Quantum Mechanics
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Lecture – 43
Tutorial 7 - Part II

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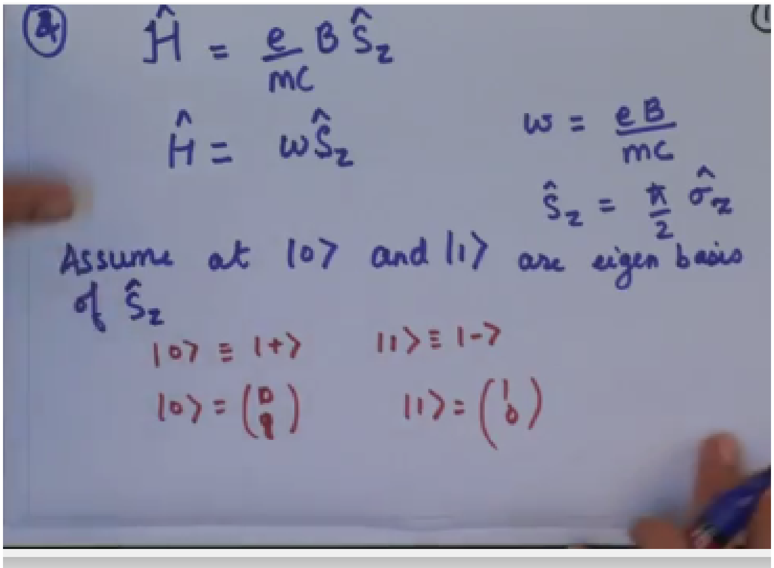

The fourth problem is that the particle is subjected to magnetic field along the z direction and it has some energy with the Hamiltonian operator given to you as e/ mc * B Sz, where B is the magnetic field and it is along the z direction and we will rewrite this in terms of omega first, so omega is e B upon 2mc, so here I will write omega Sz okay where omega is eB upon mc, Hamiltonian operator in terms of omega and Sz is the poly spin operator along the z component.

You can write this as z as in terms of poly spin operator in the z direction, so Sz is h cross/ 2 sigma this is also we all know now, we will start by writing or assuming that assume that get 0 and get 1 are the Eigen ket or the Eigen basis of Sz; are Eigen bases of Sz and reconnect in your in the lecture, 0 was denoted as + and 1 was denoted as - I think, so here 0 indicate 1 0 state or 0 1 state and 1 indicate 1 0 state okay.

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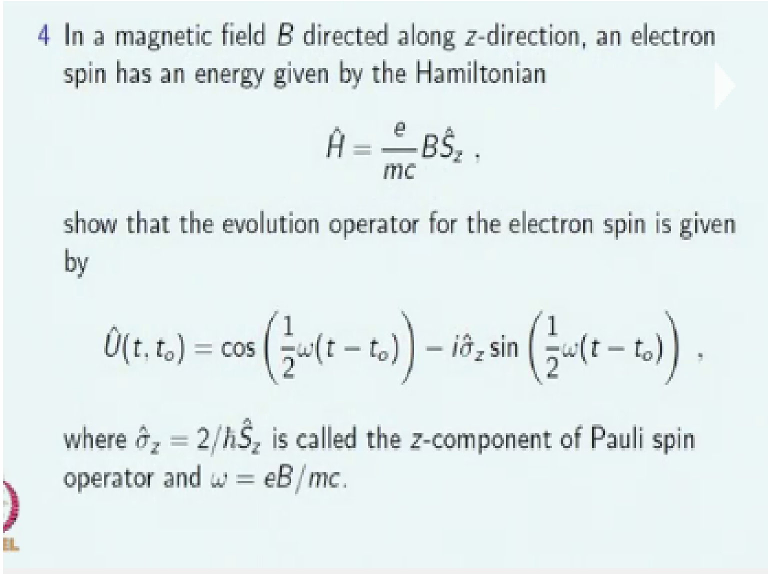

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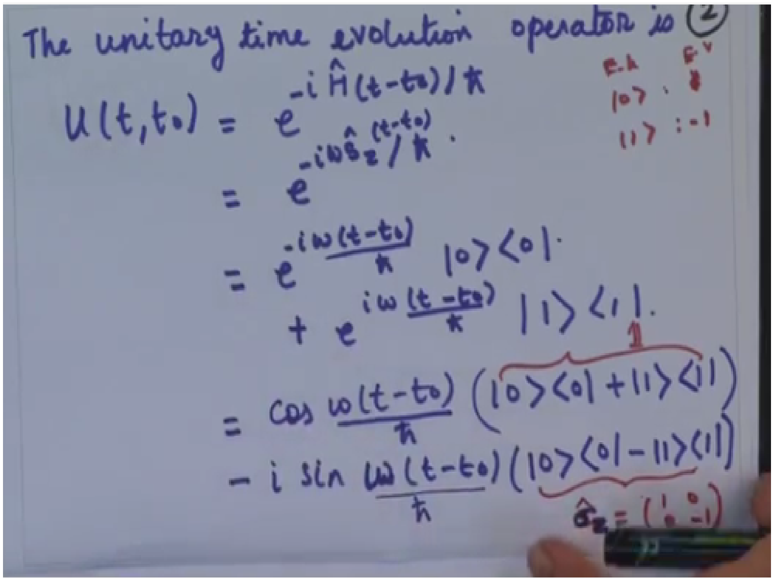

So, we denote instead of + and – ket, I am denoting it as 0 and 1, please do not get confused, so the unitary time evolution operator is; I will write it as u t, t0, so the evolution of time is from t = t0 to sometime t is written as e raised to - ih operator; Hamiltonian operator t – t0 upon h cross and we know what is the form of Hamiltonian operator, it is of the form omega Sz, so e raised to - i Sz times omega h cross into the factor t – t0 which is here.

So, this will be in the Dirac notation, we can write Sz or this exponent as omega t – t0 upon h cross ket 0 0 okay + e raised to I omega t – t0 upon h cross ket 1 1, remember the ket 0 and 1 are the Eigen basis of Sz with Eigen values 0 and 1, Eigen values, these are Eigen ket, so here we will have e raised to - I omega t – t0 ket 1 and another ket in terms of Sz basis, this exponent you can rewrite with eigenvalues 1 and – 1, I am sorry; 1 and – 1.

I made a mistake here, the exponent e raised to I theta you can write it as cos theta + I sine theta, so I write first term as cos omega t – t0 upon h cross okay and the ket from the first term of this and the first term of this will give me; because again I have cos theta that is cos omega t – t0 upon h cross okay + e raised to - I theta will give me - I sin omega t - t0, correct upon h cross times 0 0 -1 1, okay.

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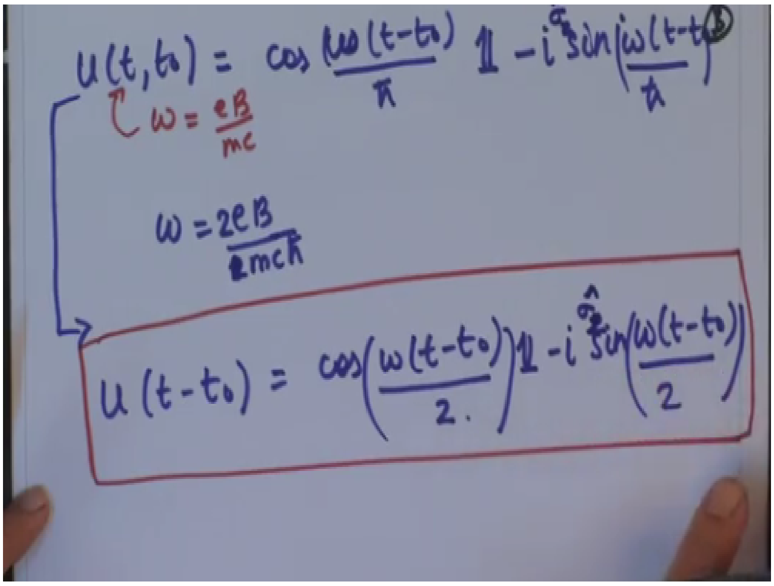

First term will give me - I sin theta times this + I sin theta this will be -, okay, then what we do next is; u of t, t0 will be; we can rewrite as cos omega t – t0 upon h cross okay, times the unitary operator or the identity operator - I sin omega t – t0 upon h cross, this is what we obtain in terms of times, this term this would denote the identity operator, top one however, this would denote Sz in the Sz basis I will have 1 and – 1.

Sz, you will know is 1 0 0 -1 okay so in terms of Eigen basis of Sz you obtain Sz, right, now here just a note that if you consider omega is eB upon 2mch cross then you will obtain, so this will become actually t – t0 t cos omega t – t0 upon 2, okay or rather you have to multiply by 2 because I have a 2 in the denominator times the identity operator - I sin omega t – t0 divided by 2, so if we start writing omega is to e by; eB upon mch cross then we obtain this result.

However, we had started this by writing omega as eB upon mc, so you obtain this result, so it is just the way you define omega okay, we will get the same result okay, here I have to write sigma z, here also I have a sigma z or Sz term, so this was Sz, here this was Sz, okay, so in terms this is sorry, this is sigma z okay and this we have included in our expression over here, Sz said okay, right.

So, the unitary operator can be expressed in terms of cos of the identity cos theta, I can call this entire term as theta, so cos theta times the identity operator - I sigma Sz sin theta right.

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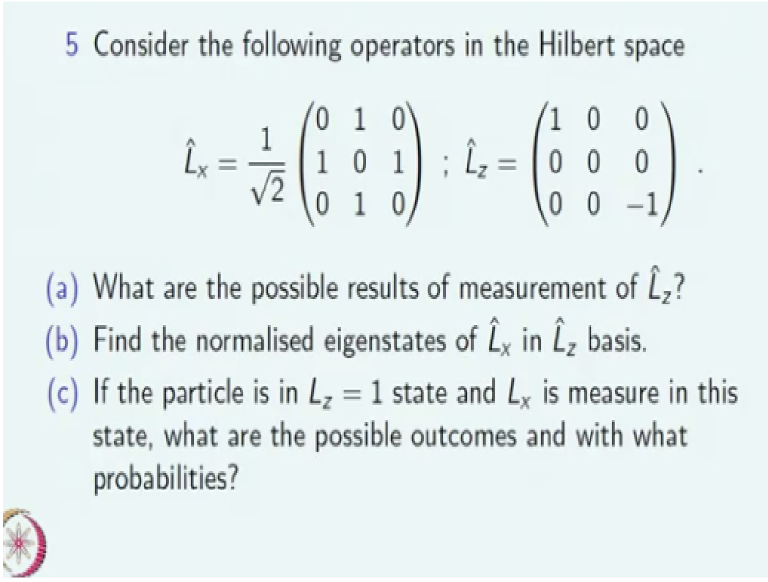

Let us go to the fifth problem okay, fifth problem is based on the; okay, fifth problem is based on the orbital angular momentum operator Lx and Lz.

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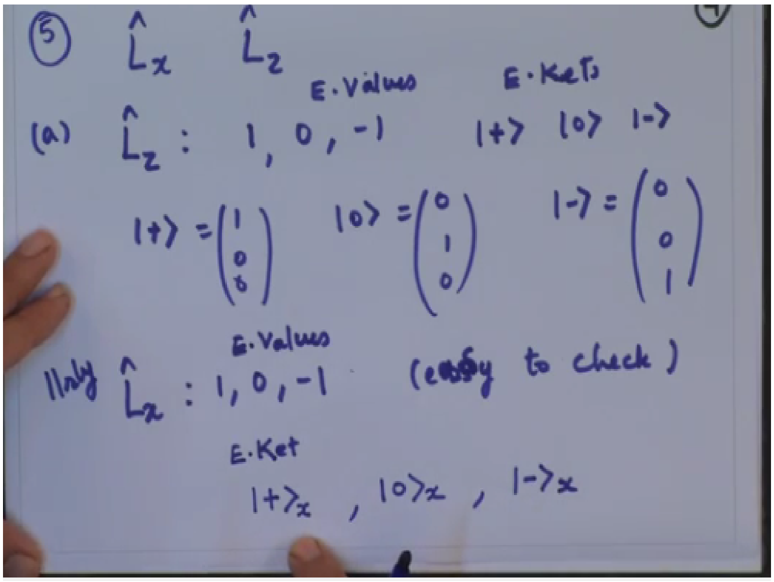

Now, we have already seen in since first tutorial, we are calculating, we are trying to calculate the eigenvalues and Eigen function, the wave functions, so you can do this computation by yourself but let me give you some hints, what are the first part is; what are the measurements, what are the possible results of measurement of Lz which is asked to you and you have to find the normalized Eigen state of Lx in terms of Lz basis.

And third part is if the particle is in Lz state and Lx is measured in that state, what are the possible outcomes that we get? So, Lx is given to you and Lz operator is given to you, from Lx and Lz, you can see that Lz has Eigen values 1, 0, - 1 these are 3 Eigen values and the corresponding Eigen ket or the operator; the vectors would be, I denote it as + 0 and – okay, so here plus is 1 0 0, this is the standard basis and -, is this.

So, the Eigen state of Lz are 1, 0, -1 and Eigen values are 1,0, -1, the Eigen state are + 0 and -, now from this you can try and calculate, what is the Eigen values of Lx, Eigen values of Lx are also 1, 0, - 1, it is easy to; easy to check okay, it is easy to check this and we have to now write, let me call this is the eigenvalue and the eigenvectors or the Eigen ket values okay, so that there is no confusion.

So, Eigen ket let me denote them by + ket with a subscript x, 0 with a subscript x and minus, now you have to find out the Eigen ket of that is + x in terms of + 0 and - that is in the Sz basis, so when once you do that you will be able to calculate the Eigen vectors or the possible measurements of Lz, we have seen that these are the possible measurement that is it can take Eigen values 1, 0, -1 and the corresponding Eigen ket we have seen.

For Lz, we have to find out so that we can write the normalized Eigen ket Lz in; Lx in Lz basis.

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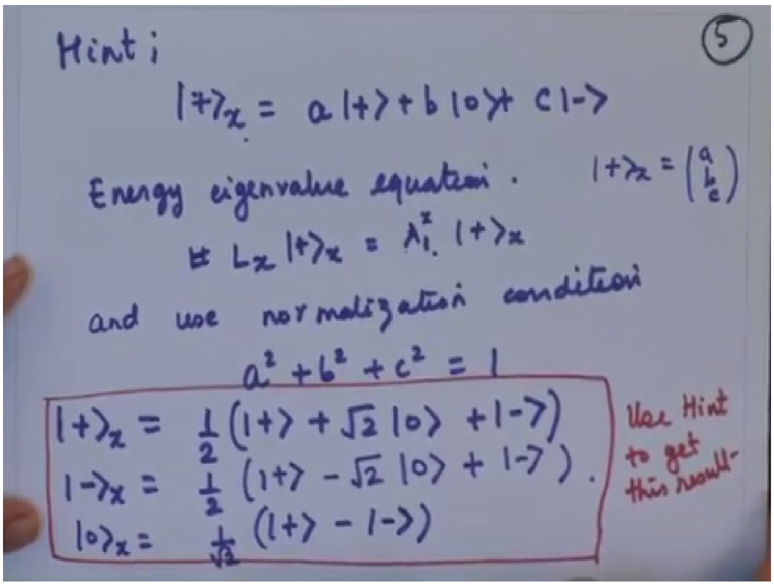

I can give you a hint that if you want to calculate ket plus of in the x basis in terms of ket + 0 and -, these are the Eigen basis of Lz, so in terms of Lz, if you want to calculate, let us write a linear combination of these such that L +; so Lx as the Eigen ket + x in terms of the linear combination of the Eigen basis of Lz, now how will I obtain this coefficients a, b and c, simply what you can do is; use energy eigenvalue equation, right that is Hi, so you have Lx + = Eigen value is say lambda 1 x + x.

And you will use normalization condition and you know that +, 0 and – ket are orthogonal, so normalization condition is a square + b square + c square = 1 and first part you can just substitute for Lx, substitute the value of lambda 1 which is the Eigen value, say if you start with 1 as the Eigen value and just write + x as a, b and c and see what you get. So, what I have obtained is + ket in x basis will be 1/2, I have + root 2 ket 0 + ket -;– ket gives 1/2 1 +; 1/2 ket 1 ket + - square root of 2 ket 0 + ket -.

And the third one is 1/root 2 ket + - ket -, so use hint to get this result, you will do it for plus, minus as well as ket 0 and obtain the corresponding coefficients a, b and c and then you write the result, so it was very simple, this was Part B of this problem; last problem and in the last part if the particle is in Lz state, which is Lz is one state and Lx is the; is measured in that state, what are the probabilities.

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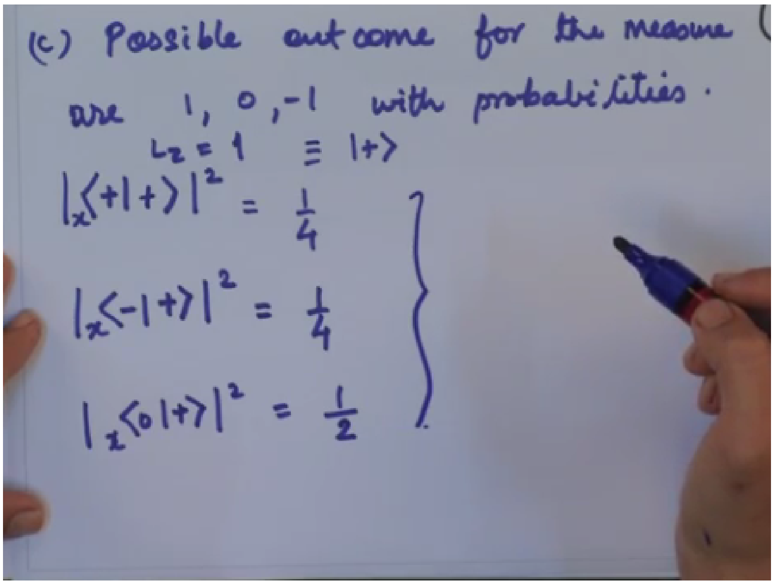

So, again you should know what are these kets; plus, minus and 0 in x basis and you have to calculate the get these ket on plus that is what I am trying to say here is the possible outcomes; outcomes for the measurement are 1, 0, -1 okay with the probabilities, what will the probability? So, for +x on so, Lz is in state 1 which will correspond to Eigen ket plus so, we have to find out the probability of measurement on state Lz = 1, this will give me 1/4, you can just do it okay.

We have obtained already the Eigen ket plus minus and 0 from; in Lz as well as Lx basis using that information, you can calculate the result, so for plus you have 1/4 for minus, the probability is again 1/4 and for ket 0 on 1, Lz = 1 is simply 1/2 okay, so from the coefficient itself you can infer this result. So, let us stop here with these exercises, you can also have more exercises on orbital angular momentum operator on commutation of these operator Lx, Ly, Lz on spherical harmonics like L; Y +; Y combination of L and ml and those problems you can try it yourself.