I release it half way if I what is if I can calculate what is the midpoint; so, we release it at x equal to b by 2 then that just comes out to be b square by 8D. So, this height the height of this is b square by 8D, ok.

You can also calculate a spatial, this remember this is already a mean, so you do this experiment many many times and you take the average of all of this. This W of x is that average value. You can also, but this is depends on some particular starting position W of x. You can also start it randomly anywhere along the lattice and you can ask what is that that mean, so spatial mean as well.

So, W of x D x, ok; so, if we start it randomly anywhere on this lattice anywhere and then let it absorb what would be the average first passage time or the average captured time in that case; and that you can just integrate this x from x equal to 0 to x equal to b and that will give you the answer. In this case, that answer is b square by 12D, ok. Since, we do two things, let me do one more. Let me do this part of the problem as well. So, that was an absorbing reflecting setup.

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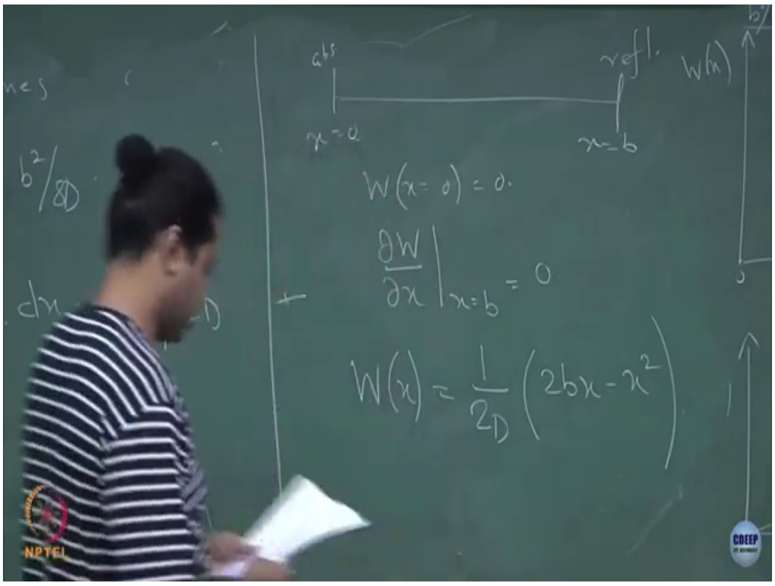

So, at x equal to 0 I have an absorbing boundary, at x equal to b I have a reflecting boundary, right. So, I solve the same equation, but I put in different boundary conditions; so, again W at x equal to 0, 0, right. It is an absorbing boundary. On the other hand, at x equal to b I will have d W d x, at x equal to b is equal to 0 that is a reflecting boundary condition, ok, right.

And again you can you can solve this, so you can solve for this W of x, the same equation, but subject to these new boundary conditions and again you will get something taking 2D, 2bx minus x square, ok. So, instead of bx minus x squared you get 2bx minus x square which means if I put x equal to b that is no longer 0, in fact, that is b square by 2D.

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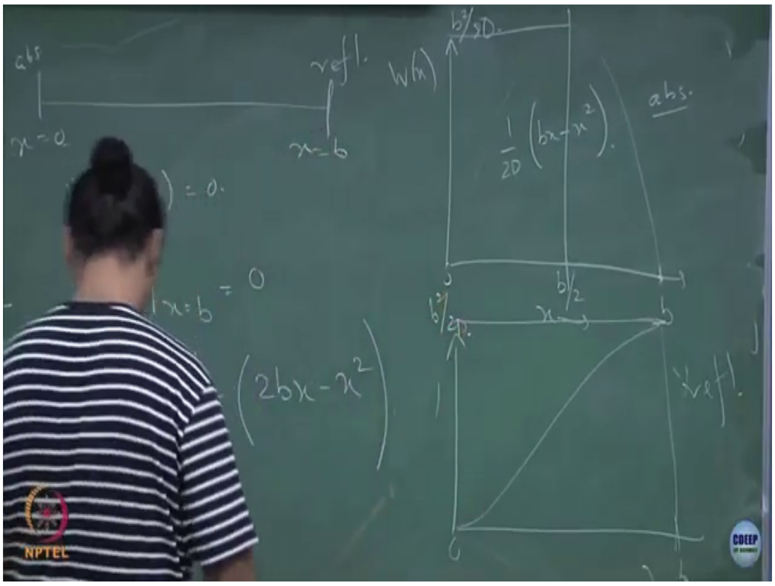

So, this value is going to be b square by 2D.

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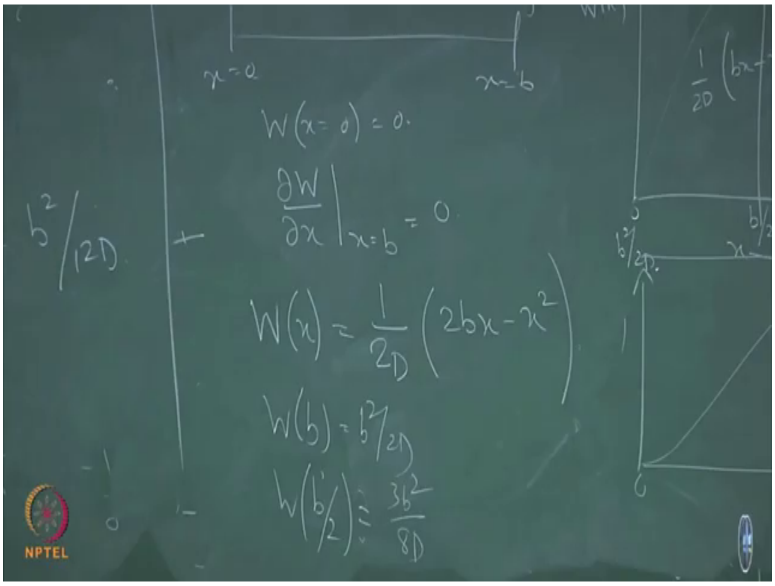

And if you release it, so W at b is b square by 2D, W at b by 2 if you release it at the mid midpoint it is going to be b square by some 8D, 3b square by 8D. So, in this case if I release it at this midpoint that is it takes 3 times as long as it took for this absorbing case where both boundaries were absorbing.

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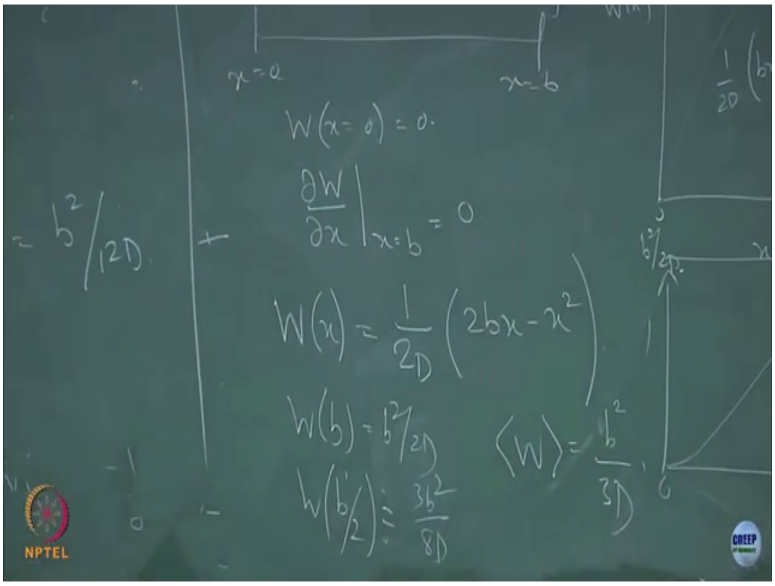

And again if you take the spatial mean as well that comes out to be b square by 3D, all right. So, in all cases, this one reflecting one absorbing boundary gives you times which are larger than this both boundaries absorbing which means if I plotted it somewhere on the same curve it would look something like this, ok. This, this is the reflecting one, this is the absorbing reflecting and this is both boundaries absorbing.

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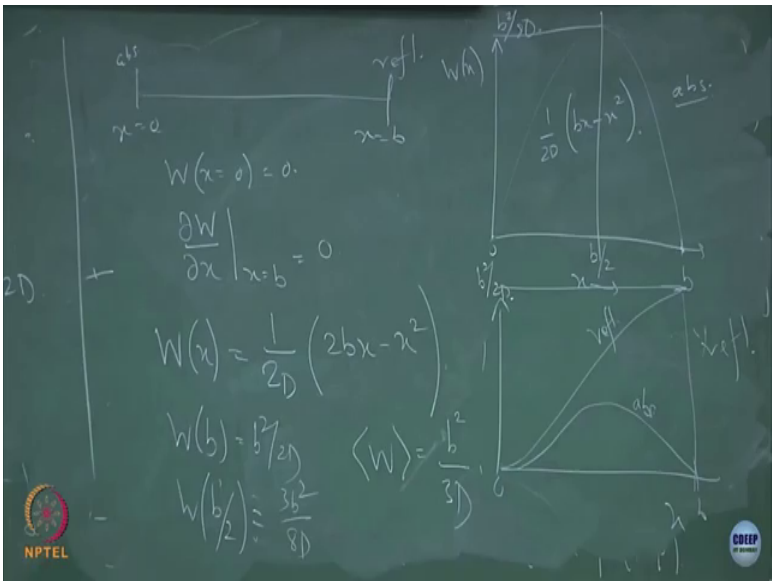

I just did this for the partition setup. You can do this you can solve this in general whatever, spherical coordinates since for this sphere problem that we were doing and again calculate what is the average time it would take for a particle to get captured, ok.

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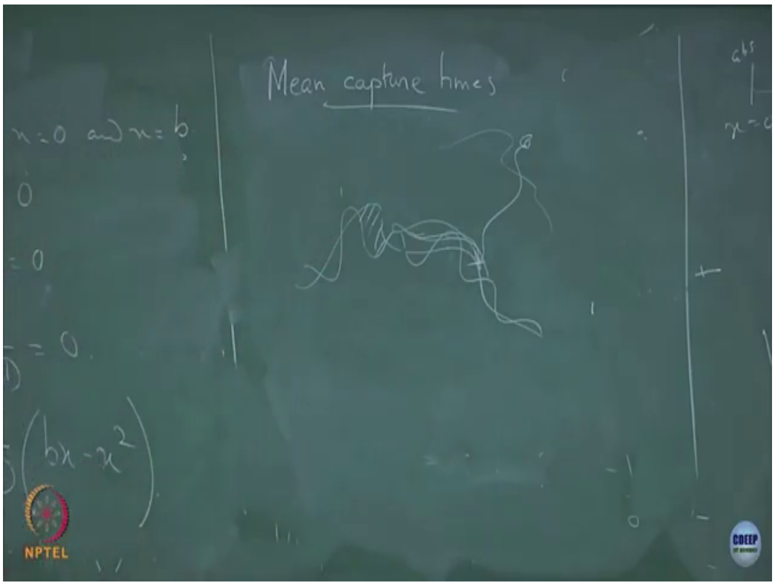

But, these are interesting in the sense that even this simple Cartesian problem is interesting in the sense not only in this context of cell signaling, but in the context of 1D. So, this 1D diffusion you can think of these proteins refusing along a DNA backbone. So, protein comes and it binds on this backbone, it can then slide along this backbone and as it searches for a target sequence, ok. This search, this 1D search happens by a diffusion and you can ask that if a protein binds at some base pairs away from this target site and it then diffuses randomly; how long would it take to get absorbed on to the target site itself, ok. So, that is a physical mechanism.

Often when proteins bind they do some part of the diffusion in 3D and then there is some part of the diffusion in 1D. So, it is a 3D plus 1Ds of the computation (Refer Time: 16:25). And this sort of a free setup can let you capture tell you what is the meantime this protein state to get captured or in 3D what is the meantime these chemoreceptors would take to get adsorbed onto the cell surface, ok.

I think I will stop here today.