Quantum Mechanics
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Lecture – 61
Addition of Angular Momentum - I

Addition of angular momentum, it is not a scalar addition that is the first important thing, so addition of angular momentum is a very important topic which you have to use in most of your atomic molecular and nuclear particle, so you have to be attentive to know what is addition of angular momentum, okay, so that is the theme.

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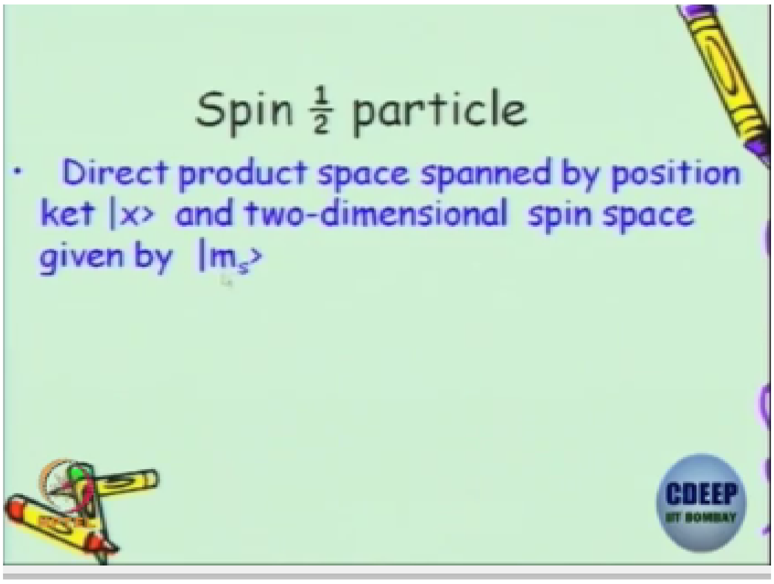

So, spin 1/2 particle we have extensively studied by looking at the Stern-Gerlach data that it has 2 states, the linear vector space is 2 dimensional, right but technically once I say is a spin 1/2 particle, you need to remember that the particle is in the physical space and then it also has an internal spin space, so physical space you can either put a position basis or you could also use the angular momentum basis; orbital angular momentum basis.

So this is what we call it as a direct product space, the direct product space or tensor product space is a product of the physical space multiplied by its internal space for this spin 1/2 particle, okay, the internal space ms; ms will be up spin or the down spin, it is a notation okay, so let me write it for you.

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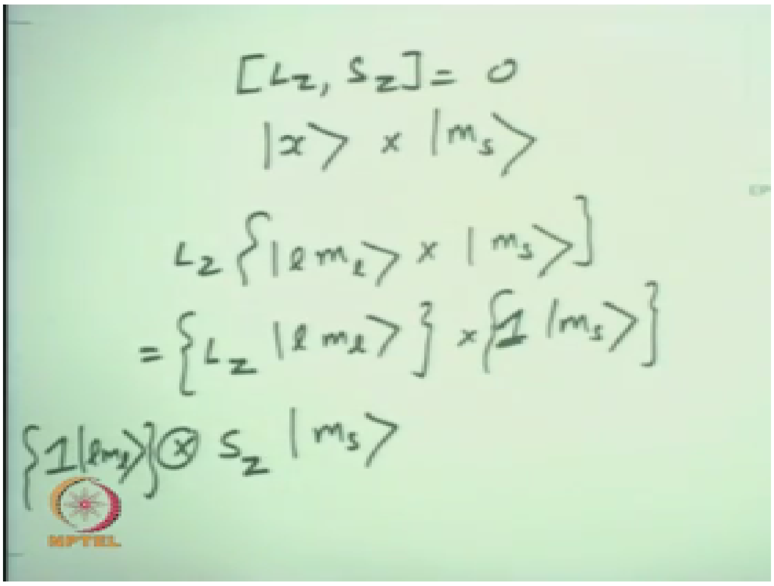

So, when we write a state, I can write this as a direct products states equivalently, you can you also write m, okay I am not putting an s because I am taking S to be 1/2 that is the only reason, these are equivalent direct product space. When I want to do Lz; Lz well operate on l ml and it will be like an identity operator on ms, suppose I do Sz, Sz will operate on ms and it will be an identity operator on l ml, clear.

So, even though I write Sz operator it will go on operate on the respective state in the direct product space so this is what is the meaning of Lz operate in the direct product space even though I will write Lz operator on this, you need to know that this Lz operator on this means this one, so this is what is the direct product space, where you have orbital angular momentum space separately and spin angular momentum space separately and we work with this.

So, if you have a Lz, Sz, so you have to operate Lz on the state and Sz on that state, it can change the order why is the reason why you can change the order tell me, they come in two different places, so Lz, Sz is 0, they are in 2 different spaces, so this same holds for L squared also, same holds for L squared.

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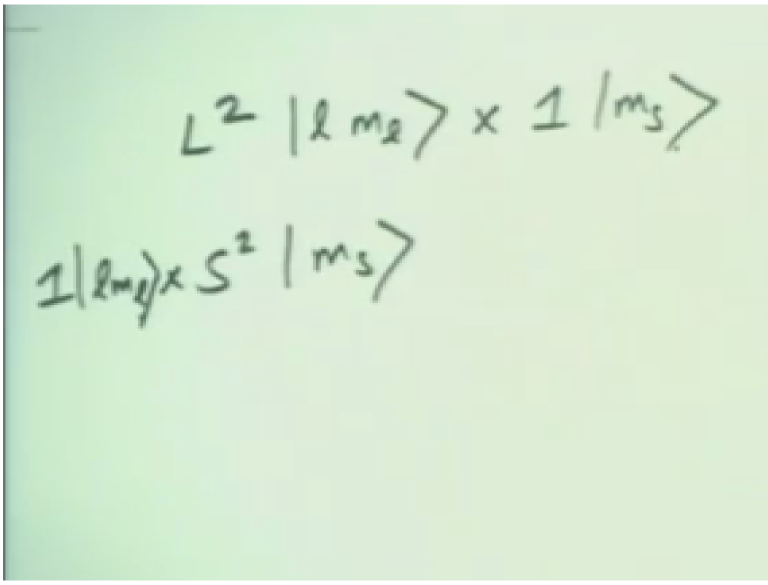

L squared will operate on l ml and identity on ms state and similarly, S squared will operate on ms state and identity on the l ml, I am just keeping this order in the sense that the first ket represents the orbital space or the physicals space and the second space represents the internal space, this is just the normal, so what is this L squared on l ml, all of you know similarly, Lz is also you all know, so I do not need to repeat it, right.

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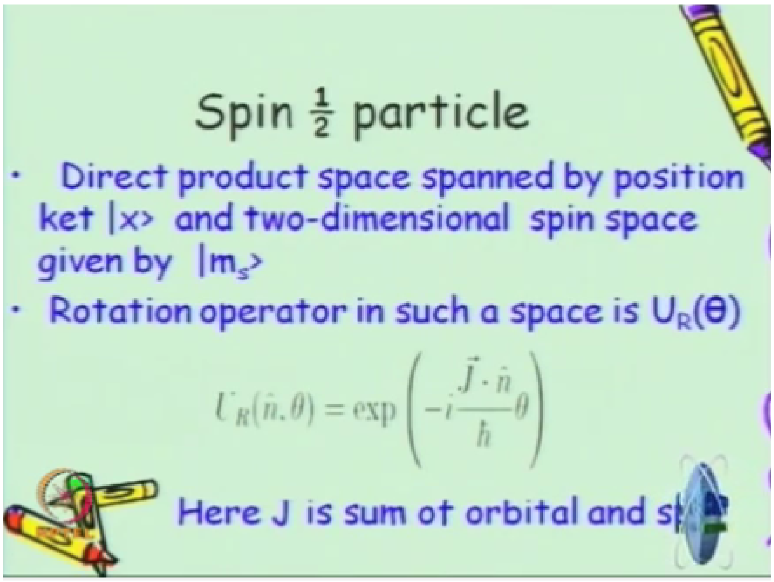

So, L squared will be l * l + 1 h cross squared, Lz will be; so the direct product space spanned by position ket or l ml and 2 dimensional spin space given by ms, what happens to the rotation operator; rotation operator formally for any spin j, how did we write; we had an exponential of, this we had extensively discussed in the earlier lectures, its j dot n but j is composed of sum of 2 orbital angular momentum and spin angular momentum.

That sum has to be seen in the direct product state as Lz will operate on the l ml, Sz will operate on the ms, have to remember that so, in the same way can be write this Ur operator in a neat fashion?

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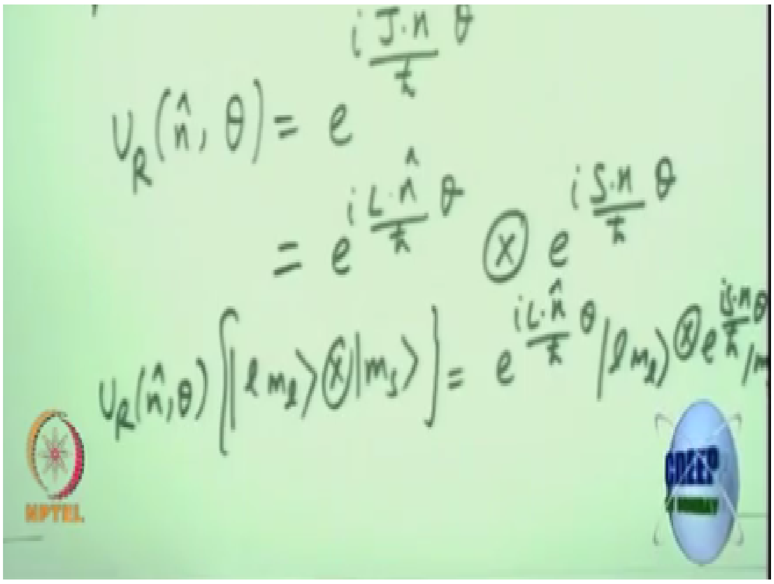

So, UR n hat theta, is exponential of i J dot n/ h cross times theta, I could write this in the direct product space as e to the i L dot n/ h cross theta direct product e to the i S dot n/ h cross theta, is that correct, so this operator will operate on and l ml, this operator will operate on; so if I want to write what is UR n hat theta on l ml cross ms that will be e to the i L dot n hat/ h cross theta on l ml crossed with e to the i S dot L/ h cross theta on ms, is this clear.

So, it separates, it operates on this, the L operator should be treated like an identity operator on the ms state and S operator should be treated like an identity operator on the; so J is formally as sum, people will write it as a sum of orbital and spin angular momentum but you should remember that it is on the direct product space.

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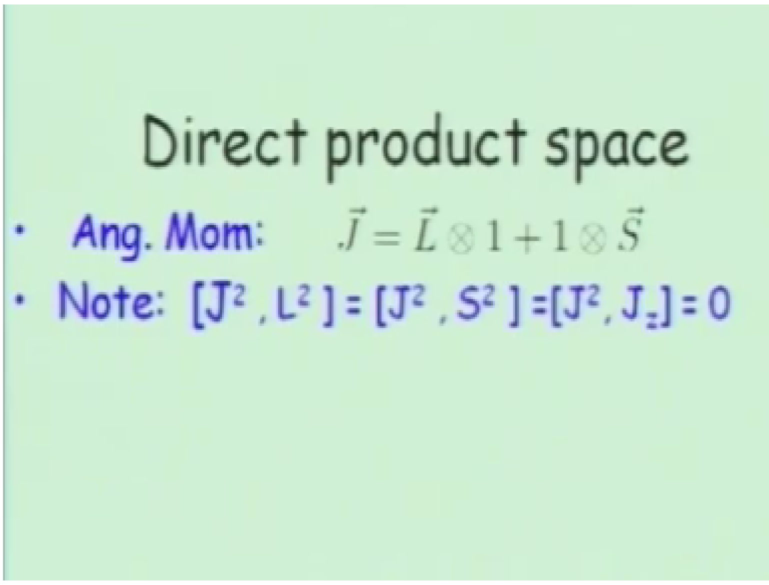

So, this is what I was trying to say formally, when I write J, the J operator on a direct product space should be treated as L operator on the l ml state and identity on the ms state and similarly, identity for the l ml state times the spin operator on the ms, many textbooks do not write this, many textbooks writes j is l + s but when you are operating on the direct product space, you have to make sure that it is it has to be done this way on the direct product space.

This is at the back of your mind, when you are doing it, they do not write this identity anywhere, many books do not write, they write l + s but technically, you should know that on the direct product space, it is identity for the internal space for the orbital angular momentum operator and identity for the spin operator in the position space. There are some things which I want you to check yourself, you can take the square of this, operate it on this arbitrary state.

A simple exercise to show that J squared, so J squared is L square cross identity + identity cross S square, you can write it like that in the direct product space and if you try to operate, you can show that J squared commutes with L squared, this is an exercise for you to try it similarly, J squared with S squared is also 0, so we can also write Jz as Lz cross identity + identity cross Sz. And also check Jz will commute with J square.

What is this set? tell me; if I have the set which is written in terms of total angular momentum, total Jz and S squared and L squared, is this a compatible set; this is a compatible set again, okay so you can have 2 compatible set.

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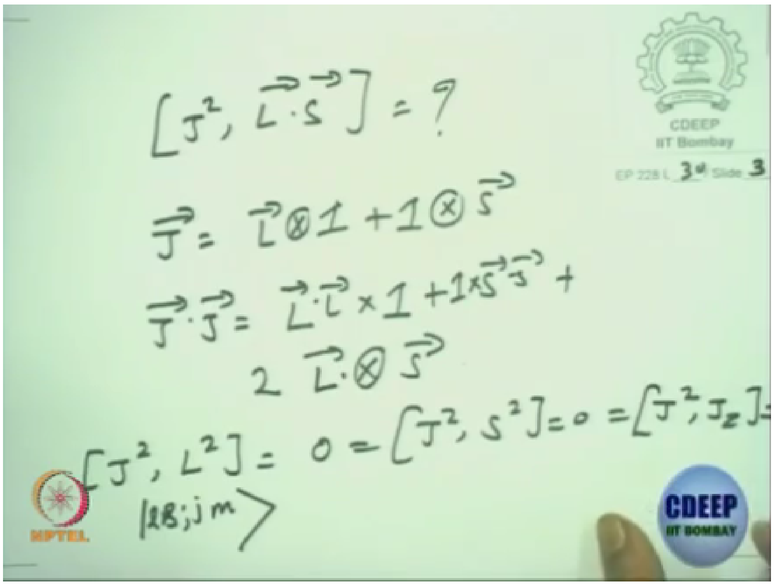

J squared with L dot S, you know that is fine but I am just trying to say that the L dot S here equivalently you can check this out for you, J is L cross + identity cross S, so J dot J when I do you are trying to say that there will be an L dot L cross identity + identity cross S dot S but then there will also be cross terms that is a good point yes, you will have the cross terms from here multiplied with this which will give you an L cross S.

And then you will also have twice right, just twice of this, sorry I made a mistake here, so you will have this term and this is in some sense, it is like the L dot, you have to remember that there should be a dot here, it is you know it is even though I write it as a; this side is a scalar, so you have to take care that it is a scalar, so this will hinge on this information that is also I want you to check what it is, okay.

So, I am just trying to tell that J squared; this is for you to check L squared = 0, J squared with S squared is 0 and you can also show J squared with Jz is; this is trivial, this we all know, so this gives you one compatible set, can I denote that compatible set neatly; I can denote it and do not need a magnetic quantum number of L and S but I need a magnetic quantum number of the total J, okay.

So, I will write jm; jm is the total magnetic quantum; total magnetic quantum number and then you can have an L and S, you can denote something like this, so this is the simultaneous Eigen state of L square, S square, J square, and Jz, okay, this is a shorthand notation of writing a simultaneous Eigen state which for this compatible state, so we have written a total magnetic quantum numbers here, we have not written separately that is not there in the set.

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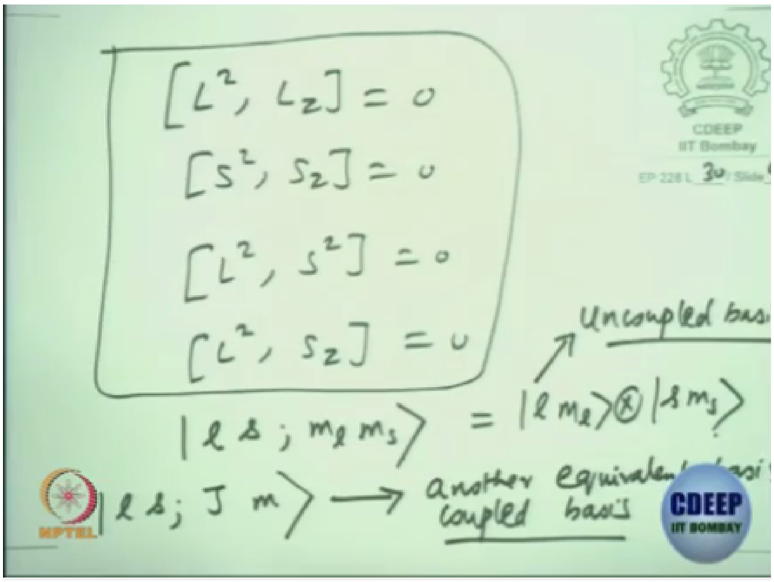

What is the other set possible, where we can get the magnetic quantum numbers, we also know L squared Lz is 0, we know S squared Sz is 0, we also know L squared with S squared is 0, L squared with Sz is 0, is this also compatible set, what is this compatible step, I can write ls, ml, ms, what is the difference between these 2 sets; this keeps the tag of orbital quantum number and the spin quantum number; the magnetic quantum numbers.

The magnetic quantum numbers of the orbital part and the spin part it keeps tag, so it is like I will say this piece is the orbital quantum number, this piece is the spin quantum number then I have this information together, right I can equivalently write it as l ml cross s ms, is a direct products space, so this direct products space is sometimes said that it is a uncoupled basis, why; I can say which one is the orbital quantum number and which one is the spin quantum number.

So, this is what we will call it as a uncoupled basis, this is also one set, so this is one set; compatible step and the basis is called uncoupled basis, the reason is that it keeps tag off separately orbital and its magnetic quantum number and spin and its magnetic quantum number. The other basis which I wrote is also another equivalent compatible set but then that one keeps track of the total J and the total magnetic quantum number which is like a combined.

It has combined, you do not know which component is orbital in it, which component is spin in it, you cannot make out okay, so in some sense we call this to be another equivalent basis which is called coupled basis okay, it is called coupled basis, you cannot have an individual identity of what is the magnetic quantum number corresponding to ml and what is the magnetic quantum number corresponding to ms.

Suppose, I say m is some value, it can be made of all possible things there, so that is why it is called as a coupled basis and the other one is called as a uncoupled basis as long as you are in a system, especially in your experimental system, if there is no interaction like as he already pointed it out, if you do not have an L dot S term okay, if you do not have; suppose, I have if you turn on a magnetic field, you have the magnetic moment which is dependent on Lz times the magnetic field.

And spin magnetic moment which depends on the spin quantum number suppose, you have a L dot z term; L dot S term, so this is what experimentally could happen.

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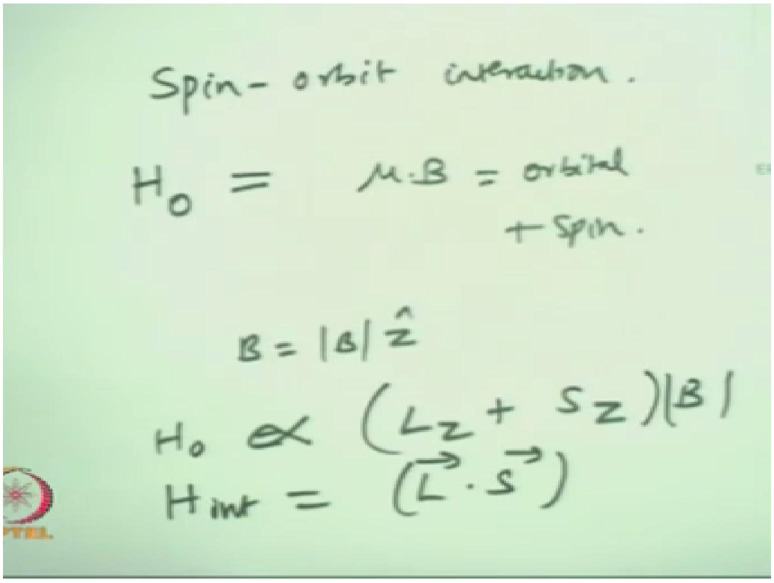

This is call spin orbit interaction, okay so typically your Hamiltonian to start with let me call the initial Hamiltonian to be a Mu dot B which could be having orbital piece + spin piece, let us take the B along the z direction, take B to be mod B z cap, you could have that H0 to be proportional to Lz + Sz times B, here Lz is separate, p is S is separate but I could turn on something an interaction which is L dot S.

So, if I do this, this interaction kind of allows us to go to the coupled basis, okay because you can try to write what is the total angular momentum and start working with it, you could do this in uncoupled basis, you could do this in couple basis both are equivalent, what do I mean by equivalent? If you remember when we did this two dimensional vector space, we had one basis, we had another equivalent basis, then those 2 basis are related by say unitary transformation by which you can relate one to the other, okay.

Similarly, for such a system you could work with uncoupled basis or coupled basis, sometimes coupled basis will be convenient and then you should also know how to go from one basis to another and that matrix is the famous matrix.

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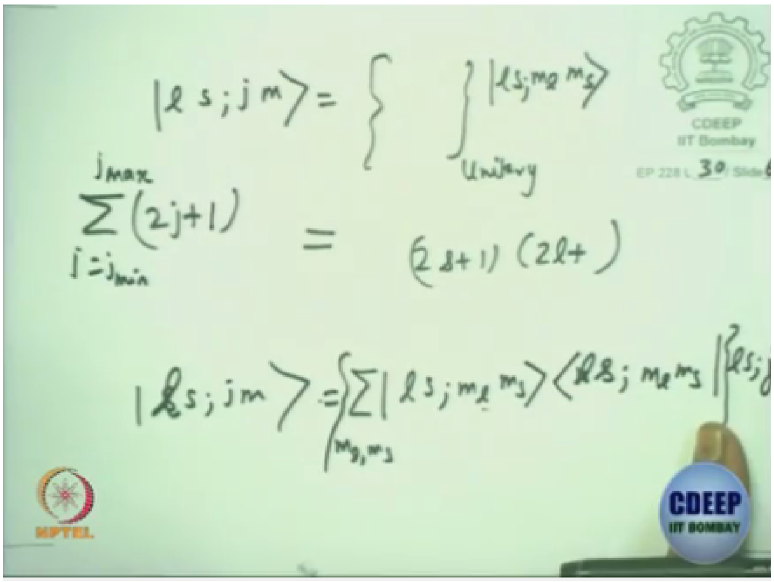

So, you have a ls, jm, you also have ls, ml, ms and you have a unitary matrix, its relates them okay, both are equivalent, what do I mean by equivalent; the dimension of this vector space should be same as the dimension of this vector space, that I have not checked but once I say it is equivalent that is what you did, when you did a rotation of the basis in a two dimensional vector space, you cannot need a three dimensional vector space, it is still 2 dimensional.

Unitary transformation does not take the dimensionality of the vector space, what is the dimensionality of this vector space; 2s + 1 2l + 1, so that is the dimensionality of that vector space which is an uncoupled vector space, so here I have not specified what is J, I am not specified what is J but definitely m will go from -j to +j that is the definition of the angular momentum algebra for every j.

But there are so many values of j here, what is the dimensionality for a specific j; it is 2j + 1 and then you have to sum up over some j, which is j min to j max, I did not say what is the values j takes, let us assume it takes from j min to j max, that we not say what that value is, what is the requirement; if they are equivalent basis, this sum should add up to the dimensionality of the vector space has to be merged.

And j has to be either integers or 1/2 odd integers, it goes always in steps of 1, okay so these are the restrictions which we have and there is a matrix which relates the same dimensionality basis to the same dimensionality equivalent basis related by this matrix, how can we write this matrix, we also know how to do this. Let us do that we can insert; so if I take this, sorry, if I take this, I can insert; what is this insertion?

It is a complete set of basis, complete set of uncoupled basis, right and I am putting the same state which I had on the left hand side, so what is this matrix element now; so this is this, it is; is that correct, matrix element is a matrix which connects couple basis to an uncoupled basis, that matrix element is nothing but the inner product of these states, I am doing whatever we knew from quantum mechanics tools.

I am not really trying to push in this concept but I am slowly trying to get you to understand that there are 2 equivalent compatible set, one you can call it as a coupled set, other one is called as an uncouple step, you can work with both of them depending on convenience I said, spin orbit coupling when you have, it is much easier to work with coupled basis, you could work with uncoupled basis also, okay, it is nicer to work with couple basis.

And you should know how to go from one basis to the other and the matrix elements are given by just taking the inner product, it is a concept clear, so 2 equivalent basis for the direct product space.

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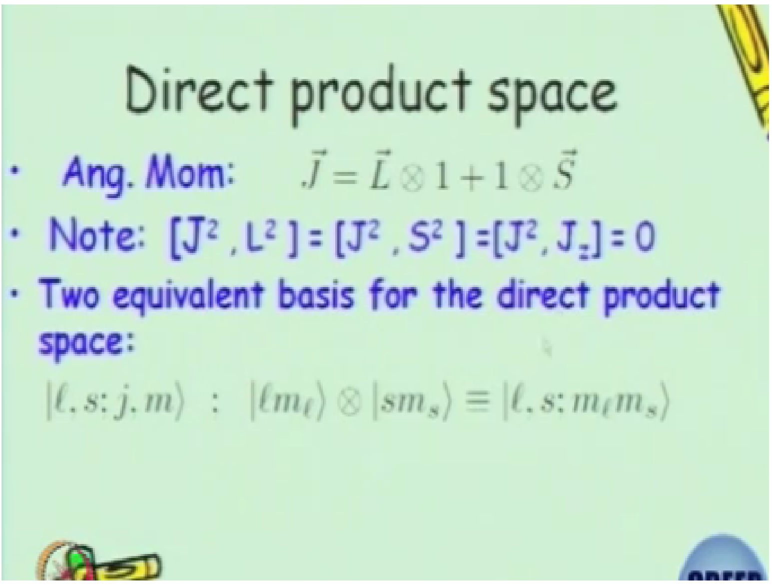

One is what I called it as the couple basis, other one is the direct product basis which is also called as a uncoupled basis, most of the books once they start saying I am going to look at a specific particle with orbital angular momentum as something L = 1 and S = 1/2 they may suppress this L and S on both coupled and uncoupled basis, many times they do not really worry about putting this orbital specifically and especifically.

They are more worried about putting the magnetic quantum numbers for them in the uncoupled and the total magnetic quantum number for the coupled okay. So, this is something which some books might not write all the 4 but it is incorporated, it is there.