Quantum Mechanics
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Lecture – 62
Addition of Angular Momentum - II

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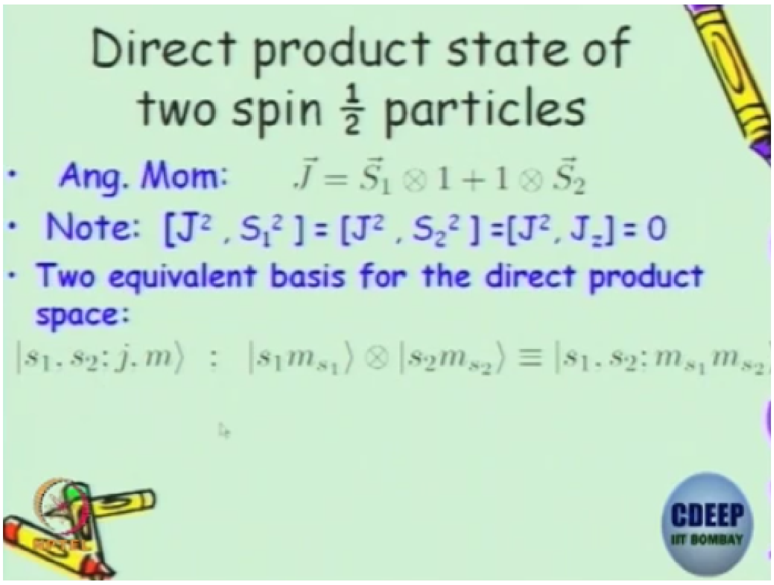

So so far we did combination of spin-orbit coupling, but we can try to take 2 particles at rest that means orbital angular momentum is 0. Then we can take 2 spin half electrons and combine them, we can do that, same methodology I told in earlier slide, what is the modification? The only modification is a direct products space will be a spin space of first particle times, the spin space for the second particle, is that correct. You can see it in that fashion.

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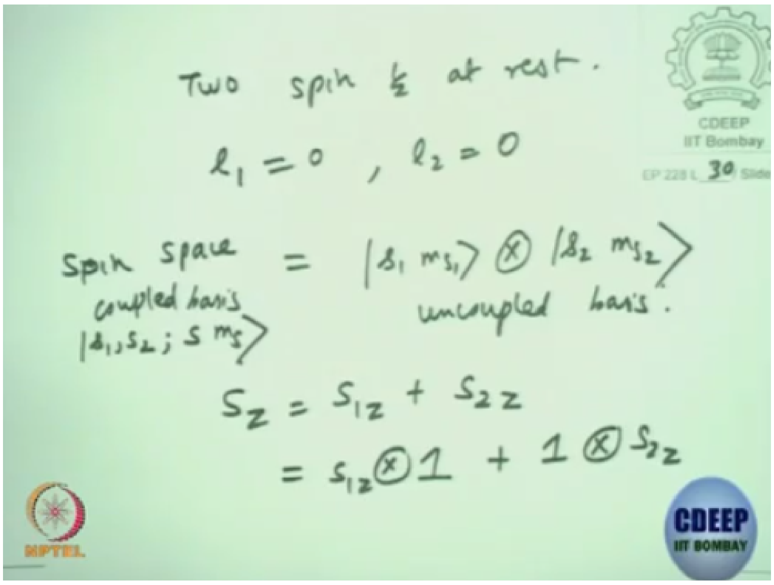

So you can take 2 spin half at rest okay, so that means the l for the first particle is 0, l2 for the second particle is 0. I do not need to worry about, if something is 0 spin it is like as if it does not play too much of a role. Adding a 0 to anything, angular momentum addition does nothing okay. So we can try to take the space as spin space now for such a particle, you can write it as a direct product space of s1, ms1, s2, ms2.

I have suppressed the orbital quantum number assuming that it is at rest, is for simplification. What is this space called? It is called as an uncouple okay. So this can be, this is what we call it an uncoupled basis. You could also write the total spin right. We can write the total sz as s1z + s2z, what is the meaning of this in this couple basis, it should be technically treated as s1z cross identity + identity cross s2z in this direct product space.

But I can also take the couple basis where it is total z, total sz and total s and I can write a couple basis as s1, s2, s, ms, this is a coupled basis is that okay, s1, s2 will be half yeah. Yeah you can do that but just for simplicity I am just saying the tag of particle 1 and particle 2, I just want to keep it, but if you have 2 spin half you can put s1 and s2 to be half (()) (03:38). Technically you can write half, half there.

So these are various scenarios where you can combine 2 spin half particles at rest, you can combine orbital angular momentum and spin quantum number of a single particle. You can have various other things, I do not really worry about experimentalist tells that this object has angular momentum J1, another object has angular momentum J2, can you combine and do the physics, then what will you take, it is J1 + J2.

All kinds of combinations and you do not really worry about once he gives the angular momentum you do not question what is the orbital, what is the spin you take that as an entity which is given to you. So given whatever is the data, you can write, this is what I have tried to motive you that you can have 2 basis, one is uncouple basis where individual particles in this case the tag has maintained.

Individual spin half particle information is maintained in the uncoupled basis, but in the coupled basis, you do not know what is the contribution of the individual spin half particles okay. You know the total quantum number, so that is why it is called coupled. So again 2 coupled equivalent space you can write as he said you can put s1 and s2 to be half just to keep track of that tag of 1 and 2 particle I have put it here.

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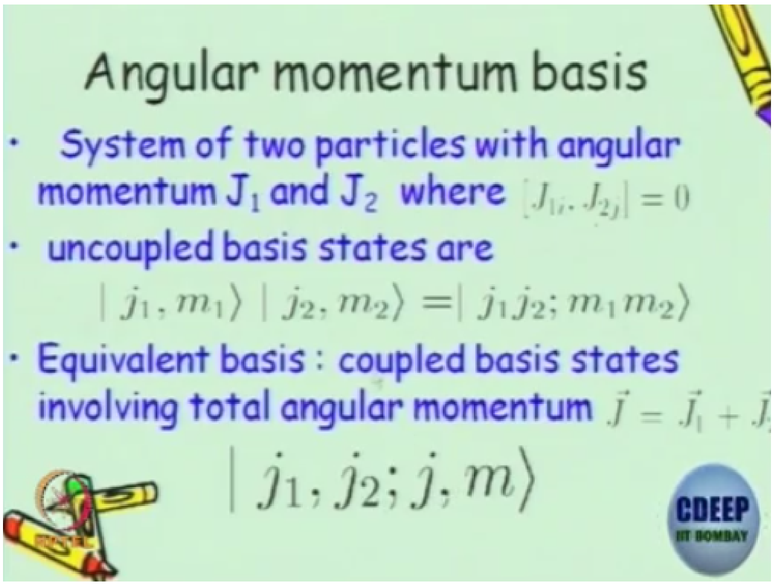

So now I try and take you into a formal that given 2 particles with angular momentum. I do not question what is the composition of this angle of momentum, the experimentalist tells me that this particle has quantum number J1 and the second particle has quantum number J2. How it is composed of orbital and spin he does not give me an I do not worry. The only thing you know is that these 2 particle angular momentum operators they have to commute okay.

When you take 2 particles even in the earlier case, s1 operator, commutes with s2 operator if they are 2 different particle okay. So you can take J1i commutes with J2j where J1 is for particle 1, and J2 is for particle 2 and take any particular component i'th component and the j'th component and that combination is that commutator is always 0. This is the meaning of saying that these 2 particle angle of momentum operators they commute.

So uncoupled basis states we can write the tensor product or a direct product of j1 m1 times j2 m2 or equivalently people write this as j1 j2; m1 m2. This also I tried to stress with spin orbit and 2 spin that you can write the total angular momentum formally as j1 + j2, but you should remember the j1 will operate only on the first state and similarly j2 will operate on the second state.

Most of the times they do not write cross identity + identity cross j2, but you should remember this. So there is an equivalent basis in the coupled angular momentum or total angular momentum which is an equivalent basis because the dimension of these 2 will match. So the change of basis, how do you do it, I have kind of indicated to you, one is the dimensionality have to be equal.

You know that here the dimension is 2j1 +1 times 2j2+1 and in this case the j will go from j minimum to j maximum and 2j +1 has to be summed up and it should add up to give you this, is that clear? What is the range of J I have not really told you this j, this j, but we will come to it, but there are constraints.

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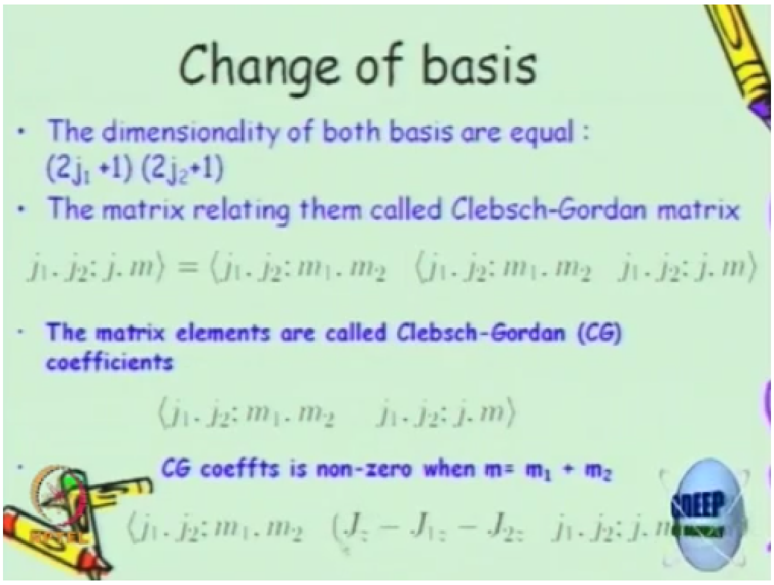

The constraint is that the dimensionality of both uncoupled basis and the coupled basis have to be equal. The matrix which relates them, this is what is the famous Clebsch-Gordan matrix okay, it is called CG coefficient matrix or Clebsch-Gordon matrix and interestingly you can put an insert and identity operator is that okay, I think I have made a mistake here yeah, you can write the identity operator.

This should be a ket, this one right, so this is a typo please correct it, it should be a ket. So you can put an outer product here and sum it up. So that this matrix element this inner product which you have is a matrix element of this Clebsch-Gordon matrix. So let me just redo it on the sheet for this is very important so I do not want you to.

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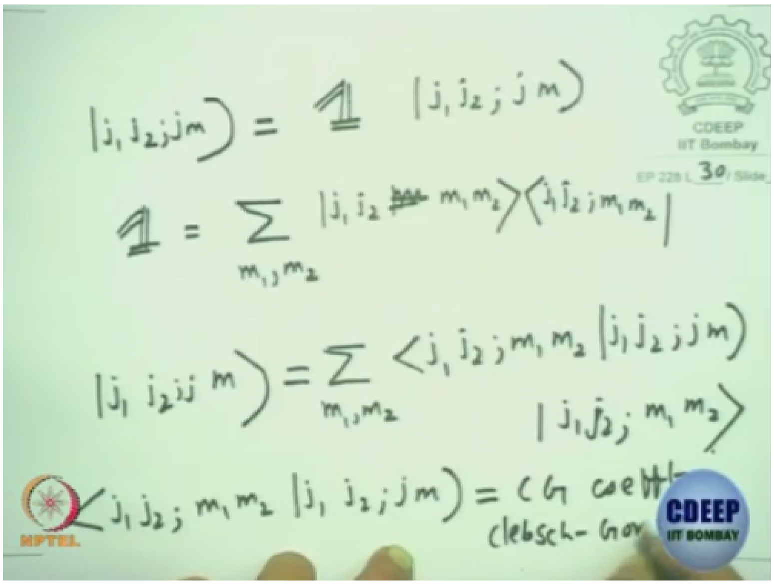

So you will have j1, j2, jm okay, so this is what we call it as a couple basis. I prefer to put this couple basis with the curved line, usually we put it as angular bracket, just to remember that it is curved, it is a coupled basis, I just like to keep tag of it by putting a curved line, not that books put it but just for our sake we will put this. Here I am going to insert an identity operator and write this j1, j2, jm and this identity operator is in the uncoupled basis which will be m1, m2 okay.

So you have j1, j2, jm will be summation over m1, m2, I have just inserted it here, wrote the inner product which is a coefficient okay, is that alright, and then it is in terms of the state. So this coefficient j1, j2 m1, m2, j1, j2, jm, this is what is called as a Clebsch-Gordon CG co-efficient, we write it for the first time, Clebsch, always miss this sch, Gordon.

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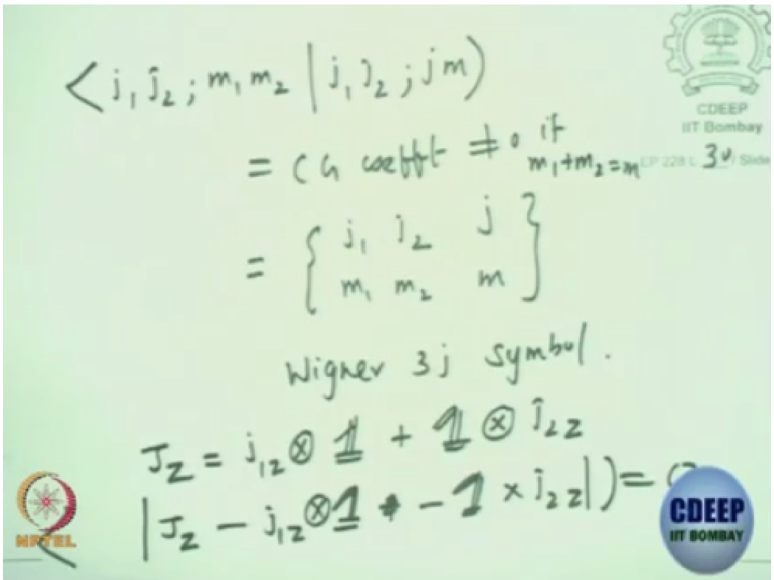

What did we do, it was j1 j2 m1 m2 j1 j2 jm, just to remember that it is coupled basis I have put this angular bracket, it is not necessary, this is the CG coefficient, sometimes, some books denote this this way okay. Sometimes they call this as a Wigner 3j symbol okay. This is just to remember that you combining j1 and j2 with a corresponding magnetic quantum number m1 and m2.

The combined j1 and j2 is the total j, that j is not just a simple scalar addition, the j is having some range, we will come to it and you also have a corresponding magnetic quantum number associated with this. This is a different notation which sometimes the CG coefficient matrix are written in this notation. Both are equivalent. So this is the matrix element of the coupled state, coupled basis state with the uncoupled basis state that matrix element is what will give you the coefficients of the CG matrix.

This is one of the elements, so this depends on whatever is the different jm and different m1 m2 will give you all possibilities. Our aim is to work out for 2 spin half particles at rest. What are these coefficients. Similarly let us take orbital angular momentum of a particle to be 1, which is the spin 1/2 particle, let us find out what is the CG coefficient for that is and the reason we need the CG coefficient is, if you know the CG coefficient you can go up between coupled basis and uncoupled basis back and forth using the CG coefficient.

Some situations the couple basis will be a convenient basis, in some situation uncoupled basis will be okay to work with, but if you want to go back and forth you should know what is the CG coefficient you should know how to compute the CG coefficients, that is what is the main you know calculational things which we are going to do. So this angular bracket has to be a ket there and the matrix elements are called the CG coefficients and I have put the same bracket here.

But you could try and just for remembering as couple bases you can slightly smudge this bracket okay. How do we prove this? How do we prove that the CG coefficient is nonzero only when this total n is restricted to be the sum of m1 and m2, how do we prove this? It is very simple. Take the total Jz operator, Jz – J1z – J2z, what is that Jz = J1z + J2z. So Jz-J1z-J2z is a 0 operator, null operator.

The Jz will prefer to operate on which basis? Jz will prefer to operate on the couple basis right. Jz is the total J, Z component of the total J, it will prefer to operate on the couple basis that will give you mh cross, j1z will prefer to operate on uncoupled basis, I will give you m1h cross, j2z will give you m2h cross. So m-m1-m2 has to be 0 because this is a null operator. Many CG coefficients will be 0 trivially by this condition.

And the good part about this is that this also fixes some kind of a selection rule in experiments. When they take a selection rule of combining interacting 2 particles with magnetic quantum number m1 and m2, they have to make sure that such an interaction should satisfy this condition m=m1 +m2.

So many processes in the lab which gives you a selection rule they are all dictated by the simple operator rule which we see in our angular momentum algebra between the coupled basis operator and the uncoupled basis operator the difference has to be 0 that forces this condition. So what have we seen here, we have seen that Jz which you can write it as j1z cross identity + identity cross j2z if you take Jz – j1z formally people do not write this.

But if you want you can put this in for at least for the first time and - identity cross j2z that is 0 and you take it between the coupled basis and the uncoupled basis. So this is uncoupled and take it to be the coupled basis. So the uncoupled basis is this and couple basis is this. If you do that Jz will give you mH cross and if you do not want your coefficient to be 0 then that tells you puts a restriction that this coefficient is nonzero if m1 + m2 is m, (()) (18:26).

What else can we get out of this, this kind of a simple operator between coupled basis operators is this, uncoupled basis operators is this, can we get some more information with J + -, we can do that also right. You can put J + here and write it as j1 + cross identity + identity cross j2+, you can do that and we can start seeing is there some kind of a restrictions which we can get. So this is the simple restriction which I have got here.

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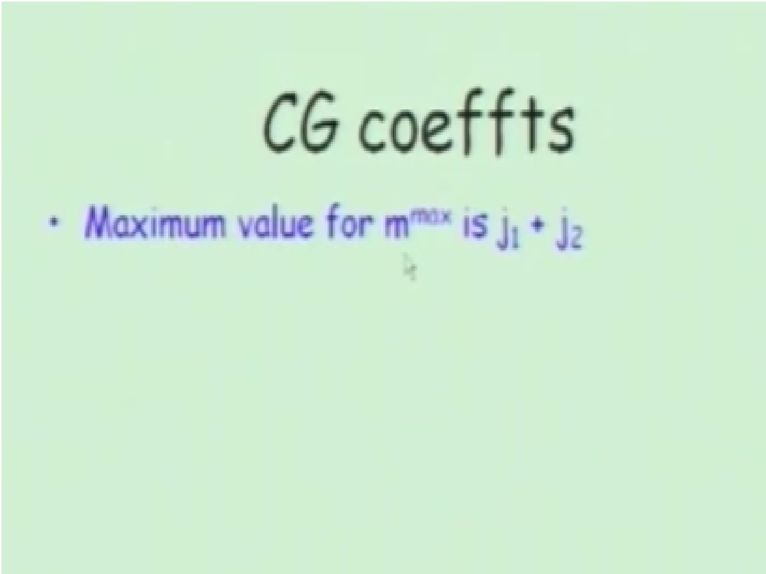

Okay, what is the maximum value of the total magnetic quantum number? The j1 maximum is m1, has to be j1, m2 has to be j2. So m maximum is j1 + j2, if m maximum is j1 + j2 what is the corresponding total j maximum, jm when I write if m max is j1 + j2, the j has to be at least j1 + j2 and if it is the maximum it cannot exceed that, that will be the maximum value for. So let us that summation which I did for J max and J min I can try to.

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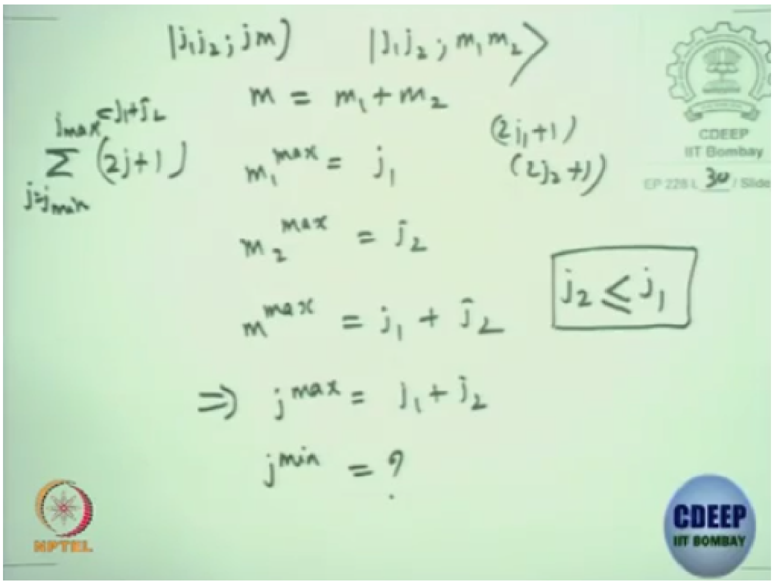

So I will write m has to be m1 = m2, m1 max is j1, m2 max is j2, so your m max = j1 + j2. This implies j max has to be j1 + j2 because this side is the couple basis, this side is the uncoupled basis and from here the j and m I can find it out. Can we determine what is j min now? You can use the equation with the dimensionality. We had here the dimension was 2j1+1 * 2j 2+1.

This side will be 2j+1 summation from j min to j max okay. Just for simplicity let us check j2 to be less than j1, less than or = j1 once you know this you can do the other way round also and then we can figure out what should be j min. okay I will leave it to you as an exercise, just check just by doing this series J max is given to you, it is j1 + j2 that is given okay, find out what is j min, what I will do is let me stop here.