Quantum Mechanics
Prof. P. Ramadevi
Department of Physics
Indian Institute of Technology- Bombay

Lecture – 42
Tutorial 7 - Part I

Let us begin this tutorial, it is a tutorial with variety of problems, problems based on the finding the energy and wave function of 2 identical particles which are non-interacting and in line, we have problems on the orbital angular momentum operator and some problems on spherical harmonics.

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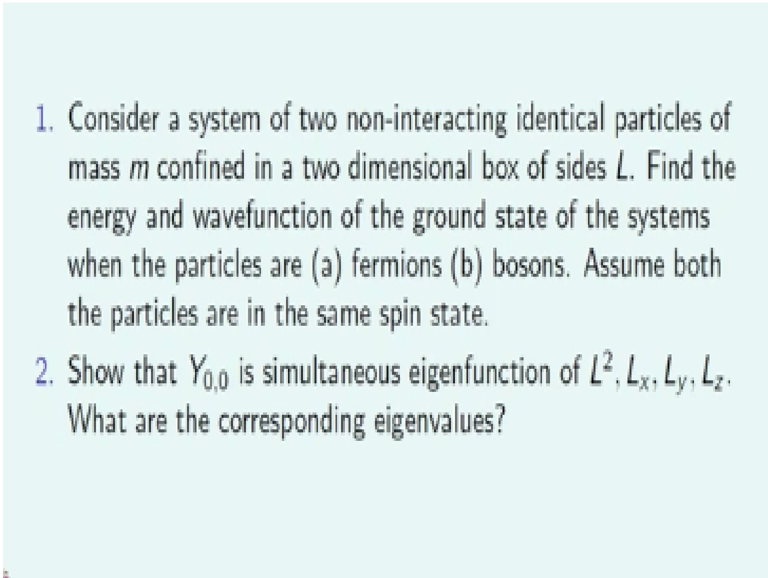

So, first problem consider a system of 2 non interacting identical particles of mass m confined to a two dimensional potential box of length L, so find the energy and the wave function of the ground state of the system, when the particles are a; fermion b; bosons, assume that both the particles are in the same spin states.

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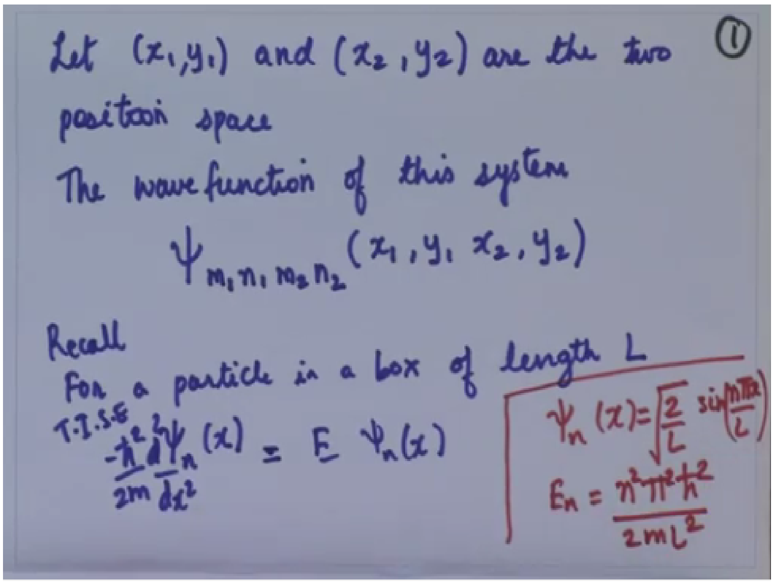

Let us start let us assume that x1 y1 and x2 y2 are the two position space are of this spin system of this interacting particle system and to start with now, the wave function of this composite system of 2 particle will be psi m1 n2 m2 n2 as a function of x1 y1 x2 y2, just recall before we go to the finding the wave function and energy recall for a particle in a box of length L, the wave function was given by or rather time independent Schrodinger equation was given by - h cross square upon 2m psi n x d square/ dx square.

This is the kinetic energy operator plus if I consider a free particle, it will be just right, here n is the quantum number, x is the coordinate space, so for one dimensional particle, we just have the wave function as psi n x which is 2/ L; square root of 2/L sin n pi x/ L, okay and the energy; corresponding energy is given by n square pi square h cross square upon 2m L square, okay. So, psi n is the solution of this equation and En is the corresponding energy.

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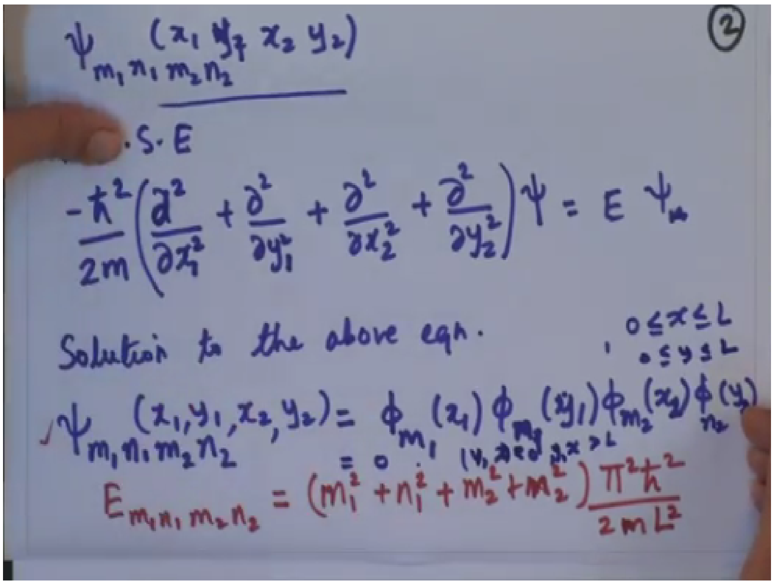

So, in the same line we have psi m1 n2 m2 n2 x1 x2 x1 y1 x2 y2 will be the wave function for the particle in a box; for 2 particles in a box in two dimension, so the corresponding time independent Schrodinger equation will be - h cross square upon 2m d square; dou square/ dou x square + dou square/ dou y square, we have x1 y2 for particle 1 + x2 and y2 for particle 2 psi that is = E psi n.

So, Schrodinger equation for 2 particles in two dimensional box here, 1 indicate the first particle, 2 indicate the second particle, m and n represents the quantum numbers for particle 1 along x and y direction; for x and y direction, so solution to the above equation can be written as the wave function, this is the wave function, we had already written but this can be written rewritten as x1 y1 x2 y2 wave function of individual particle.

So, phi n; phi m1 x1 phi m2; n1 y1 phi m2 x1 phi n2 sorry x2 and this is y2, so each of these wave function would correspond to these values, so for first one phi m1 x1, the wave function would be 2; square root of 2/ L sin m1 square pi square h cross square upon 2ml square and similarly, the others would go and the energy E m1 n1 m2 n2, I denoted by this subscript will be = m1 square + n1 square + m2 square + n2 square pi square h cross square upon 2m L square.

So, this is the energy eigenvalue; energy of the composite system which has the wave function this, okay and remember that the wave function is of this form between the regions x < 0 < L > 0 that is inside the box however, this wave function would be 0 for x < 0 and x > L, this is the same condition which we have for the particle in a box, so this; in this case it is x and y both, so this will be same for y also, x, y okay.

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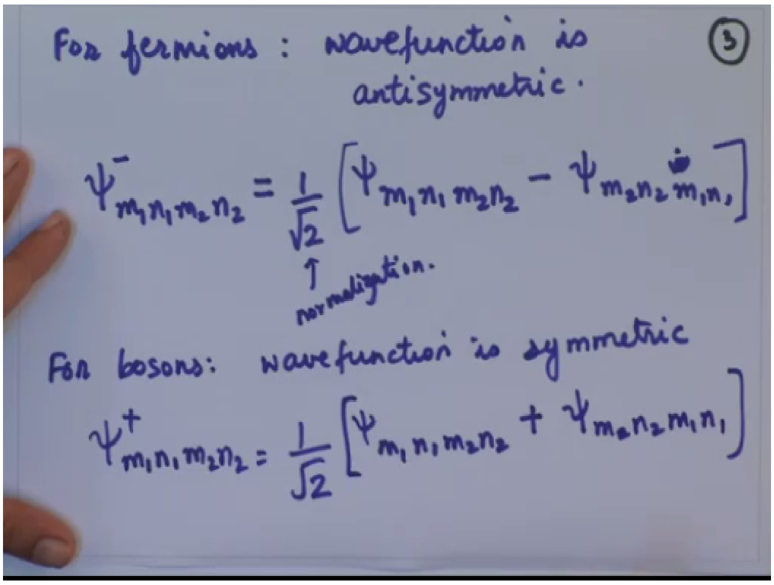

Now, for the case of fermions you have seen in the lecture already, for the case of fermions; for fermions, the wave function would change in sign, so for fermions the wave function should be anti-symmetric or it should be like the odd function, so for fermions wave function is anti-symmetric under exchange of particle, so when we exchange particle 1 with particle 2, will have an additional minus sign.

That is to say that the wave function is now written as psi -; minus indicates that the wave function is anti-symmetric, the same notation I am using as it was used in the lecture, so here I have psi m1 n1 m2 n2 - psi m2 n2 m1 n1, okay. So, when I exchange particle 2 by particle 1 when there is a flip, there is a change in sign and this factor is the normalization factor. For bosons, if the 2 particles are bosons, then the wave function is symmetric.

So, when the wave function is symmetric, we represent it by a plus sign m1 n1 m2 n2 will be square root of 2 psi m1 n1 m2 n2 + psi m2 n2 m1 n1, so if you interchange particle 1 by particle 2, there is no change in sign, it will remain symmetric.

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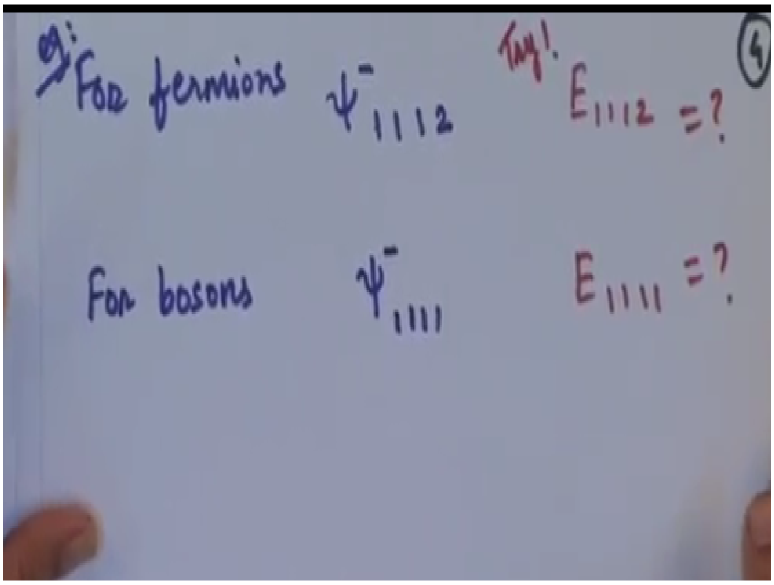

So, as an example just for fermions, this is example you can try for fermions, is a wave functions psi – 1 1 1 2 is given to you then, what would be the possible; what will be the possible energy? So, you calculate E 1112, what do you get and you can take one more example for bosons, if the composite system is of 2 bosons then, let me take same quantum numbers, what will be this, so just try this.

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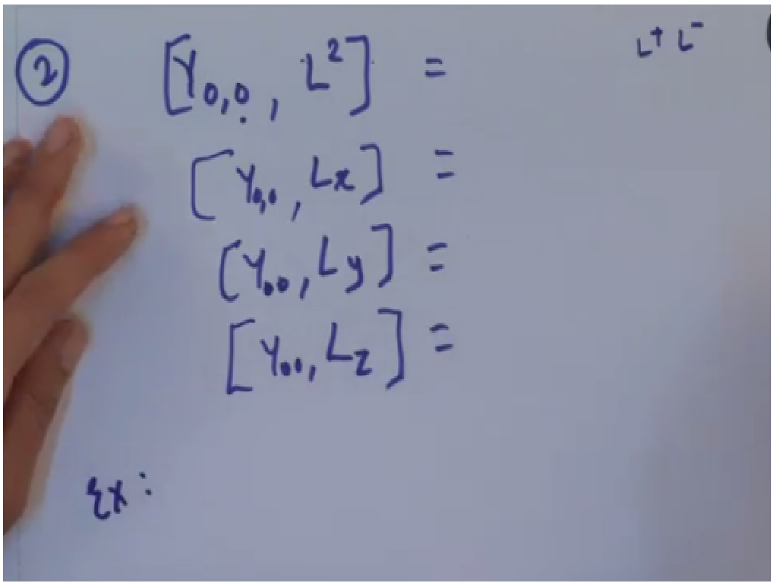

Let us go to the second problem spherical harmonic, second problem; you are given the spherical harmonic y 0, 0 and you have to find out whether y 0, 0 is a simultaneous Eigen function of L square Lx, Ly and Lz, what do I mean by this; you have to find out commutator of L0, Y 0, 0, L square, Y 0, 0, Lx, Ly and commutator of Y 0, 0 with Lz, so this you need to evaluate which I think, when you write this in the; in it's; in polar coordinates or in terms of L square in terms of Y, you can easily do this by just writing down the value of Y 0, 0 and L square.

You can either use ladder operator you can write in terms of ladder operator if you wish or you can just simply write, you can write the spherical harmonics and L square in polar coordinates and try to see what you get, so this is for you exercise, you can try.

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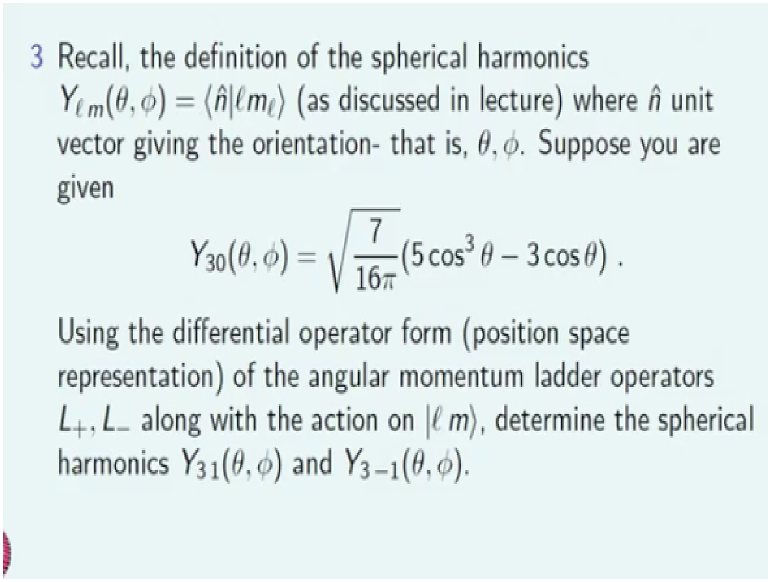

Then going to the third problem, recall the definition of spherical harmonics which was discussed in the class okay, so what is the definition?

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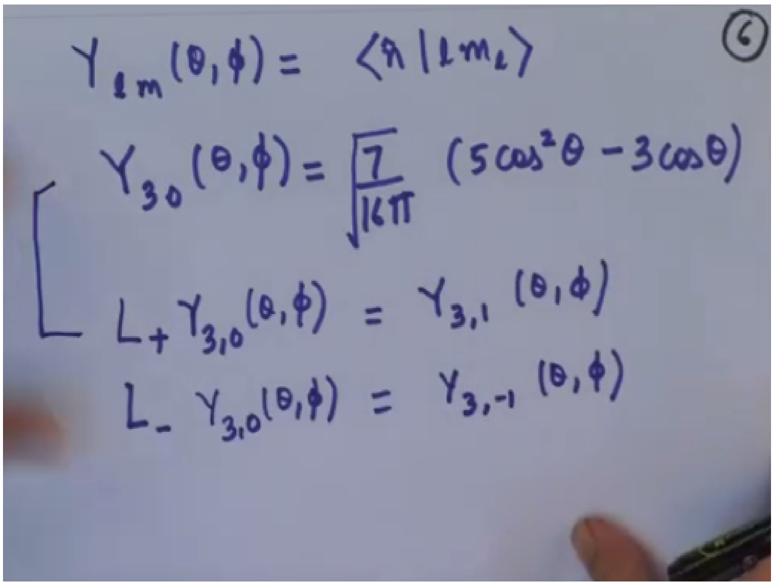

Psi lm theta pi, you can write this as n cap l ml and using this definition or the notation, you have to find out what is Y 31 and Y 3 – 1, what you are given is Y 30 as 7/16 pi 5 cos square theta - 3 cos theta, this is given and you have to use the ladder operator, so if I want to go from 30 when operated on L +; we can obtain, right similarly, if I go from Y 30 to Y3 – 1, so when the quantum number changes from 1 to – 1, 0 to 1 or 0 to -1, you will use ladder operator.

So, this will give us 3 - 1 theta phi, using this you have to obtain the result, so we use these two.

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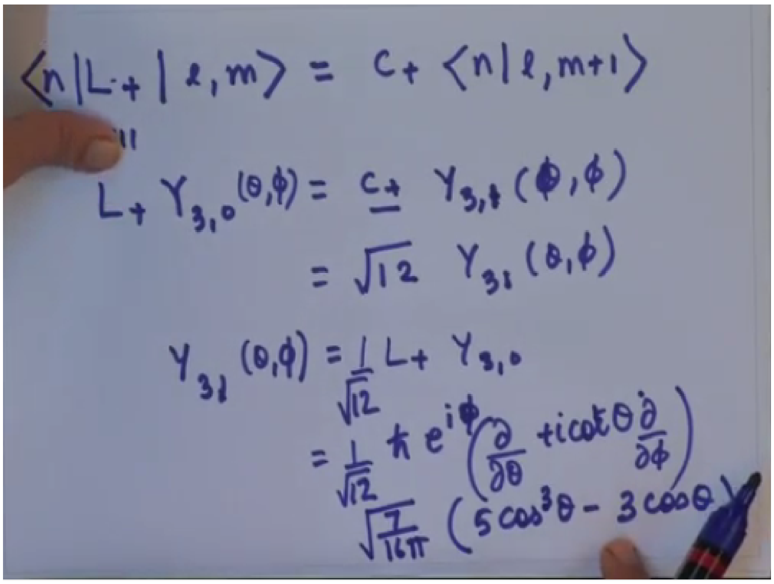

L + on l ml would give us C +, you get a Clebsch Gordan coefficient for L + which will be; I let me call it as C + and then you have n l m + 1, so when you do this, this is equivalent to writing L + psi 3, 0 where m is 0, L is 3 will become C + and here you have 3 + 1; 0 + 1, I get 1, so I have to evaluate the value of C, so for L = 3, m = 0, I will have C + as square root of 12 and this will be as it is theta phi, theta phi, theta phi.

So Y31 theta phi will be L +; I will write in terms of 1/12 Y 3, 0. L +; I will write in polar coordinate as h cross e raised to i phi dou/ dou theta + i cot theta dou/ dou phi. And when you operate this on Y, Y is nothing but 7 upon 16 pi 5 cos cube theta - 3 cos theta, so when I do, when I operate this on this function, this term will not contribute because this operated on this function will give me 0, this will give me 15; -15 cos square theta sin theta - 3 sin theta, so - will become +.

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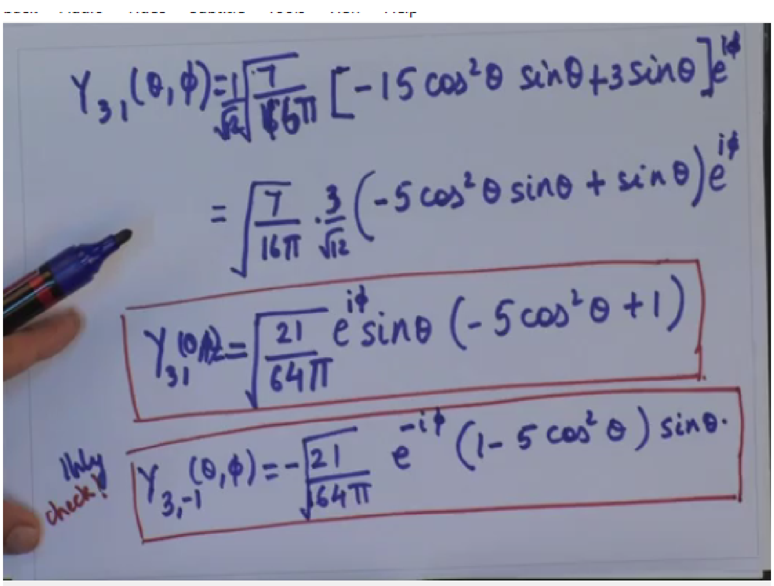

Y 31 theta phi would be 7/ 64 in the square root because here, we had 16, then here you have square root of 12, 7, 16 pi, so I will have here on simplification, I will get this derivative will be - 15 cos square theta sin theta + 3 sin theta, so just check after simplification if you get this result, 16 * 12 , okay so, 3 I have taken out, this will be 16, 1/root 12, here if I take 3 out, I will have 7/ 16 pi in the square root times I have 3 outside, so I have - 5 cos square theta sin theta + sin theta.

In fact, I can take sin theta also outside and then simplified further, I can just take sin theta outside 1/ 12 and I have a 3 over here okay, so this will result in 21 upon 64 pi sin theta - 5 cos square theta + 1, in the same way and see to it that you are carrying out this properly here, I have e raised to I phi which I was not writing throughout, so I will just include it here, e raised to I Phi which has to be included everywhere.

Similarly, Y 3 - 1 theta phi would be 21 square root of 21, 64 pi e raised to -i phi 1 – 5 cos square theta sin theta okay, so this is the result you can check whether you obtain this, so the crucial part here was that you write L + in terms of the polar coordinate and then you operate this on the Y 3, 0 given to you and then you can easily see the result, so with this 3 problems, we can discuss the remaining problem in the next part of the tutorial.