Quantum Mechanics
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Lecture – 53
Tutorial 9 - Part I

This tutorial is a special tutorial because, I say special because this tutorial is made for you on more number of practice problems. So it contains more practice problems which you can do on your own. These practice problems are simple ones which I will not discuss. But you can try it. It will be 1 or 2 line problems which you can attempt based on Pauli spin matrices and orbital angular momentum operator. Those are very simple exercise which has been included in this tutorial so that you can try more number of problems.

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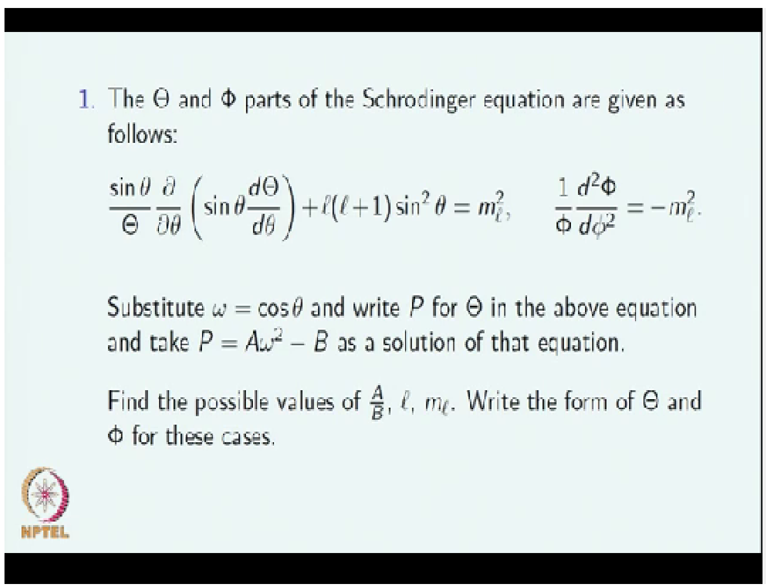

So I will discuss now the first problem which you have already seen in this Schrodinger equation solution in terms of R theta and phi using separation of variable. So you are given a Schrodinger equation for theta and phi equation. And you are asked to make some substitution where in omega or w is cos theta and you have to substitute for capital theta as P and try to solve this equation.

And in this equation, you have to solve this by substituting or taking P as A omega square-B, a general solution for P or theta in terms of omega. Omega is cos theta. So we have to find a solution of this equation and in doing so, you have to find the possible values of the fraction A/B which are some constant l and ml. So we have to write the form of capital theta and phi in terms of these solutions what we get for A/B, l and ml. So let us get started.

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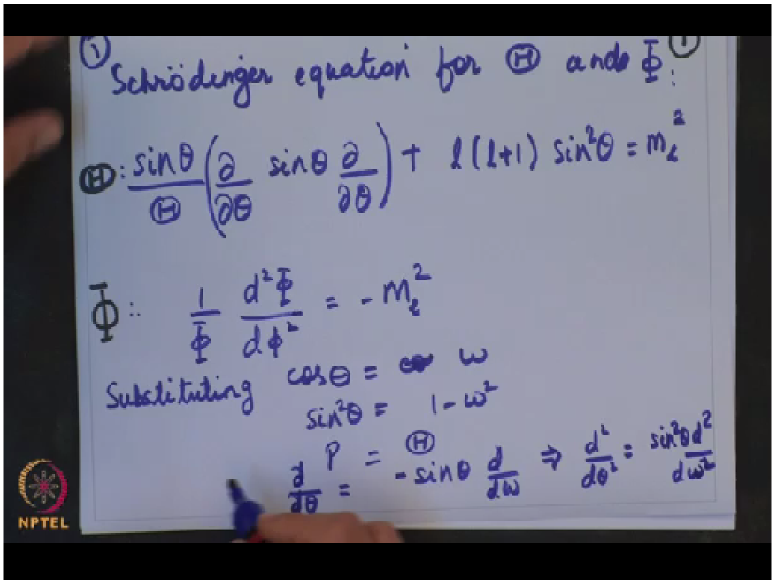

You are given the Schrodinger equation, first problem, Schrodinger equation for capital theta and phi are given by capital theta, it is given to you, sin theta/capital theta do/do theta sin theta do/do theta*l*l+1 sin square theta=ml square, okay. And this you can actually rewrite by taking capital theta on the numerator, okay which later we will do. So this is for equation for capital theta. Then we write the equation for capital phi.

Equation for capital theta. Equation for capital phi is nothing but 1/phi. So this Schrodinger equation is a spherical polar coordinate. You get this form. d square phi/d phi square=-ml square. So do not get confused here. This is capital phi and this is a small phi, okay. And now when you rewrite this by substituting. So substitutions made are, substituting cos theta=omega, okay. So what will be sin square theta?

It will be 1-omega square or w square, you can call it as w or omega. So this will be sin square, theta is 1-omega square. P is capital theta and I want to rewrite do/do theta in terms of do/do omega. So that will be rewritten as do/do theta will be nothing but -sin theta do/do omega, okay. Now this is a function of theta only I can write it as complete differentiation instead of partial one.

So this will be d/d theta is -sin theta, this I have got from here because this is nothing but d omega/d theta which is nothing but -sin theta. This would imply that d square/d theta square will be equal to sin square theta d square/d omega square, correct. Now these substitution we will make in first and second equation. Second equation is very simple.

We are not suppose to touch anything here. It can be solved easily. But we need to know what is the value of ml. So value of ml you will get by solving the first equation. So let us make the substitution and in doing so, we substitute for capital theta and take it in the numerator. And in this step, we are also trying to rewrite this in terms of omega, okay. So let us do that.

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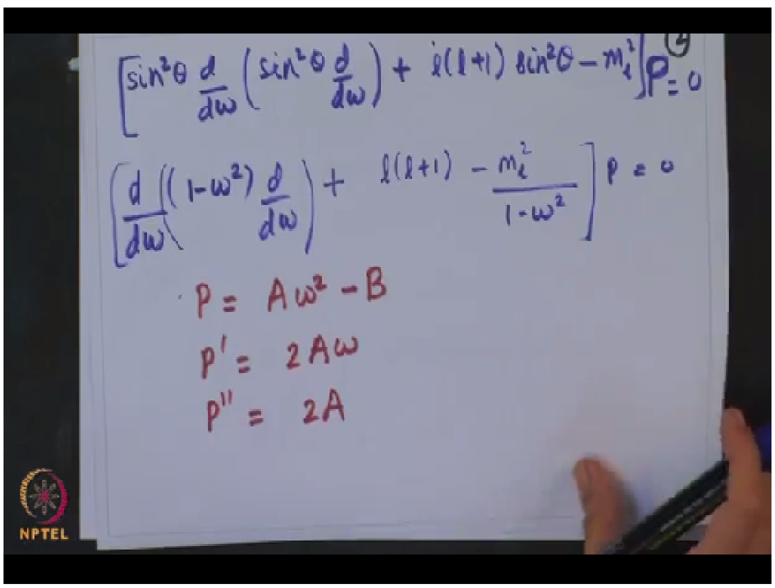

So this will become sin square theta, okay, do/do omega sin square theta d/d omega. Here also d/d omega+l*l+1 sin square theta because I am multiplying throughout by capital theta. I will have additional term here. So I will write throughout common as capital theta, correct, here which is nothing but P=0, okay. This is the equation we have now. Now what we do is, we divide throughout by sin square theta, okay and make the substitution for omega which we have not done yet because we have to find out the terms in terms of differentiation.

So d/d omega, sin square theta is 1/omega square, d/d omega. This is entire term like this,

+l*l+1, I am dividing my sin square theta. So I will have ml square/sin square theta which is nothing but 1-omega square. And here there is a P. Now it is very simple. Here you will do the differentiation. So here you will have, let us do it. And at the same time, we make a substitution for P. P is A omega square-B. So from here you can write what is P prime.

P prime is 2A omega, that is dP/d omega is 2A omega and 2 times when you differentiate, you get 2A, right. Now I will substitute these values in this equation. So again when I do the differentiation, I will have 1-omega square term. d square omega/d omega square, d square P/d omega square which is nothing but 2A.

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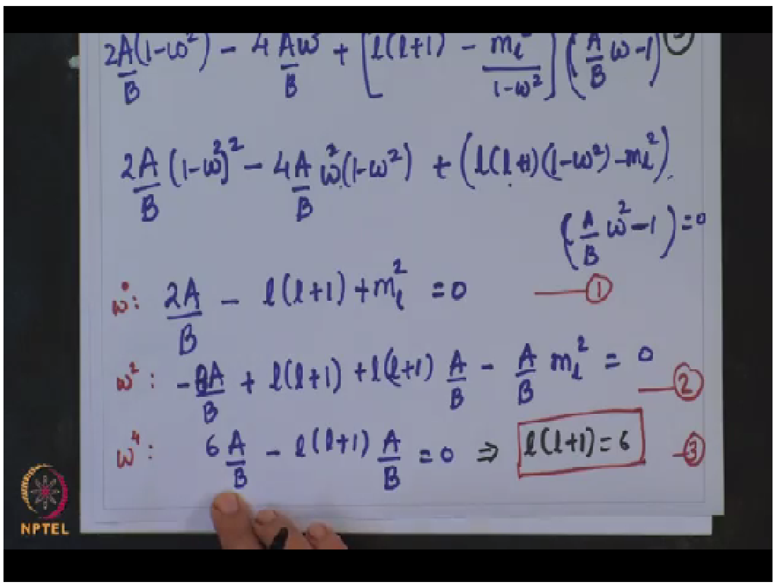

So let me write here 1-omega square*2A, correct, -, again come back here, I will have, when I differentiate this with respect to omega, I get -2omega and *2A omega so that becomes 4A omega, correct, +l*l+1-ml square/1-omega square. And here, I have to substitute for P. P is A omega square-B. So A omega square-B I divide in this equation itself by B throughout. So here I have A/B omega-1.

Again, I feel little inconsistent. So I will just, not inconsistent but it will be easier if the omega term is in the numerator. So let us bring it back. In the previous step, I had divided but you do not need to divide here. So let us bring it back here. I have 1-omega square-4A/B omega 1-omega square. Then here you have l*l+1 1-omega square-ml square, this *, here you have A/B omega-1, okay.

This entire thing equal to 0. What we will do is we will compare the coefficients of omega and write the coefficients of omega individually. So each coefficient will individually go to 0. So the omega is to 0, that is when the coefficient of omega that is 1, omega raise to 0 is 1. So coefficient of omega 0 will be the first term. From here, I will have 1 term that is 2A/B, okay. 2A/B-, here I do not have any coefficient of omega.

Here I have 1 coefficient of omega that is omega raise to 0. That is l*l+1. So I have here l*l+1, okay, -ml, correct. So this entire thing multiplied by -1 for coefficient of omega raise to 0. So this when we have multiplied throughout by -1, what do I obtain is this - and - becomes + and there is a -l*l+1*ml square. This goes to 0. So this is my equation number 1, okay. Then coefficient of omega square.

There is a power of omega square. Here I will get 2*2 omega square that is -4A/B and from here also I get -4, here it is omega square, sorry it is omega square here, right. Here also it is omega square. So here I have omega square term which will be -4A/B. So this becomes 8, correct, 8 omega square A/B. So coefficient of omega square I am writing. So it will be -8A/B. Next will be term coming from here.

So we have already multiplied by -1 for the first term. So from here I will have -l*l+1. So -l, so this becomes +l*l+1, correct. And then I have, here also I have omega square, sorry. So here I have now again the first term of this when I ticked the cross term, I have l*l+1. So +l*l+1A/B. And here I will have 1 more term which is -A/Bml square=0, okay. So this is our equation number 2.

Now, let us find out equation for omega square term, omega raise to 4 term. So omega raise to 4 term, let us find out. So this will be one contribution. This is 2A/B omega raise to 4. Here I have - and here I will have omega, - and - +. So 4A/B. So it becomes 6A/B, okay omega raise to 4, one contribution I got from here. Another I will get from here which is -l*l+1A/B=0. So this is the third equation.

But what does this imply? This would imply that l*l+1 is 6, okay. So here I have solution for l from the third equation. Now I will make the substitution in these equations. I will use first and second to get the value of ml and the ratio A/B. So here we have 1 equation. Let us do it. So this becomes, I will use this substitution here. So this becomes 6+6A/B.

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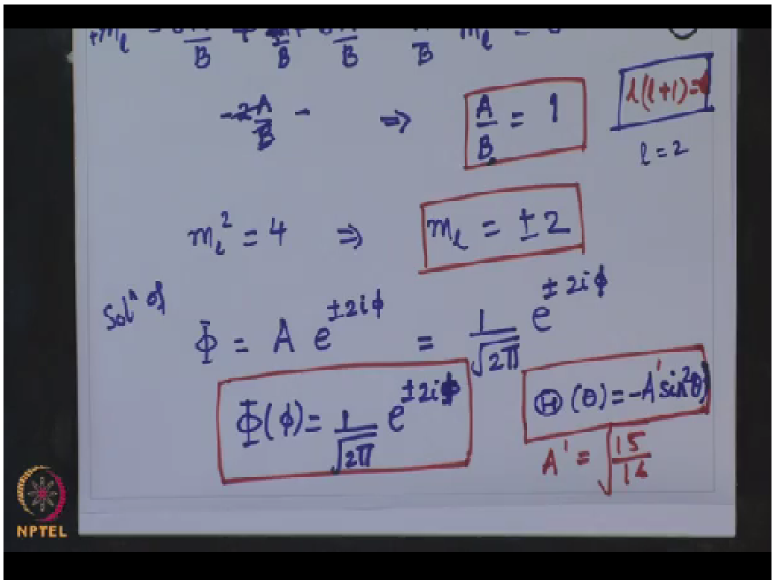

So this will be -8A/B+, this is 6 and this is 6A/B. So 6+6A/B, correct and then I have -A/Bml square=0. So -2A/B. I can also make here substitution for l*l+1 which is 2A/B-ml square, that is one way to look at it. So I will have here, this can be rewritten as -, this here I can write it as 6A/B. So this is, here you can see l*l+1, this term I can write it as 2A/B-ml square. So 2A/B-ml square.

So I will have, I will write here ml square and, so this will give me ml square and I will have ml square*, this will give me, on solving we can actually write this term as 8ml square and then that would imply that, look at this. This is 0 and I have, if A/B=1, then only this is possible, right. So I have made the substitution for from this equation. So 2A/B I have here, +ml, okay. So I had -A/Bml square=ml square which means A/B is 1.

So when A/B is 1, here this is 2-4, -6 that is -4. So I will have, let me, and we have used l*l+1 as 6, correct. Now substituting this, I obtain ml square is 4 which is nothing but ml is +-2. So you have got the value of mlA/B and l*l+1, okay. So this is one set of possible solution. You can have more solution, one more solution I have actually. You can work out and find out. So for this set of solution, one can obtain the value of capital phi as, solution of phi as, e raise to +-2i phi.

When you find out the normalization factor, you obtain, this is nothing but 1/square root of 2pi e raise to +-2i phi. So this is the solution. Let me write here as 1/square root of 2pi e raise to +-2i phi. And similarly, you can obtain the solution for theta, this is capital theta as the solution of the form -A sin square theta, okay. Let me box it. So this A or I can call it as A prime is, when you find the normalization, you obtain this, okay.

So one set of solution for the ratio A/B is 1, ml is +-2 and l*l+1 is 6. You can infer the value of l, correct. l is 2. So with this we discuss this possible case. Now let us go to the next exercise. Next exercise is you have to find out some relations of Pauli spin matrices. So Pauli matrices as we have already seen that they follow certain relations, okay.

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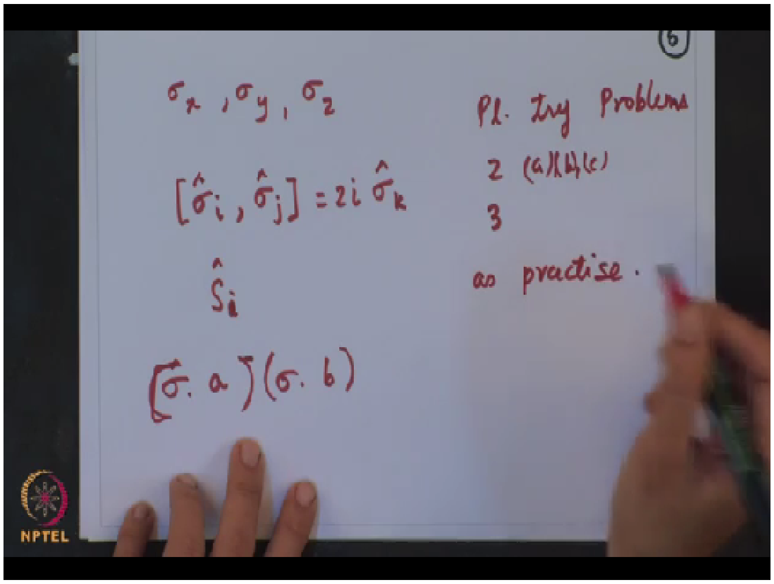

So you know this sigma x, sigma y and sigma z are Pauli spin matrices. And you know the properties of Pauli spin matrices. So if I write sigma I, sigma j, this is nothing but 2i sigma k. This relations we all know. And even these Pauli spin matrices, this operators you can write in terms of Si also, okay. That you all know. So using these properties and you know that these properties can be used to calculate the problems given to you 2A, 2B and 2C.

So these exercises you can try out yourself. It is just 1 or 2 step problems. And for the first one, there is a sigma dot a, you know we can write it as and sigma dot b. So you can expand it in terms of sigma xax+sigma yay+sigma zaz and the product and you will get a dot scalar product as well as the vector product. So you can just obtain this by making some substitution. Those exercises are for you to practice.

So problem 2, a b c and problem 3, these are all practice problems. So please try problems 2 and 3, okay, a, b, c as practice, okay. If there is any difficulty, we can always have a discussion. and the third problem is what will be the nature of operator lz-h cross*lz+h cross. So you have to assume the eigen state of lz as + and - state. You know you can write, understand now that how to write it in terms of +, - and 0 state.

We discussed in the last tutorial. So using this information, you have to find out the nature of this operator which is again a few step, 1 or 2 step problem. It does not take much effort. So these exercises will give you a way to look on these Pauli spin matrices and the operator notation here are now very much comfortable I hope and are familiar with.

So this is the part of tutorial which I wanted to discuss today. And I hope that you all are practicing these problems on your own because these tutorials will be very beneficial for you all in solving the assignments, weekly assignments. So hope you are getting enough time and you are solving these exercises.