Quantum Mechanics

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Lecture - 66
Tutorial 11 - Part II

Now next problem is you write the x y and xz and also x square-y square as the component of second rank tensor, so you need to recollect here in this problem number 3 that in the tensorial notation and these.

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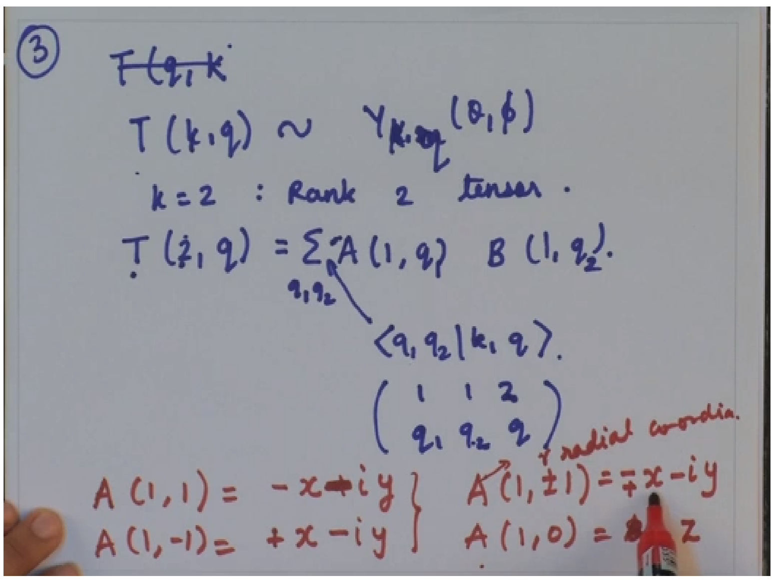

If I write T q,k sorry T k,q okay these can be written in terms of the spherical harmonics. l m theta phi so these things I think you all know you all have seen in your regular class that the tensorial notation or these tensors again be written in terms of this spherical harmonic. So, here you have you can write If write here k,q this is k,q okay theta phi and this tensor for example now I have 2,q this can be written as this is rank 2 tensor.

When I write k=2 implies it is a rank 2 tensor okay, so this is a rank two tensor and one can obtain a rank two censor by taking a tensorial product of the two that is A some 1,q times b 1.q and you take a sum over, so this is q1 q2. Sol if I take sum of q1 q2 so q1 can take any values and the co efficient here can be written as so this can be written as the coefficient which I missed here q1 q2 and k q.

Or another way of representing this would be 1 1 2 q1 q2 q so this q corresponds to k as 2 q1 corresponds to k as 1 q1 corresponds to k1 as 1. So in order to write the tensorial representation we need to have matrix, or we can write it in terms of our matrix but before that just remember how the coordinates or x y z are represented in terms of the irreducible tensors. So in order to calculate the irreducible tensors of the product of the two irreducible tensors.

We need to know how they are represented in terms of a reducible form. So, reducible for you write in terms of irreducible form and then you can prove this W or Omega is a product of these two irreducible tensor or scaler So actually they are in the next problem you will prove that the are scalers. So, that will be a generalized statement which we will see in fourth problem. So a point to remember here is if I have A okay.

I can write A 1,1 as -x+ -x-iy sorry -x-iy and a1-1 as -x will become +x so this will be +x-iy. So, in general let me write here A 1+-1 is written as -+x-iy and A1,0 is the z component Az okay. So, with this we can actually evaluate the direct way in which you can actually write x or here you can just write it as Z okay. So since I am writing in terms of the radial coordinate where is some radial coordinate?

Okay so when A is a radial coordinate you can write in terms of x+iy and A1,0 will be the z component. If I am writing A as some momentum operator, then it will be px py pz if I am writing in terms of vector potential it will be Ax Ay Az again the form will remain the same x will be replaced by Ax or Px Y will be replaced by Ay or Py X will be replaced by Az or Pz. So this definition you have to keep in mind before you actually start doing the problem.

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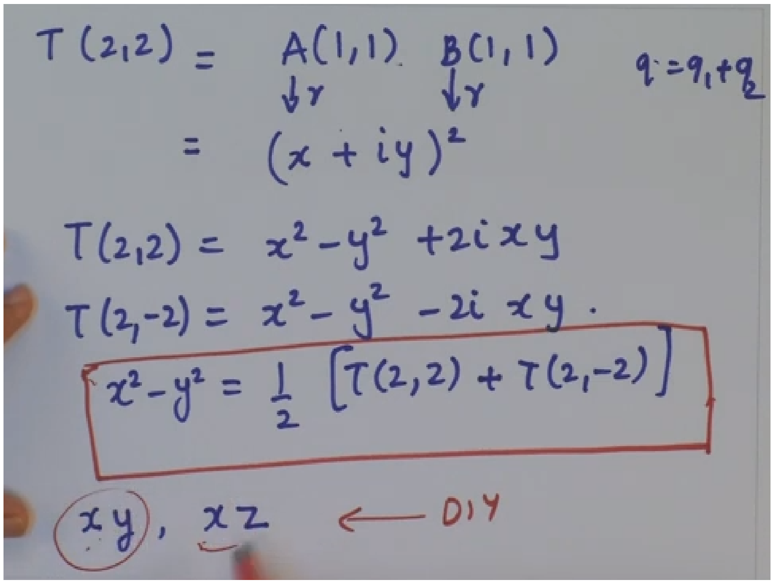

So, we will go to the next point wherein I will write this T now here k =2 which is the second order Tensor rank two tensor and q =2. So this I can write it as A 1,1 B 1, 1 okay so we take the example wherein you have a q cross q or 1 1 1 as a component So q we have a constraint over q that q should be =q1+q2 so we can have 1,1 over here okay. So with this I can play around I can write this.

So if I assume that these are the radial component only A goes to a radial component B is also a radial component Then my A 2,2 when I write in terms of the previous thing which I showed you 1,1 okay. So, I will have simply this as x+iy the whole square correct so I will have x+iy*x+iy correct this will be -x-iy so when I have this, we square it and I get x+iy the whole square. Now you know how can how we can write this 2,2 as x square-y square +2ixy.

And similarly 2-2 is written as x square -y square -2i xy from this simply you add these two and when you add these two you get the value of x square- square. So your x square- y squared will be 1/2 T2,2+T-2- 2 this is what is expected of you who can play. In the similar manner using the same line in the same stream you can calculate the values of x y and xz. So, this is another Diy okay you can do this you have to make some guesses.

Now x and y you can see directly from here I can subtract the two and obtain. So, I have already given you for this now think how will you get xz okay this thing and you will get it. The next part of the problem whatever we have done in this third problem a similar line of thought will help you to solve the fourth problem.

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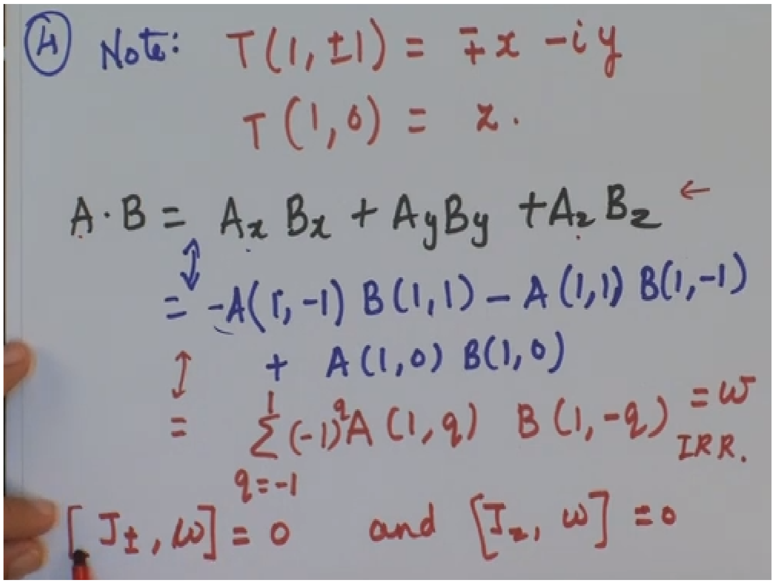

So, when you can start but again the reminder to you note that T I had written in the previous case but again I wish to rewrite it -+x-iy T 1, 0 is nothing but the Zt component and remember how we write the dot product of A.B whenever we write in Cartesian coordinate how do we write? We write it as Ax Bx+Ay By+Az Bz right. So let me write it for you all Ax Bx +Ay By+Az Bz this is the way I write.

Now let us do it in terms of the irreducible tensors this can be done you have to think a bit and convince yourself what I am writing here. A1,-1 B 1,1-A1,1 B1,-1+A1,0 B 1,0 so this I am not getting blindly you have to just rewrite this, or you can start by writing the form in which you can have A and B the possible values of A and B. So A can have the Tensors or the terms which involved with A or -1 1 0 1 1.

Similarly, for B and you have to make a combination such that you get Ax Bx + Ay By+ Az Bz so this you can try how this transition or the other way around you can start with this and get the term. Now let me write this term in the compact notation so what is the compact notation I can think of? Here is A I have 1 is there in all and q is changing q is -1 o or 0 either it is 1 0 or-1. So, I can take summation/q going from 1 to -1 times B 1-q.

You can see here that b is taking different value so I write it this way and I have a factor of - one times q. So, I will have this in the form of compact notation, so I have three terms you can again check this to this step it is very easy and in order to prove that the A.B is a scalar what do you have to do is you have to write the scalar product in terms of the irreducible tensor. So now you know that this is a reducible tensor reducible operators or terms which can be we have written it in terms of irreducible representation.

And I give you a hint that you use if you can show this was called as omega right? This term in the expression in the question is omega If we can prove j + - omega=0 an Jz omega=0 then you are indirectly showing that omega is a scalar so when you have a commutator of an operator with a scalar then it is always 0. This by notion you know but to do this job you should know what is the commutator of J+ J- and Jz with these reducible irreducible tensor. So I will give you another hint which would make your problem much simpler.

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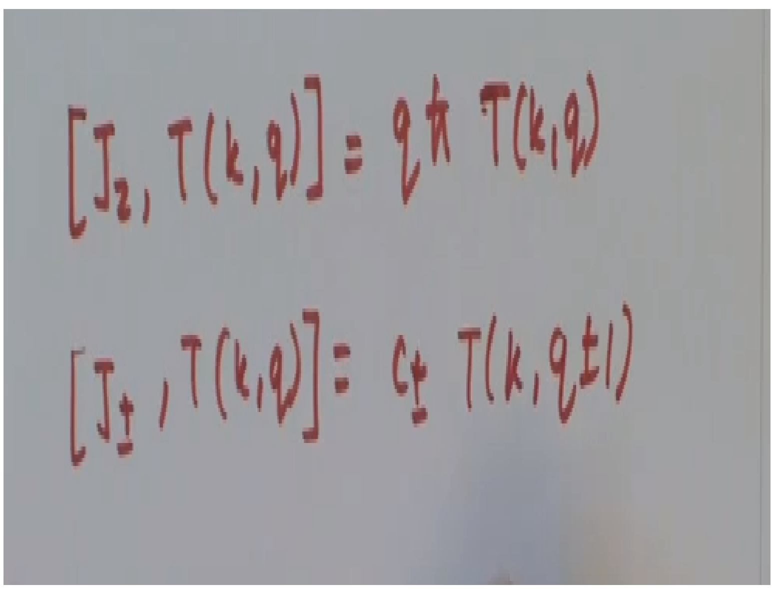

So this is J k, q is q h cross we have come across this before also you have seen this in a problems where we have a linear operators and we discussed about these kind of commutator bracket, so it is not a new thing for you but just trying to recollect it for you all so that you can directly apply it in the problems. Here I am not sure about the + and - so I just write it as c + - you will have this h cross square root of q k+q*k-q+ or – so you have to check that part. Okay.

But you have already with you, but this is just a hint an outline so that you go about for the fourth problem Okay fifth problem at this point we can actually skip but since it is a part of the tutorial, I will discuss it if it is not very clear then in the next tutorial, we are going to repeat such exercise such problem, so you need not get much panic if you are not sure what is going on because Wigner Eckart theorem will be discussed in the coming lectures.

So there will be a recapitulation of everything in the last tutorial, so the problem given to you is in in our rotationally invariant system in three space dimension the matrix element of the z component;

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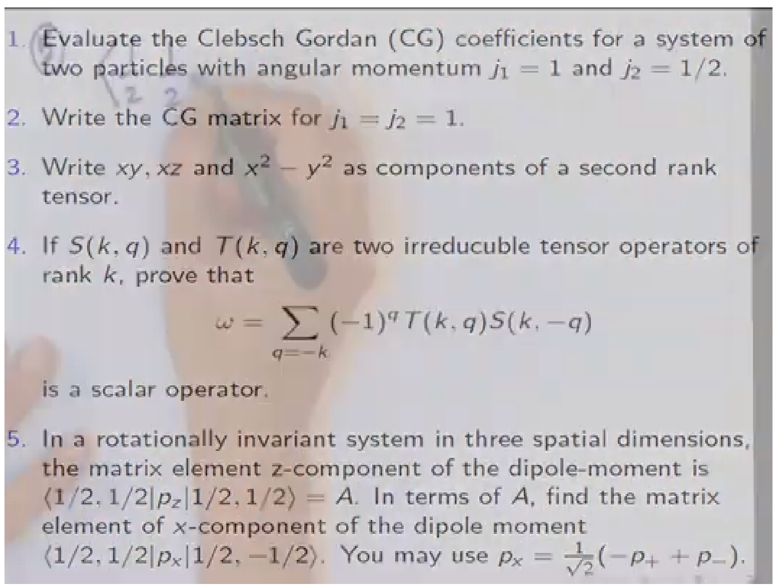

Of the dipole moment that is 1/2 1/2.

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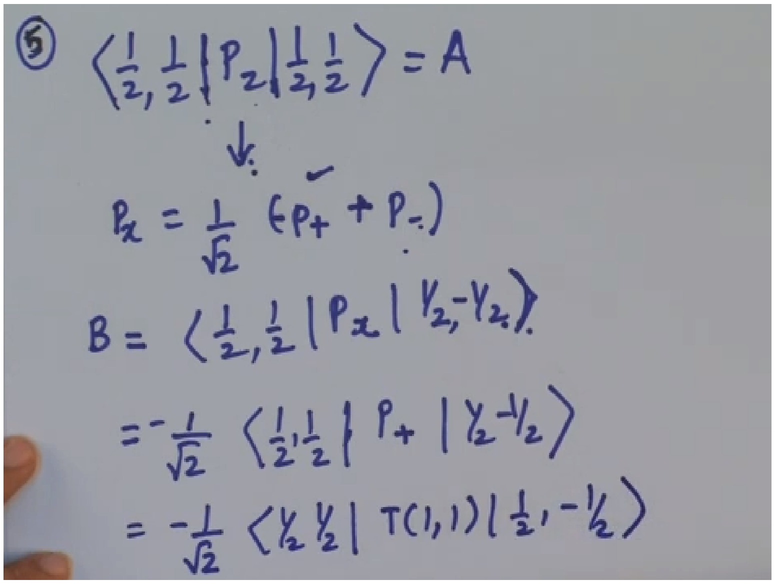

The Z component of the dipole moment is given to you as some A value and in terms of A we have to find out the x component of the dipole moment and some hint is given to you. Okay before we actually attack this problem what I want to say is that I will what I will do here is I will give you one example wherein you will have you will understand what I what exactly is being done in this problem.

So the thing is that when you are doing some measurements you are given the output of the z component the experimentalists give you the output of the z component and you want actually when you are performing the experiment you want the data of some other quantity. Here you have Pz so in order to obtain Pz you cannot make the measurement but there is some way in which you can obtain Pz Px so Px can be obtained by using Wigner Eckart Theorem.

When you use Wigner Eckart theorem you will not get the exact value of the observables mind you, but you can rewrite in terms of the known observable. So that means that in this problem you can write Pz such that all you take the ratio of Pz and Px. So, here you are given a hint that Px is 1/root 2 P+P- but there is a -sign here and P-. So when you are attacking such problems you have you can write let me call B as 1/2 1/2 Px 1/2 1/2 okay.

This is what is asked right here there is a -sign okay I have skipped the commas, but you can understand so this I can write it as P+ and P- obviously P- will not act on this P+ will act on this.

So this will be the this will only give us a non 0 contribution, so this will give me -1/2 1/2 P+1/2 -1/2. Now again you have to write this in terms of irreducible tensor so when I write this in terms of irreducible tensor what do I obtain.

I will have remember I will write here the Wigner Eckart theorem or let us move first to the reduceable Tensor part and then I write the Wigner Eckart theorem. So this would be nothing but 1/2 1/2 here I will have a 1,1 1/2 -1/2 so with this you can move further and write the Wigner Eckart theorem. So, Wigner-Eckart theorem is;

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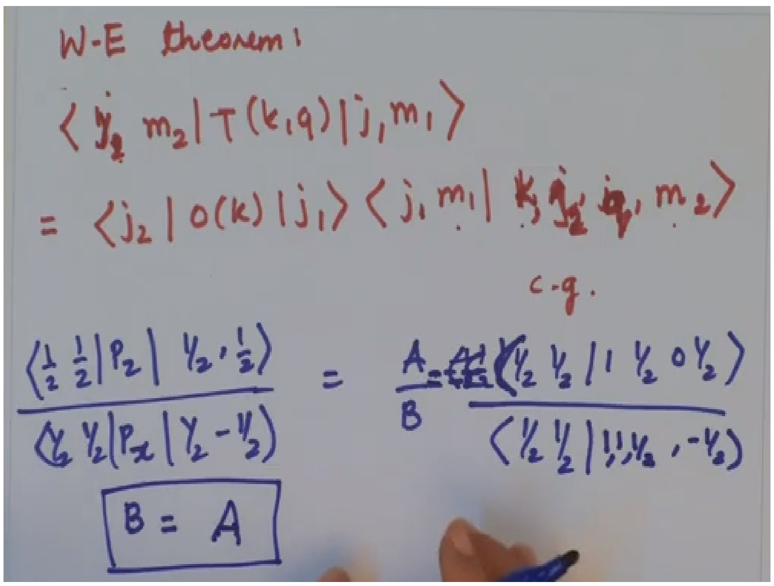

W-E theorem I am writing the short form W-E theorem is that you can write this 1/2 I can write in terms of m1 and m2 j2 m2 T k, q j1 m1 can be written as the constant part j1 and j1 m1 k, q here you have j2 q, m2 this is a k this is j2 this is q, and this is m. So, I have j2, m2 ,j1,ml and this is my reducible irreducible a tensor and I can write in terms of two pieces one constant part another which will be of the form of cg coefficient which you can evaluate.

So this will be j1 m1, k, j2, q, m2 this is the way it is written down so for the case of Pz that is my Pz was 1/2 1/2 Pz 1/2 1/2 / 1/2 of Px 1/2 -1/2 this is A/B and this quantity here this can be reduced or rewritten as I will have two terms which exactly cancel these constant term. So when I write A/B I will have 1/2 1/2 1 1/2 0 1/2 okay just take care that here I have k is 1 q is 0. So, this is the result and, on the left, obviously I have 1/2 and 1/2 okay I have written that.

So, this is the bracket, then I have 1/2 1/2 and here I have 1 and then I have a 1/2 then I have a 1 and a -1/2 okay so these quantities you can evaluate. So, now our B would be you have to now evaluate using the cg table you can use a CG table and evaluate these quantities. So, here your k is 1 and your q is also 1 okay B you will find that =A. So if this is not very clear you will get the essence of this problem in the next tutorial. So let us stop here and we will continue for the next tutorial in the next session which will be the final session.