Quantum Mechanics
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Lecture - 67
Clebsch – Gordon Coefficients – III

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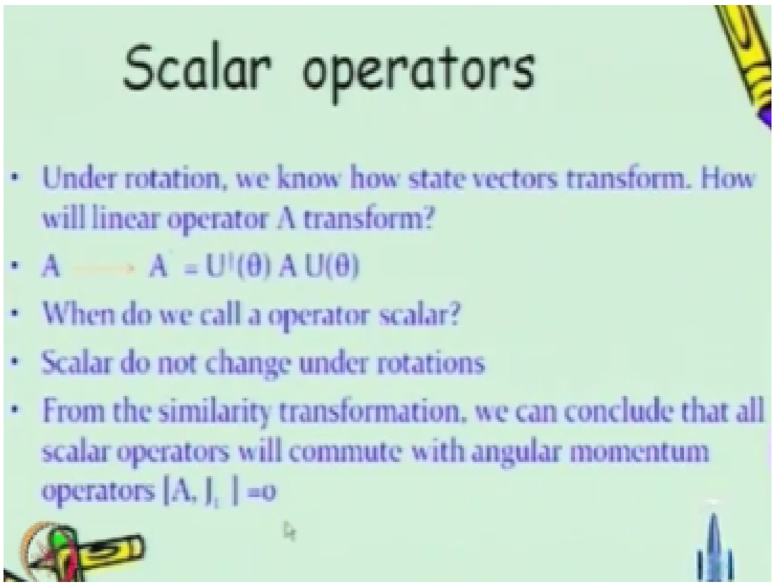

So now I am going to slowly take you in to understanding operators which should also be categorically say that this is like a spin 0 operator or angular momentum 0 operator and angular momentum 1 operator and so on so that is a very neat way in which you can see 1 is a trivial operator which is a scalar operator when you call it as a scalar in classical mechanics why do you call it as a scalar?

You will only look at rotations because angular momentum is the generator of rotation under rotation a scalar operator remains same radial co-ordinate for example is actually a dot product of 2 r.r vector under square root the radial co-ordinate does not change under rotations right that is why you call it as a scalar now we want to associate with such scalars operators you can have a position operator.

The radial operator in central force problems so this radial co-ordinate operators are scalar operators that is what we will say but what is the formal definition under rotation we have seen how state vectors transform and we need to see how a linear operator transforms under rotations any linear operator if the state vector transforms under rotation you know how will it transforms U to the i J.theta by its cross.

The corresponding operators can transform so when do we call an operators scalar? When A prime = A no change when A prime becomes = A we call it as a scalar operator when can that happen? Only when the commutator of the scalar operator with angular momentum this is 0 all of you have checked that the L LI with r co-ordinate or function of r radial co-ordinate is 0 you checked mechanically.

Now I am just trying to say that scalar operators they commute with angular momentum generate that also tells you other operators will not commute with angular momentum when this changes they will not come classical mechanics tells us if you have a vector operator or position vector you know how the position components of that vector transform under rotation all of you know in any vector operator like you take vector potential.

All electric field momentum they have to transform the transformation properties are not different from your components of your position vector that is why you called them to be vectors even when they announce the vectors they are some distinctions what are the distinctions? Those which change in the parity those which does not change in parity they are polar vector axial vectors let us not get in to that at least for proper rotations.

There is no difference and under proper rotations any vector will transform like components of a position vector this you know from classical mechanics.

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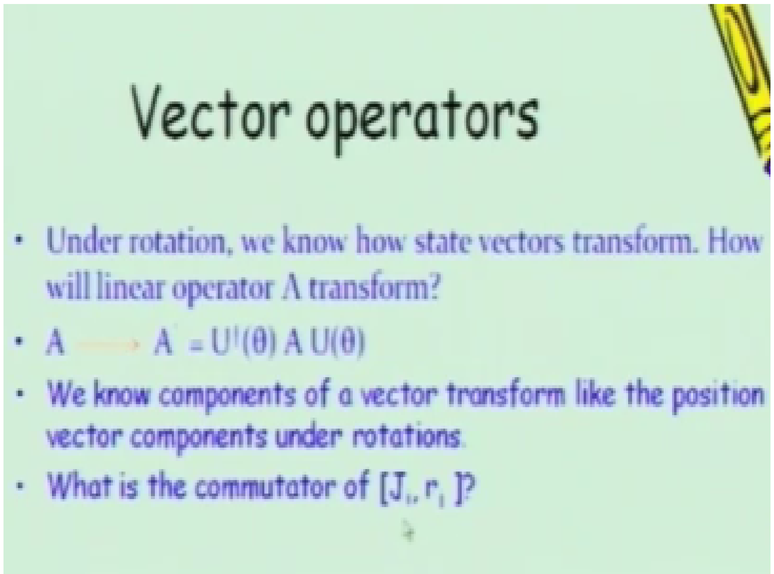

Clearly this above equation A prime is not A you know how A prime changes also under rotation from their you can deduce what is the commutator of angle momentum with different co-ordinates position co-ordinates whatever I write for the components of the position operator it should hold for any vector operate so you would check this if you have not you should go back and check this.

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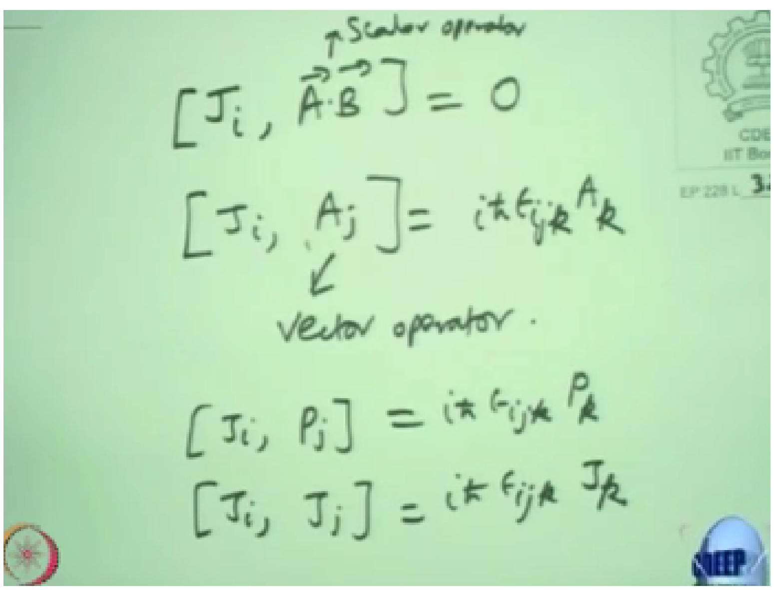

J i with any dot product that is 0 and you can also show that so this is why it is called as scalar operator and Ji with A j will be if it is position co-ordinate you know what it is i h cross epsilon ijk A k so this is what we called it as vector operator okay called them to be vector operators if it behaves exactly similar to the way the components okay position vector behaves under rotation so you can go and check ji with momentum j components what it is.

It will be i h cross epsilon ijk P k angular momentum Ji itself a vector operator this satisfies your familiar angular momentum algebra you know how the components of the position vector changes it is also delta r do that the same way delta A will also change under rotations and if you try to write that out find at the commutative bracket of that because delta r if you remember it may be delta omega cross r for infinitesimal rotation do that for any delta of a vector operator.

If you do that if you compare both sides the only way both the sides can match if it is commutative keep this theta to be small they will be a commutative bracket and that commutative you can should it to me exercise for you practise exercise I did this elaborately for position operator, please go and do the same thing for any vector operator should behave exactly same like a position vector so this is a vector operator this is a vector operator.

And this is a vector operator they all transform whatever is the vector operator should be seen on the right hand side and what is the next operator which we should consider we did scalar we did vector then we should look at tensors one of the familiar tensors which you all studied is a moment of inertia tensor has two indices sure you will have done also in the labs working out moment of inertia also lot of problems you have done in moment of inertia.

It is a matrix that much you know the matrix and the matrix can be some mij matrix or iij matrix whereas a vector has a only one index and a scalar has 0 index okay I am just slowly and gradually taking you to the concept of the behaviour of different operators and rotations are different if you take the moment of inertia with 2 indices and if you do rotations what happens to it is what I want you to go and look it up if you take iij. And i am sure you would have all done this but in case you have not done this let me just.

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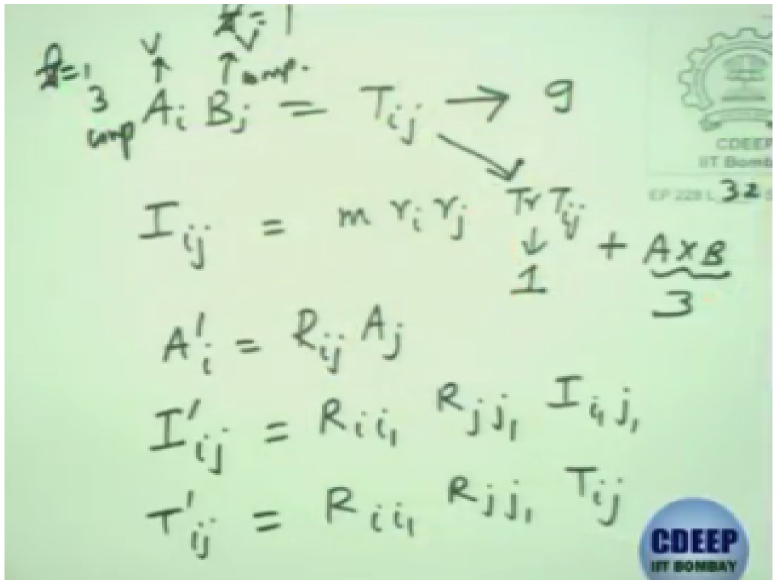

So if I take Ai Bj and call this to be a Tij so moment of inertia tensor this 1 let me write it as m ri rj and let me forget about all those subtraction and so on and just for example if I take this A and B are same allow when i do a rotation on 2 index object when I do a rotation on 1 index object i will get A prime i to be Rij it is a rotation matrix times Aj when I do it on I prime i j are lets even take T prime ij.

Under rotation what happens Ri i1Rjj1 similarly this one also a same R matrix how many elements are there in the 3 dimension. How many are there? 9 and if I look this up as a vector r i as a vector operator how many components it has 3 components this is another vector with 3 components combining 2 vectors is like taking tensor a product so I have combined 2 vectors this also combines 2 vectors.

This is a vector and this is a vector this vector has 3 components this has 3 components combining is a tensor product what is the dimension of this composition 3*3 they are 9 independent components you said 9 independent components of the composed object is made of 3 dimensional vector multiplied by 3 dimensional another vector does this rings you a bell something what I am trying to drive at we did uncouple basis and couple basis.

In the context of states, I can also compose operators take 2 vector operators and compose them and get a 2 indexed object and now I would like to resolve this is uncoupled okay I would like to resolve this into pieces which I will call it to be like similar to a couple case1 is trace of T matrix what is Trace of T matrix do under rotations trace remains invariant under similarity transmit the trace of T matrix is behaving like a scalar under rotation.

It is 1 dimensional this is 1 dimensional what else can I do I can take a cross product of these two vectors cross product of 2 vectors in 3 dimensional is a vector A cross B is a vector so I could write anti-symmetric combination which I call it as a A cross B how many components are there 3 across B has only 3 components and anti-symmetric T12 T13 T 23 and others are anti-symmetric 3 components but that will behave like a under rotation it behave like a vector what is left.

I have pulled out a trace I have pulled out a anti-symmetric operator what is left symmetric I have not taken it out and symmetric and you have to subtract the trace so that is the symmetric how many components in 3 dimension symmetric matrix and if you remove that trace is 1 condition you are not removing the diagonal elements so 5 components dimensional state in your angular momentum is called what? J = 2 is 2J + 1 which is some resemblance here.

What did I do? I got a composition as a raw which is 9 dimensional I would like to pull out a invariant under rotation which is a lower dimensional 1 another piece which is invariant under and it is not invariant it is behaving like a vector under a rotation and then the other 1 is j = 2 so these are J=1 and J= 1 because they have 3 components when you combine 2 operators of j=1 and j=1.

What are the possibilities we saw in angular momentum combination you have to get j=1 j=2 j=0 it is exactly what I get but I have not really done that algebra for you I just gave you some kind of a feel so all the operators of course all the operators are going to have only integer angular moment let us put it that way okay all my vector operator is physical momentum and angular position momentum operator everything is and these are all integers?

Yes, I should call this to be technically as l if you want because it has 3 components and it is a vector operator number of components it is an analogy there is 1 to 1 correspondence between operators also to behave the way we did the Clebsch – Gordon matrix this is just for you to think about it all your spherical harmonics you did all your spherical harmonics you never wrote it as never took quadratics.

And if you remember we did this we said that we can write all polynomials in the variable x and in 3 dimensional also you could do that is what you took you took a linear combination R theta Phi specially theta Phi in a way that you can call l=1 l=2 remember the way you did this you never took I am just trying to kind of give you a feel that.

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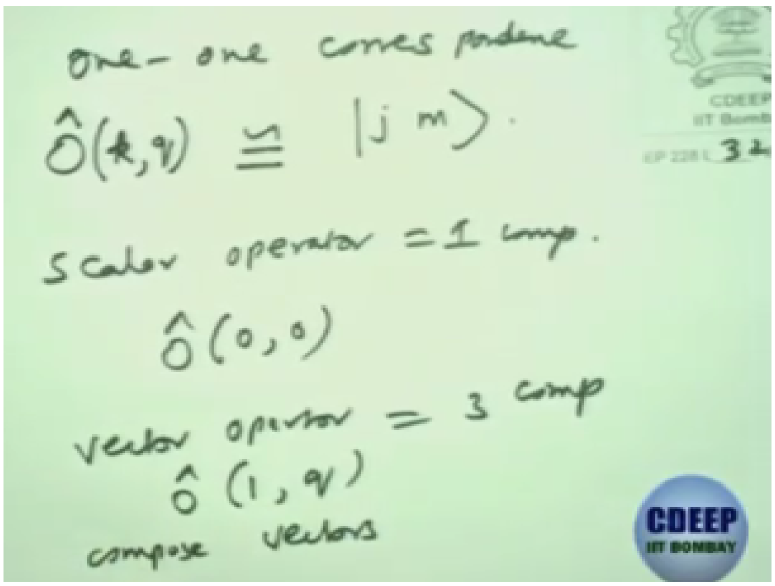

There is some kind of an one-one correspondence that you have a state j m there is an equivalent way in which you can say the operators with one index which I call it as k and other index which I call it as q so scalar operator is 0 0 vector operator is 1, q which has 3 components this has 1 component and the next thing is you can compose vectors to get the higher rank tensors can compose vectors you can compose 3 vectors to get a 3 rd rank tensors.

Whatever I did today is AiBj but you can take AiBjCk and get a 3 index form so we will go to this tensor operator in the lecture and see that the composition if you combine 2 spin j s how things happen will also happen in this the same Cj co-operations helps you to resolve let me stop here.