Quantum Mechanics

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Lecture – 60
Tutorial 10 - Part II

Now, we continue with this tutorial 10, where will we have discussed 3 problems and we will move ahead with the fourth and the fifth problem basically, the fourth problem is very simple, I can give you hints and you can just work out, problem 5 is little lengthy and it requires a lot of hints, so I will be giving you in steps the hints and apart from that I have been all the time telling that you must practice the problems.

And the problems which we choose just 5 or 6 problems or maybe less problems, so those problems will just give you the insight or the flavour of the part of the lectures you have seen, so when you want to do it the numeric part or when you actually do the problem solving, you understand theory in a better way. So, it has been always being a punch line or a statement in all the tutorial that you must attempt the problems.

And many a times you end up by in not solving the problems because of some certain reason but you can sit back with the hints and work out the things, so that would take lesser time and lesser efforts, so these tutorials basically we have, we are trying to make it effortless and more fruitful to you all. So, with this note and enthu, let us start with the remaining part of the tutorial.

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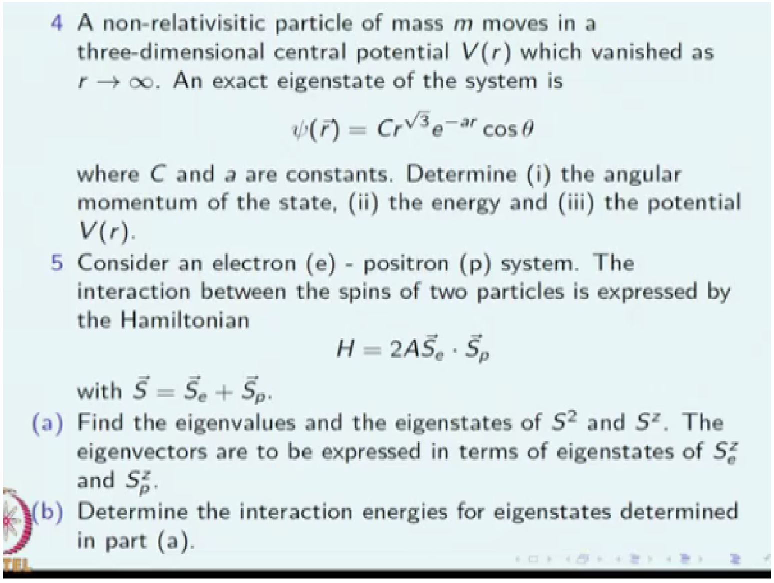

So, in the fourth problem what you have is the question; in the question you have fourth problem, a wave function is given to you normally, you have seen wave functions in the problem, you have seen in this problem you have a non-relativistic particle of mass m which moves in three dimensions central potential, so this is I think the first problem based on central potential and at infinity, the potential vanishes.

So, central potential problem; in this problem you have the wave function or the Eigen state is given to you in terms of our r e raise to exponential of r and cos theta, where C and a are constant and you have to determine the angular momentum of the state energy of the state and the potential V of r, so this problem is divided to 3 parts.

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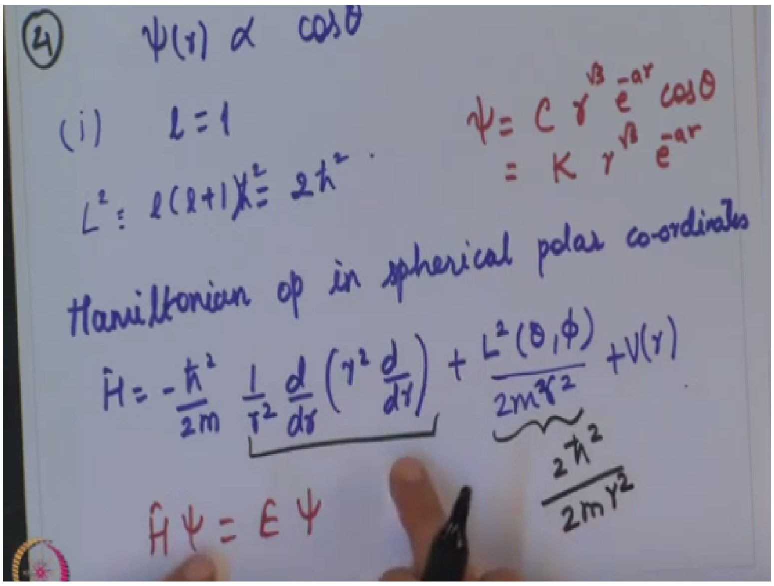

Since the wave function given to you is psi of r okay, psi of r is given to you and this wave function is proportional to cos theta, okay and from here you can infer that l is 1 okay, so l * l + 1 will be 2, so this is your; the Eigen value or the Eigen value corresponding to L square will be of this form approximately, okay and the Hamiltonian you can start by writing a Hamiltonian operator in spherical polar coordinates, okay.

So, how will I write the Hamiltonian operator and spherical polar coordinate; H = - h cross square upon 2m, then I will have a Hamiltonian will be function of r, theta and phi, so what we do here is you can see the wave function is of the form, so your wave function is psi C r raised to root 3 e raised to - ar and cos theta, okay and I rewrite this as K r raised to root 3 e raised to – ar, okay where K is C times cos theta.

We are not going to operate the theta and phi part, only the radial part is what we will need to operate on, so here you can see that the Hamiltonian is written as d/dr r square d/dr okay + L square theta phi okay, we have clubbed the theta and phi in this 2m r square + V of r, so this is the kinetic term and this is a potential term okay, so from this you can guess what you have to do, so first recollect the eigenvalue equation; H psi = E psi, right.

And when you operate the wave function which is given to you on the Hamiltonian, what you have to do is; the only part that will be affected or changed would be the radial part, so here you will substitute for the radial component and differentiate it with respect to r.

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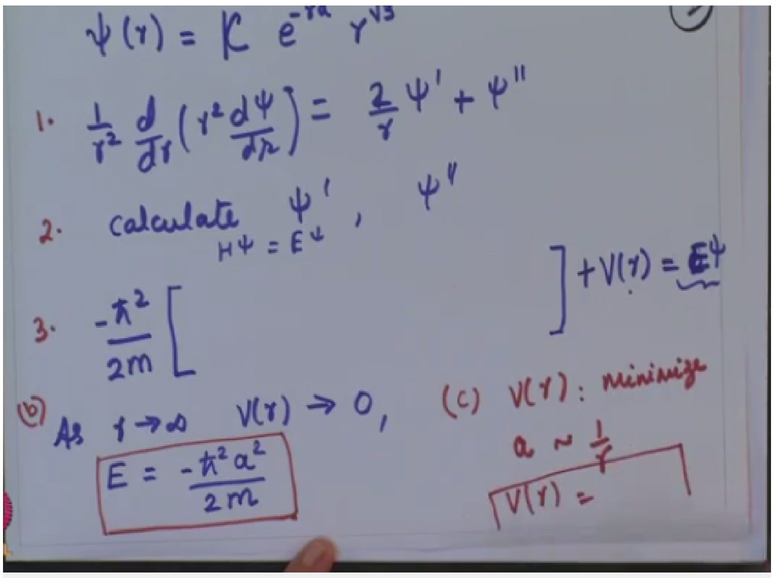

So, here you have let me rewrite this psi of r = C, now I denote test K e raised to - ra and r raised to root 3, this is what I have, okay and coming back here, here you can see when I operate H psi, I what I get is; I mean first worry about this part, okay so let me write this in terms of steps or points, so point number 1; what you will do; you will evaluate 1/r square d/dr r square d psi/ dr okay, this will be, I call this as psi prime.

And when I differentiate this, what do I obtain is; 2/r psi prime + psi double prime, so this is point 1, so this differentiation for this you need to differentiate, so point 2 will be calculate psi prime and psi double prime, you have psi as a function of r and you will evaluate these 2 quantities and then substitute these quantities in this point 1 and then after doing that you go back and substitute this in the Hamiltonian.

So, the point 3 will be; what I will do here is I will have h cross square upon 2m, the part which you will calculate okay and remember the part of the kinetic term that is h square; h cross square upon 2m 1/ r square d/ dr okay and r squared by; d psi/dr that part you will evaluate, your L square as I told you before is; L square is 2h cross square, so this term would turned out to be 2h cross square upon 2m r square, you will substitute for this using point 1 and 2 and this here okay.

And then you equate this to 0, plus you will have a V psi term, V of r term okay, so what do you obtain in this will be e psi actually not 0 because we are doing H psi = E psi right okay, so we have to evaluate this E term okay, after doing all this and in the question you are given that this potential V of r is the central potential and what do you know about it; we know that as r goes to infinity, V of r will go to 0.

So, after doing these substitutions 1, 2 and from 3, we infer that E comes out to be - h cross square a square upon 2m, okay so just verify this and check whether you get the same result, so this was part b of the problem 4 and in the last part of the problem, you are asked to find out the potential V of r, this you do not even require a hint for this, you will try to minimize the potential term and then calculate the potential by getting the value of a in terms of r.

So, I think you will get value of a as something equivalent to 1/r or so and then after making that substitution, you get the potential V of r as the required; as the required term which is asked, okay. So, this would be a very simple way of evaluating, so c part; V or r you will minimize V of r and then get the value of a in terms of r and then you get V of r okay, so this would be the broader picture of how to go about in the; in solving the problems.

And now, we go to the third rather the last part of the tutorial that is the fifth problem and this problem is a bit lengthy problem but it will help you understand 2 particle system and in this the 2 particle system is a particle is an electron and a positron, so consider an electron and a positron system and the interaction between the spins of 2 system is expressed by a relation or the Hamiltonian in which the Hamiltonian is proportional to the spin of electrons and spin of proton coupled.

So, Se is the spin of electron, Sp is the spin of proton and when you consider the couple state, the total spin is added as Se + Sp and in this problem, you are asked 3 parts; first part in part a, there are 2 parts basically, so the find the eigenvalues and Eigen states of S square and Sz is one part and in the next part of this would be the eigenvectors when they are expressed in terms of the z components of electron and positron.

And in the third part you will have in part b, you will have to determine the interaction energies of the states determined in this system or in the part a, so before we get started to the problems, a rough idea you might already be having about how the addition of angular momentum takes place, it is not a simple addition, it will be when you add the spin of electron and positron, we use tensorial addition that we will see in this tutorial.

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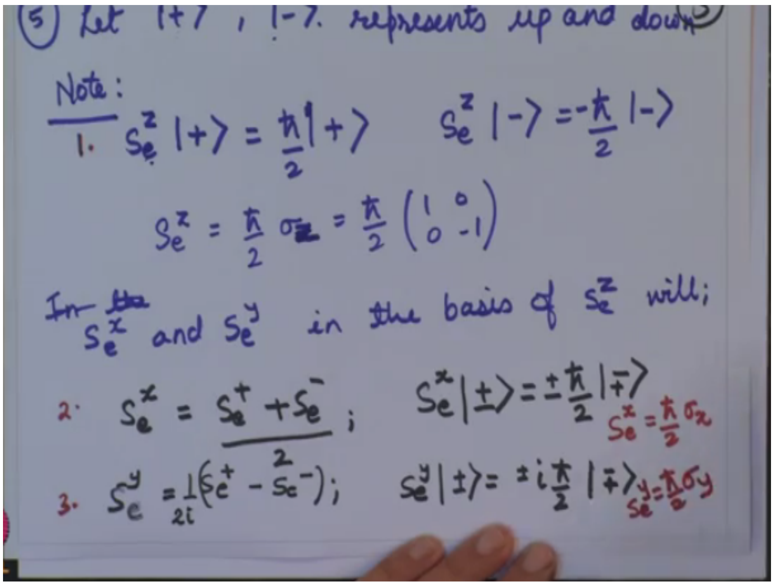

And I will again write down jot down the points in which you will go through the steps to get the result okay, so let us write, so we already know that + and – okay, represents up and down state okay, so in some places you will find 0 and 1, in some places + and -, + 1/2 – 1/2, so these notation differ from books to books and people to people but the meaning of this remains the same that is a plus state or ket plus is the up state which will give the eigenvalue when operated on Sz, it will give you the eigenvalue 1/2 h cross.

And minus state will give you the eigenvalue – 1/2 h cross, so + 1/2 h cross/ 2; 1/2 h cross/2 and – 1/2 h cross/ 2, okay, so just note that whatever I said just now, okay let me call this as point number 1 and you all are aware of this very well but just to put it on paper, so Se denotes that we are talking about the spin of electron and when I write the subscript Sp, it is the spin of proton; positron okay.

So, Ez; Sez will be the spin of electron in the z component; z component of the spin and Sez - would give - h cross/ 2 - okay and remember Sez is or can be written as the poly matrix that is 1 0 0 -1, okay. So, in the basis, in the or let me write it as Sex and Sey in the basis of Sez will be; remember we can do this, we can write this as let me write in the form of highlighted manner, Sex can be written as Se+.

Again, you can see that I have come back to the ladder operator notation which is most convenient way to work with, so this is Sex can be written in terms of the ladder operator + and -, so when you operate the x component on the Eigen ket of Sz, so if you are operating the plus component, you will get h cross/ 2 and you will have a flip in the sign okay. Similarly, if you operate on down state, it will project out the - h cross square / 2 and it will flip the state, okay.

So, continuing with this the third point, I would emphasize here, would be now coming back to the Se; Se+ -Se - and here I will have a factor of 2y okay, again I need not say but you just recollect this that Sex in the similar manner as we did for Sez that is the z component similarly, you will have for the x component okay, so this was the z component, let me correct this, this is sigma z, this is sigma x, sigma x will be 0 1 1 0 okay.

And here again, when the yth component is operated on plus or minus component will also project out h cross/ 2 but the imaginary part will appear, so these 3 points okay, this will set up the basic assumptions okay then we; what we have to do here is; let me also write for Sey is h cross square/ 2 sigma y which I did not write before.

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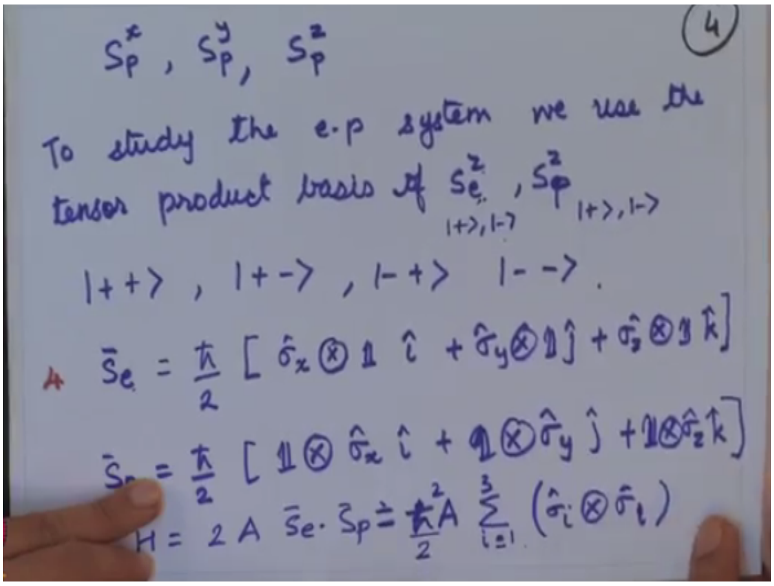

One can also write in the similar manner for Spx, Spy that is the positronium; positron you can write the x, y and z component okay similarly, you can demonstrate the x, y and z component. Now, to study the electron positron system, we use the tensor or tensoral product basis of Sez and Spz that is the tensorial product of the basis of the electron spin along the z component and z component of the proton, the positron spin.

So that would be; if I call for the electron as plus state, up state and so I will have plus minus state, plus minus state, plus minus state, so there are 2 spin state; plus minus, plus minus that is up down up down, so with this you can make 4 combination which is plus plus, plus minus, minus plus, and minus minus, you can make these 4 combination, so these would be the Eigen vectors or the Eigen kets.

Now, next step would be you calculate what is Se, okay, so Se will be there be a tensorial product is written as sigma x, okay, I is the identity operator or the identity matrix, sigma x we know that poly matrix for x component, i + sigma y j + sigma z k jet, so this was for sigma; for spin of electron similarly, for positron, I can have spin state, so here you can see that sigma x1 corresponds to the electron spin state along the z direction.

And the second component is unaffected, so for positron you have 1, the second component will be sigma x, i cap + 1 sigma y j cap + 1 sigma z k cap all right, now next what you do in this is; you calculate since now you know what is Se and Sp your Hamiltonian which is given to you now, you can write it as Hamiltonian is 2A Se dot Sp. Now, when you do this dot product, you will obtain, you will do this multiplication for the tensorial multiplication, this will give you, I will use here, so this will be 2 and this will be 1/2.

So, this 2 will go away, I will have just h cross A okay, this will be h cross A, alright and this term I can write in terms of summation i = 1 to 3 sigma i cross sigma i, when I do this multiplication, I will have h cross square which is here 1/4 and this is 2 so okay, now it is correct, so sigma 1 crossed with sigma 1 that is sigma x sigma x sigma y sigma y sigma z sigma z summed over.

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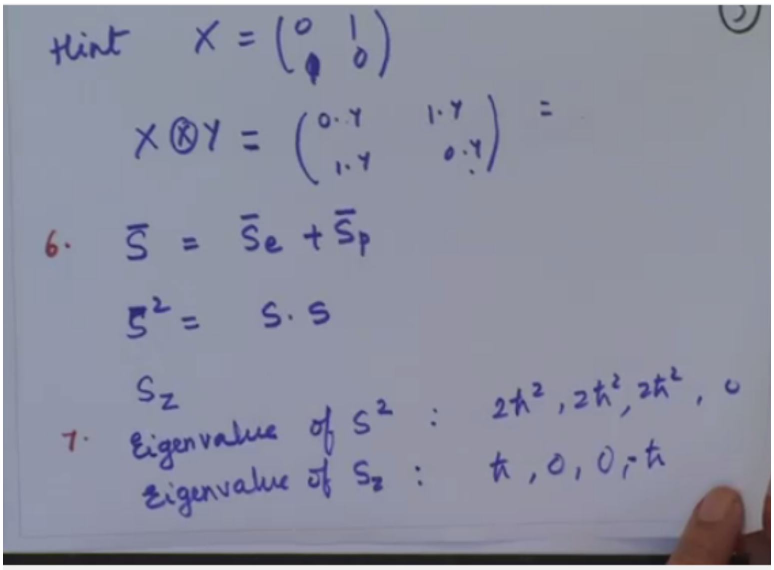

So, i is going from 1 to 3 means, i is taking value x, y and z that you should remember, okay so in this you will need one more hint for evaluating if you know, you just skip this part but if you have forgotten this is just to recollect, so if x is this matrix and you want to calculate x on y tensorial product of x and y, so you will write it as 0 dot y, 1 dot y, 1 dot y, 0 dot y, so where one will be your identity, so this will be 00, yy, okay.

So, using this hint you can do that multiplication, so evaluation of Se and Sp, this you have done in the next step that is the sixth point would be you have to evaluate S but what is S; S is the total spin of electron and positron system such that S is Se + Sp, so once you know this, you can evaluate S, you have to sum over these 2 quantities, which we just now evaluated and then you can calculate the value of S square also okay, you have to just square this S dot S okay.

So, in this point you will be able to evaluate this, it is not at all hard, it is very simple, so once you know and you know what is Sz, you already know what is Sz okay, in terms of the eigenvectors, you have to evaluate the Eigen values and the Eigen vectors, so I will give you here the Eigen value of S square, 2h cross square, 2h cross square, 2h cross square and 0, so it is triply degenerate.

And Eigen value of Sz is h cross 0, 0, h cross; - h cross because you will see that Sz is a diagonal matrix, I think with 1 0 0, the diagonal elements of 1 0 0 and – 1, so you can try and evaluate that part also and the final part that is part b, final part is the b part which you have to evaluate.

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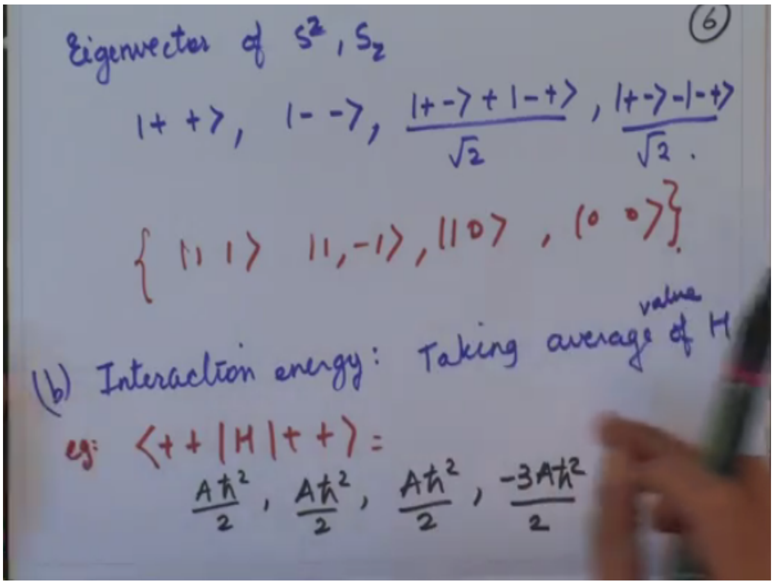

And before that I will just give you a hint of the Eigen vectors of S square and Sz, so Eigen vectors of S square and Sz would be plus plus, minus minus and plus minus plus with a normalization factor of 1/2 and plus minus minus minus plus, the normalization factor of root 2, not 1/2 so, you can actually write these in terms of up down state as 1, 1 you call them as 1, 1, 1- 1, 1 0 and 0 0, another way of writing this.

And then we finally go to the b part okay, in the b part you are asked to calculate the interaction energy, interaction energy how will you obtain; by taking the average of average value of Hamiltonian and the average value of Hamiltonian for different eigenvectors, so you can start for example, I can give you one term and then you can enjoy doing the rest, you can of try and obtain this, okay or either in terms of plus plus notation or in terms of these notation, you can calculate this will give you one component.

Similarly, you can calculate the other components and check whether you get the interaction energy as A h cross square/ 2, this is just by inspection I can, I am writing this but you must check whether you get the same result, so interaction energy we obtained by taking the average value of the Hamiltonian under the these matrices or the eigenvectors and when you do the averages, you will obtain the interaction energy.

So, you will take the plus plus or 1 1, 1 0, 1 – 1 and 0 0 and is the kets; initial and the final kets, so we end here and this particularly, this tutorial 10th part 1 and part 2 that is these 2 sessions of the tutorial 10 requires for you to work on try your hands on the problems, so in this sketchy way we have planned because this will give you since you do not have answers, so you will just have to work out the in between step and verify some of the result and some of the answers, I have displayed here but some you have to work by yourself.