Quantum Mechanics
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Lecture – 59
Tutorial 10 - Part I

Now, we are coming to the end phase of our tutorial sessions, you might be enjoying the lectures and by now, you might have mastered how to use or you have given; we are given lot of practice problems on commutative bracket, eigenvalue equation, finding out the Eigen value, Eigen vectors and you have gone through a lot of exercises on ladder operator and things like that.

So, I will be a bit sketchy in this tutorial were in I will just give you the hints or just the track or the road map kind of a thing to do the problems and then you can just work out them, so I will be giving only hints in this tutorial and I; we would expect that you would try the problems and do them by yourself and hope you enjoy this tutorial also. So, let us start with the first problem of the tutorial which is about the interaction picture of the L and Z that is the angular momentum and spin interaction.

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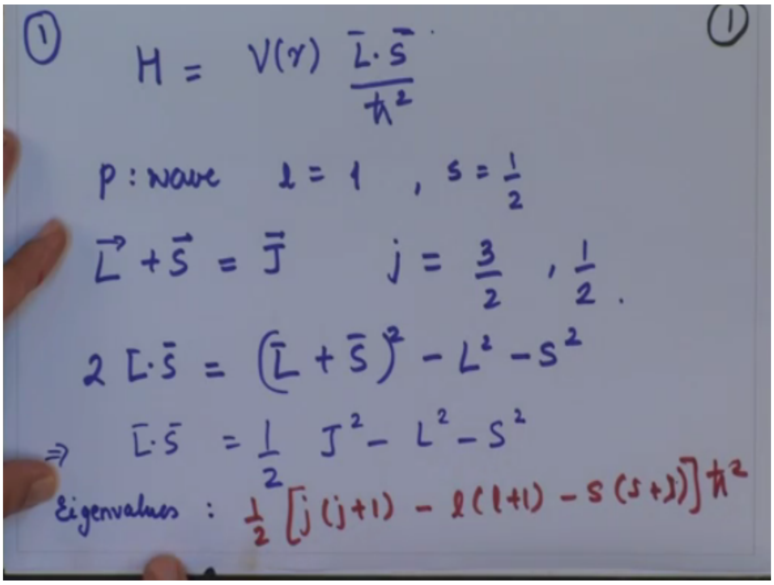

So, you have to suppose, to find out the energy of the Hamiltonian and here the problem given to you is that a Hamiltonian of the fall on h cross square LS is the LS coupling term, okay and in this problem you are asked to find out the energy eigenvalues or the energy values for the coupled system, so let us start by writing so, in this problem you are asked to find out the energy of the LS system that is the coupled system for P wave.

So, what do you mean by P wave, when you talk about P wave, l = 1 okay, so we have l = 1 s is 1/2 we know that spin can take value + 1/2 or 1/2; + 1/2 or – 1/2 okay for this coupled system and so, the orbital state for P wave is 1; l = 1 and when you talk about this coupled system, L + S is J, so now with L = 1, S = 1/2 J can take values 3/2 and 1/2 okay, now we need to write L dot S in terms of L, S and J.

So, how we can do that? So, if I write L dot S; 2 times L dot S is L + S the whole square - L square - S square, I can write like this correct, so this would imply that L dot S is 1/2 this is nothing but J square - L square - S square okay, therefore one can find out the eigenvalues for L dot S, so Eigen values corresponding to L dot S, you can write them into the eigenvalues corresponding to L dot S will be j * j + 1 - l * l + 1 – s * s + 1 h cross square with a factor of 1/2 which is coming from here.

So, by putting the value of j, l and s you can calculate the eigenvalues for j = 3/2 and j = 1/2 okay, so let us write down explicitly.

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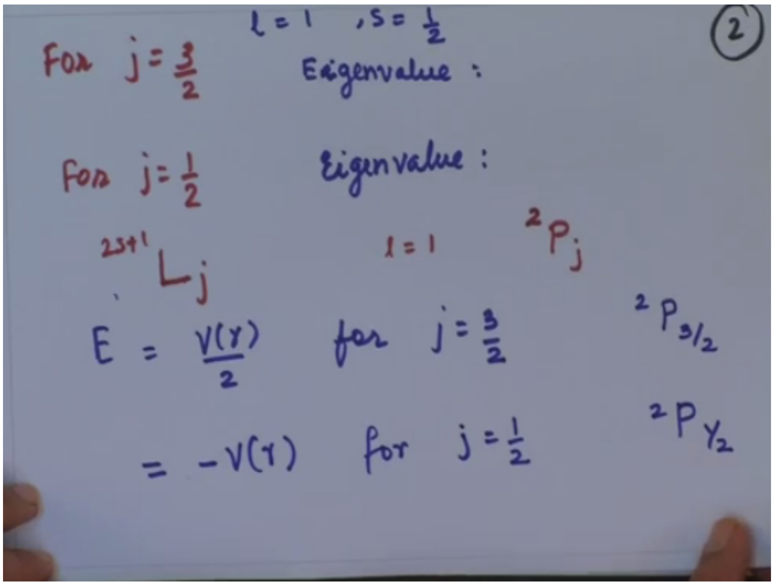

So, for j = 3/2 okay, we are using L = 1, s = 1/2 okay, we are using this, so for j = 3/2, the Eigen value corresponding to j = 3/2 , you just evaluate this part okay by putting the value of j, l and s and obtain the Eigen value and as I told you before the tutorial that I will not be giving the results, I will be giving a roadmap how to go about and solve the problems, so you can calculate the Eigen value for L =1/2 okay.

So, the energy corresponding to V or L dot S upon H cross will be; we have to calculate that okay, for different values of J, remember before we go to that the spectral notation you all might be aware that is written as j 2s + 1, so L correspond to the state which whether it is a P wave, S wave, D wave etc. so for us l = 1, so it is a P wave, so you will have a P okay, then for 2 different values of J, you will find out what is the J here and 2s + 1 in our case is 2, right.

So, now the energy E is V r upon 2 for j is 3/2, so you can; this you have to fill this blanks and just match it with what I am giving you is the final result, so E is Vr/2, so for j = 3/2 in spectral notation, how will I write; I will write this as 3/2, okay, this you might all be knowing and this will be V of r for j = 1/2 so, the corresponding spectral notation would be this alright, so this is what you are supposed to evaluate, you can also calculate the wave function and things like that by your own.

So, now let us go to the second problem, in the second problem a Hamiltonian is given to you and you have to evaluate some commutation relations and some more results okay, so this problem is divided into 3 steps.

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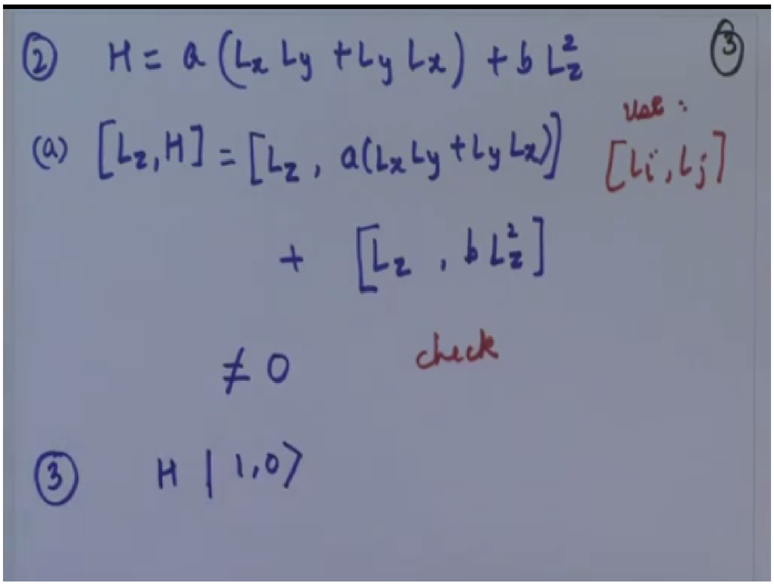

So, second problem H is a Lx Ly + Ly Lx + b Lz, first part of this problem you are asked to evaluate Lz, this is a very simple task, so we know what is a commutator of Li Lj, you have done in the last class, Li Lj is ih cross epsilon ijk Lk, okay, so use that you recollect what is Li

Lj, okay, you have to use this commutator of Li Lj and you will substitute for H over here, so let me write one more step and then the bit for you all to evaluate, Ly Lx, this will be the first term.

And the second term would be b, okay, so this will be the second term and you can evaluate these terms right, for a second term you know will give me 0, first term will give some result, so this result is != 0, you will check this and obtain the nonzero value okay, you have to obtain the nonzero value also, so this was the A part, now we have to evaluate the B part okay. So, in the B part, you have to; you are asked that you have to check whether Hamiltonian is the Eigen state of ket 1, 0, whether it is an Eigen state of ket 1, 0.

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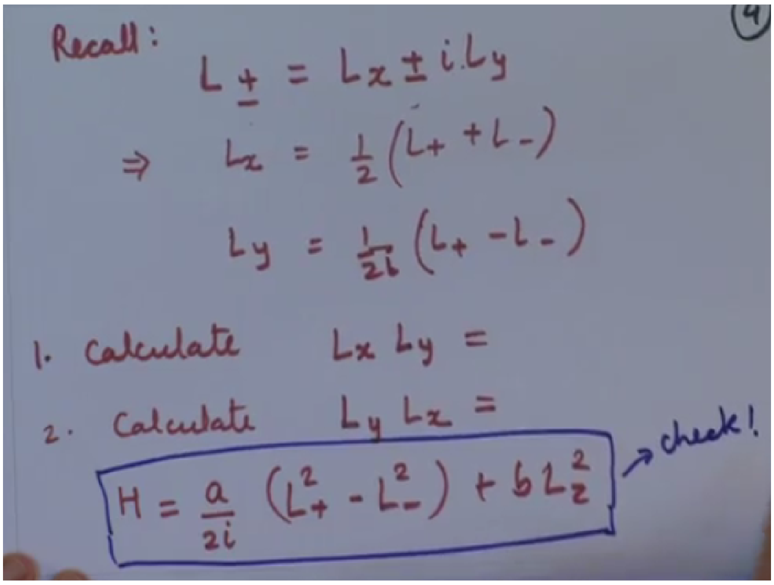

So for that I will get take you now to the; just try to recall that okay, let me tell you that it is very convenient to use ladder operator while doing such calculation, so you have learnt ladder operator for angular momentum, you can write it as L+ and L - in terms of Lx and Ly and when you operate these ladder operators on the ket, it will take the state from a step; one step above or one step below.

So, L + will take you to the next step where your M will become M + 1 and L - will take you in the step downwards, so when you use these ladder operator your; you get, you are able to simplify and work with, so L + L - by definition is Lx + iLy + - okay, so L + is Lx + i Ly, L - is Lx – iLy, so from this what do you infer; Lx is 1/2 L + + L – Ly is 1/2 and there is a factor of i L + - L -, so these would be very useful in obtaining whether the Hamiltonian is an Eigen ket of 1, 0.

It will be very simple to operate L+ and L- on the Eigen ket rather than Lx and Ly, so what we do here is; we first calculate, so the steps would be step 1 would be calculate Lx, Ly okay, in the Hamiltonian what are the terms you have; Lx Ly, Ly Lx, so next is; then second is calculate Ly Lx, you have to obtain these 2 and then you rewrite the Hamiltonian in terms of the ladder operator.

So, what you get after doing all this is just check whether you are convinced with this result okay, so you must check this result check okay and now you operate the Hamiltonian operator on the ket 1, 0.

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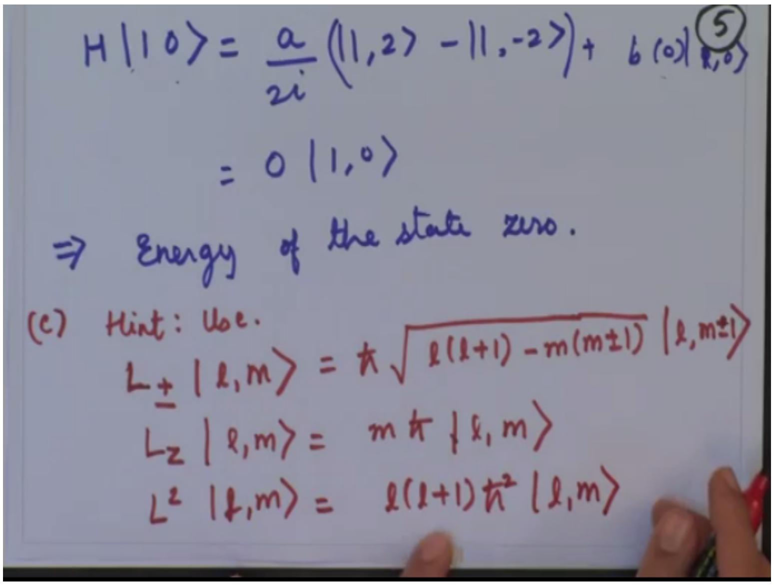

So that would give you when I operate ket 1, 0, I have 2i and then L + will take the ket from 0 to 1, 1 to 2 okay, so you are going to; you cannot write it in the linear combination, so basically you must be able to write the Hamiltonian as a; or the operation, after doing this operation you should be able to write this in terms of ket 1 1; 1 - 1 or 1 0, so L + and L - will is there is a square, so they will raise it i 2 steps, L – will blower it 2 steps okay.

So, basically what you get here is something like this correct, plus you have a B term which will give you 0; 0, 0; 1, 0, okay, so this is 0, this is = 0 times 1, 0, so this implies that energy of the state is 0 and the eigenvalue is 0 for this; so H 1 operated on 1, 0 that is this is an Eigen vector or Eigen ket of the Hamiltonian operator with eigenvalue 0, alright now, in the next part that is the C part of the problem, you have to calculate the energy eigenvalue and eigenvector.

So, calculating the Eigen value and Eigen vector you need to know again some of the basic things, so the hint here is you will use is to use the following first L-; or I can write it as L + L - in 1, I can club it okay, so when you operate the ladder operator plus or minus operator on the state l, m, what I obtain is h cross square root, so I will have a C + C - which is nothing but the CG coefficient Clebsch–Gordan coefficient.

I will have l * l + 1 - m * m + - 1, l , m + - 1, okay so L + will raise this state m/+1 and L - will lower it by -1, okay this is a compact form I have used to calculate to represent the ladder operator operating on the Eigen vector or the vector l, m, so this is one thing you are going to use and next thing would be Lz, m, l which is m h cross l, m, L square l, m is l * l + 1 h cross square l, m.

So, these hints you have to use to calculate the C part of this problem okay, so in this let us go a step further and use all this hint, so I will just give you in a sketchy way how will you represent I mean how will you write the Hamiltonian operator in matrix representation that you can calculate.

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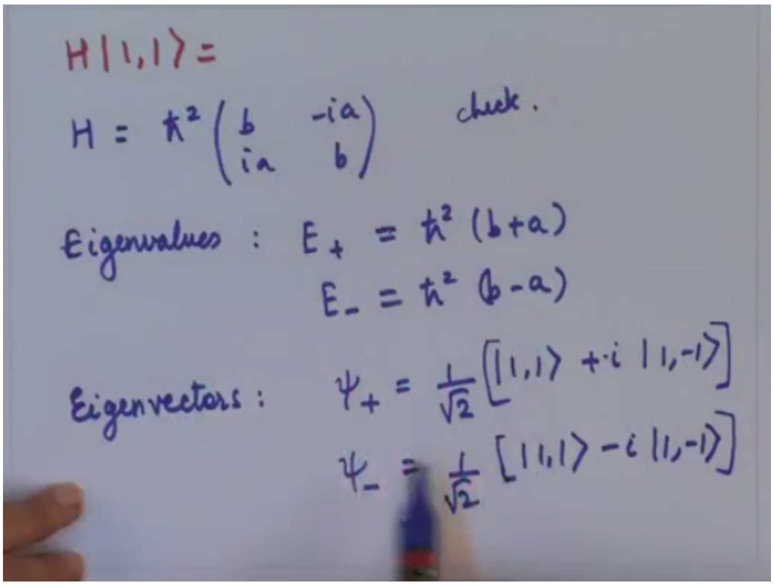

So, what you will do is; evaluate this okay and then what we can do is; you can write the Hamiltonian in terms of matrix, so just check if you can obtain this or do you obtain something else, just check this, so you have to calculate the Eigen values and Eigen function, so the Eigen value is; I can write it as E + for; so Eigen value basically is h cross square b + a, so this is one Eigen value and the other Eigen value is h cross square b – a, I call this as E +, I call this one as E – okay.

And the corresponding Eigen vectors or Eigen ket's would be I call one of them as psi +, the other is psi -, so this you can check by yourself that one can obtain this result okay, here at this point I need not say but for those who are not regularly solving the problems for them specially I would say that here the hint would be to calculate the eigenvalues and eigenvector, the hint would be to use Eigen value equation H psi = E psi, it is not difficult you have been solving this from day 1, from the first tutorial.

So, it will not be that difficult so try and evaluate this, problem 3 is you are given a set of Hamiltonian; the commutator relations and using those commutator relations you have to dig down and search the results.

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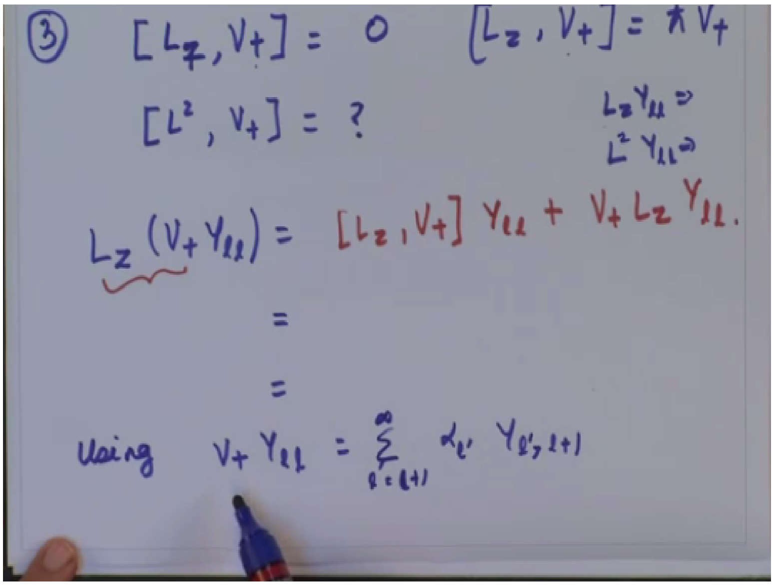

So, one of this relation given to you is Lz V + is given to you and L square V + is given to you okay, so these 2 relations are given to you and you have to evaluate or rather calculate what happens when Lz V + Yll okay, here it is L + V + is 0, Lz V + is h cross V +, okay and this you have to tell me what do you expect okay, so the relations given to you are L + V +, okay so V + is a kind of raising operator similar to your L +.

But it may not be exactly like L+ x, it is a new operator, so when Lz is operated on this state what do you expect or what do you obtain is the question, now since you know this, so the hint here would be rewrite this as Lz V +, okay Yll +; we know the relation of Lz Yll, we know this okay and okay, these things we know, when Lz is operated on Yll, it will give me ml h cross square Yll, this would give me l * l +1 h cross square term okay.

So, these relations we know, so going ahead this is given to you, you can evaluate from here and after doing one or two steps, you will get to the result of what you have to prove, so after one more step, you can get to this result. In the second part using the same analogy, using the fact that V + when operated on ll would give some alpha l Yl * l + 1, okay, if this ladder operator takes you from l to l + 1.

And in some; where alpha l prime I have here, this is a l prime okay, so we assume that it is not a trivial operator and it may change l to l + 1 and there is some l prime going from l + 1 to some infinity, so using this hint you can evaluate the second part or of the relation that you have to obtain. So, with this sketchy demonstration of the solution, I hope this would benefit you to solve the problems and get to the results.

And in the second part also, we will try to write the; I will try to write the steps involved in solving these problems, so if you have not been doing rigorously the problems then please try and do it again and again I mean repeating this all the time but it is very important to understand both the theoretical part as well as the numerical part which you have been doing in the tutorial.

And there is a whole bunch of problems available to you in various textbooks and we try to bring a set of 5 problems which are; which will give you flavour of different types of exercises you can think of and do, so try and attempt these problems and we will continue next time.