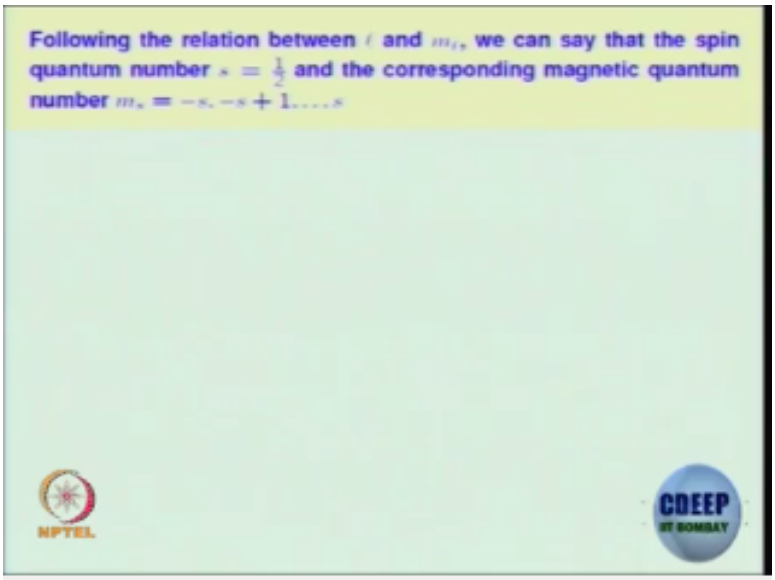
Quantum Mechanics
Prof. P. Ramadevi
Department of Physics
Indian Institute of Technology- Bombay

Lecture – 40
Identical Particles & Quantum Computer – I

(Refer Slide Time: 00:40)



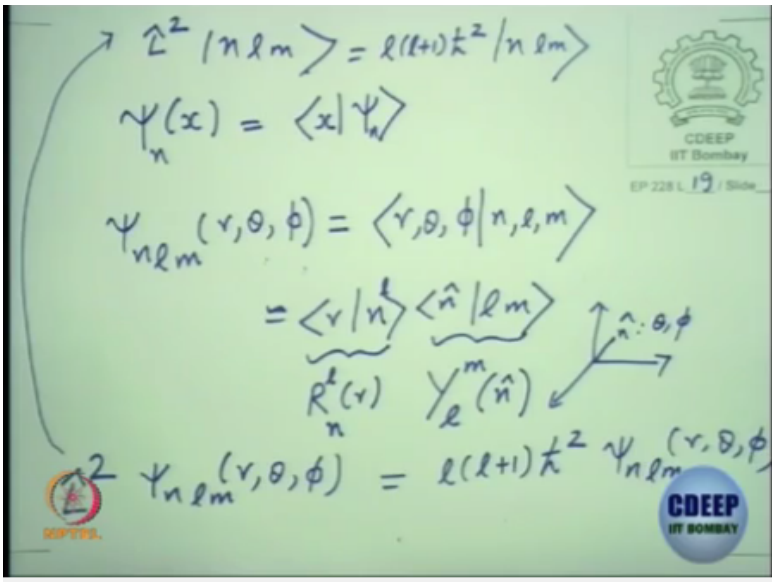
So, I try to give you enough motivation in the last lecture that for L = 0, this Zeeman splitting should not be seen but L = 0, you still see too close very close lines in the lab or in the experiments and you cannot account if you live only with orbital quantum number l and ml, the magnetic quantum number ml, it is not possible to account for l = 0 having 2 distinct lines showing up in the experiments.

So, the only way out is that there is an additional quantum number which is the spin quantum number, this is put by hand as for as non-relativistic quantum mechanics is concerned, there is no formal way in which because of the experiments, in order to understand the experiments and that there are exactly 2 lines for L = 0, you are forced to put in a spin quantum number or the hydrogen atom which is s = 1/2.

So, this is in addition to the 3 quantum numbers which you had which is n principle quantum number, l; Azimuthal quantum number and ml which is the magnetic quantum number, okay I am sure even in the middle of the sleep, when you are trying to see the periodic table you know

what they; what each of these quantum numbers are and we see from the Schrodinger equation that these 3 quantum numbers are essential for the wave function.

(Refer Slide Time: 02:12)



See, when we wrote this position space wave function in one dimension, we said that we take the state psi and take the projection with respect to x, right, so this is what we said as the position space wave function where this is the Dirac state of a system. When I write psi nlm, which is function of r theta phi, how do we do this, okay, so suppose I had an n here you could have put a n here for particle in a box, n is the; nth excited level of the particle in a box.

N = 1 is the ground state, n = 2 is the first excited state and so on for the particle in a box, this I am assuming that all of you are; now, the hydrogen atom spectrum we found that this is the solution, so there is a neat way in which you can write this as if what do you do; you can take r theta phi and take it with n, l and m or I could put a psi subscript but both are equivalent, you can forget about the psi and write a n there, okay but you can write it.

What further; you can also write this as r with n yeah, so theta and phi is basically giving you the unit vector direction suppose, you have in 3 dimension, if I put a unit vector; some unit vector that tells me what is the theta and phi, you all agree, so you could try to split this and write it as if it is r times n and I can put an unit vector whose magnitude is this radial value but whose direction is giving you the angle theta and phi with lm.

What is this we are going to call; we are going to call it as the radial part of the wave function, what is this; this is your spherical harmonic, which you can write it as Ylm of theta phi or you

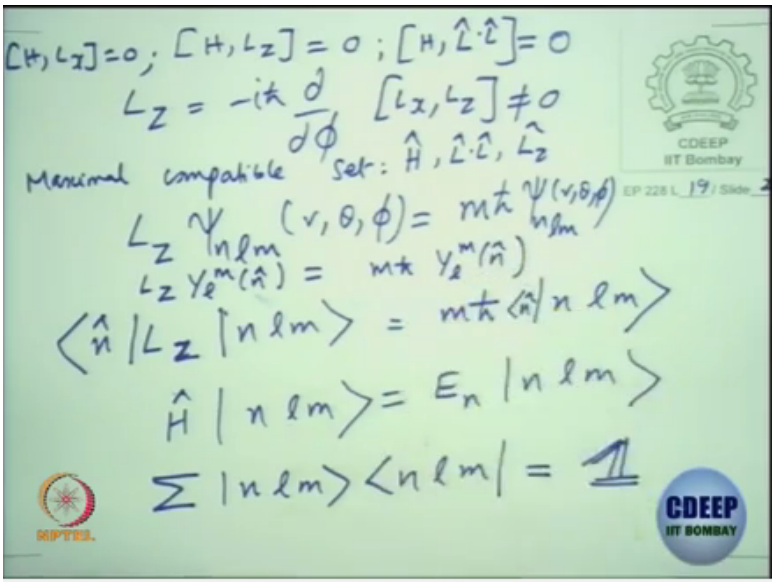
can also call it as hat n with the; where hat n is the unit vector that is so, it is not, it is I should say it is approximately I should keep in the back of my mind that there is this L dependence which is there, okay.

So, this is a very; this is correct, this one has to be taken that there is the L dependency on this place, good point okay, so let me put that also here just for us to remember, so this we do not know often but because we know the solution we know this is the way, what else did we do? We also said there is a L hat square operator on psi nlm r theta phi, what was that; l * l + 1 h cross square psi nlm r theta phi.

If suppose, we use that definition, it operates on position or on this, the quantum number seems to pick up the; like if you had the energy operator here it does not operate on the position basis, it operates on this and gives you En times psi, so similarly here what will you say from here, we could try and say L squared on nlm will be; so the Dirac notation is coming into play but whenever we write the quantum state as a ket vector, we can try to denote it by the 3 quantum numbers; nlm.

And it is an Eigen state of L square with Eigen value l * l + 1 h cross square and similarly, what happens to Lz; what happens to Lz, is it an Eigen state?

(Refer Slide Time: 07:08)



So, we saw Lz is -ih cross del/ del phi and you can write Lz on psi nlm r theta phi to be mh cross psi nlm r theta phi, this was the data which we studied in the last lecture, we introduced orbital magnetic moment and I explicitly said del/ del phi will operate only on the e to the im

phi, I will give you the m phi, right h cross is coming because you put a position representation for Lz which is r cross p, where p has the - ih cross del/ del that is why that h cross has come.

So, now in the Dirac notation, can I write this; is it an Eigen state with eigenvalue, so what are we seen; Hamiltonian on nlm will give you nlm, so the energy eigenvalue is independent of the other 2 quantum numbers and we also see the same state is an Eigen state of Lz, if you have simultaneous Eigen states should also be complete okay, some 2 states matching does not mean that it will be simultaneous Eigen state.

When I say it is complete, how do I write this completeness relation; summation over nlm, nlm with appropriate summations, all possible n's going from 1 to infinity, l going from 0 to n – 1, m going from – l to + l, the constraint thing which you have to put in here, completeness condition tells you that it should be identity, so if this is valid for all the states, it is true for any arbitrary state I have written.

What can you compare from here, what can you say are they compatible or not compatible? They are compatible observables, Hamiltonian commutes with Lz and they share the simultaneous Eigen states, it is the simultaneous Eigen state of both Lz, the complete set, both Lz and h Hamiltonian conversely you can say that the Hamiltonian commutes with Lz, we have seen all these things in the rotationally invariant central potential problems.

But I am just trying to put it in a notation where will I slowly take you on to the spin quantum number and how to go about doing this Dirac notation from this picture, okay. What about L squared; L square also satisfies this, right which what we saw in the earlier transference, so can I also say L squared is actually L dot L, the operative form of L dot L, they also share the simultaneous complete step, so this will be 0.

What about h with Lx, it is just for z but if you want to do Hamiltonian with Lx, that will also be 0 but will this be an Eigen state of Lx, why? Lx, lx is not 0, you can either write Eigen state of Lz or you can write Eigen state of Lx, you cannot write the simultaneous Eigen state of Lx and Lz, right, so you need to find from this set even though H with Lx is also 0, what is the z which is compatible if I ask you, the total z, you have to find the intersections, what is the intersection possible given this data?

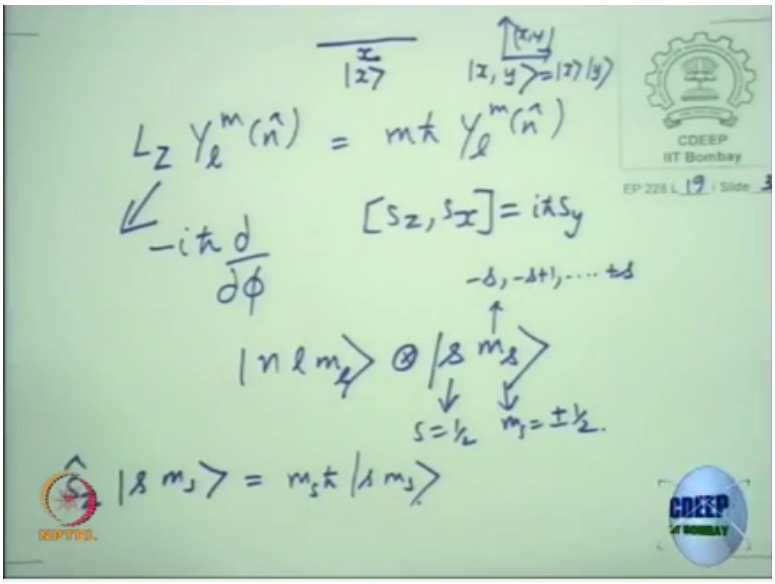
It is not an Eigen state of Lx, everybody agrees know, you can go on check and see that it is not an Eigen state, so it is a simultaneous Eigen state of Hamiltonian Lz and L square that is the maximal compatible set you can write, you cannot look at all the commutators and say Lx is also compatible, Lz is also compatible you cannot say, if you say that you have to also make sure Lx, Lz is compatible which is not, so only one over the 2 should be taken into consideration, either Lz or Lx.

And we conventionally, we take Lz to be the conventional axis; z axis to be the simultaneous Eigen states, okay so maximal compatible set will be Hamiltonian, L dot L and Lz that is it, even though Hamiltonian commutes with Lx, Lx, Lz do not commute, so it cannot be a simultaneous Eigen state, here the state which I write nlm is the simultaneous Eigen state of Hamiltonian, L dot L and Lz, clear.

So, this is a way to go about either you can see a compatible observables write the simultaneous states or conversely since I have found this states and I have explicitly verified to be an eigenvalue equation, I am trying to argue for you that for central potentials, Hamiltonian commutes with all the components of angular momentum but all the components of angular momentum do not commute among themselves.

So, the maximal compatible set which defines a simultaneous Eigen states which is nlm, it is the simultaneous Eigen state of H, L dot L and Lz, clear, you can take projections here with hat n and what will this be called? This will be Lz operator on the Yn; T lm part, okay, so we can try to write lz on Ylm of hat n is mh cross Ylm of hat n.

(Refer Slide Time: 14:24)



And you can give a position representation for Lz just like we did for the momentum; linear momentum independently, you know what these functions are and you can find what this satisfies this equation and you can say Lz, the position representation will be del/del phi in spherical polar coordinates, if you do a del/ del phi on the spherical harmonics, you will get an mh cross, is the connection getting clear to all of you, such a beautiful elegant concepts of Dirac formalism that you can throw away putting the projection onto the position coordinates.

And play around with the states as nlm in the hydrogen atom problem but at the back of your mind, if you want to write what is the position operator for Lz, you can figure it out from here, you are clear with this then the rest of the Stern-Gerlach and all will be very smooth, you will understand it, they are all finite dimensional vector spaces, these nlm for a specific n, if you took a specific n this lm, the degeneracy is n squared.

Now, if you want to introduce spin quantum number, what do I do; I have to multiply or call it as ml, then you will have a you know a separate space, internal space which they call it as a tensor product or a Cartesian product like if you are in a line; one dimensional line we call the coordinate as x, if you are in a two dimensional plane, you call it as x, y as a point, right but formally when I write a state here, I would write it as x.

When I write a state here, I would write it as x , y but you could also write it as x * y, no harm, it is the same thing, you have a space which is principal quantum number, azimuthal, magnetic quantum number but you also have, you are forced by experiments to introduce the spin quantum number and ms quantum number which is independent and because you see 2 lines in

the experiments even for L = 0 in the presence of a magnetic field, we try and say that this is 1/2 and ms can take values + or – 1/2.

It goes from - s to + s, is the motivation clear, how this s = 1/2 is pushed in from the experimental data because you have 2 splitting's only seen not more than 2 splitting, yeah, yeah, yeah, so that is why, if you take s to be 1/2 the magnetic quantum number corresponding to the spin quantum number will go from - s to + s, in steps of one, right if this one should take from – s, - s + 1 dot dot dot 2 + s and I want this to be only 2 states.

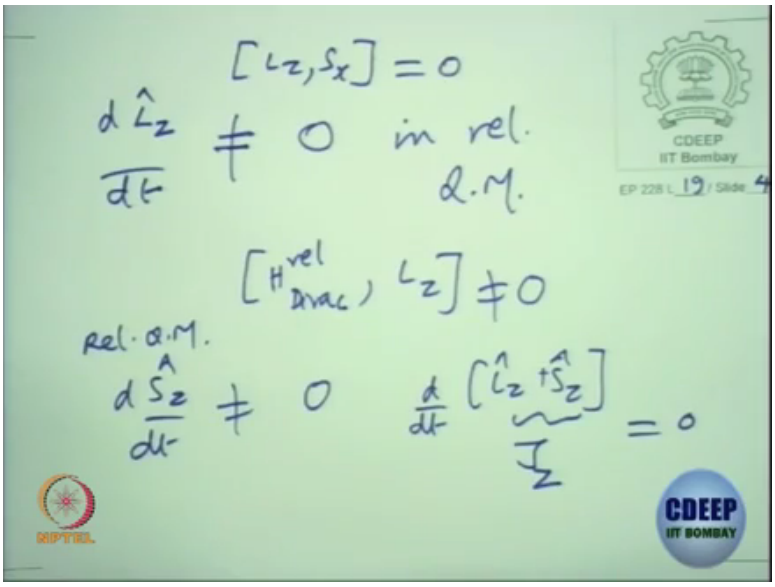
There are only 2 lines splitting seen, what is the solution, s has to be 1/2 you can work with arbitrary s also has a theory person but then if you want to compare with what is happening in the lab, you need to make contact and here put making the contact with all these Stern-Gerlach and the you know anomalous Zeeman effect seen, you have to have s = 1/2, yeah just like your orbital quantum number, the properties of the spin quantum number is exactly similar to; yeah this is ad hoc.

In non-relativistic quantum mechanics, we are just saying that if you have an s and an ms, there is a sz operator, which will give me ms h cross sms, exactly like the orbital quantum number, I want my spin quantum number to be here and also there are other kind of data like the spin orbit coupling data, it shows that these couplings cannot be done, if you do not allow your spin operator sz with sx to be exactly similar to the way, the commutator bracket.

It has to be similar to ih cross sy exactly, like your Lz, Lx, the commutator bracket of Lx with Ly that properties be is kind of seen also for the spin quantum number, so you cannot; it behaves exactly like the orbital quantum number as far as properties are concerned but it is just that this quantum number is in a different space, there is no way you can make this behave differently, if you try to do it, like it something similar to f = ma when I write, if left hand side is a vector, the right hand side has to be a vector, right.

It as tight as it is, the L quantum number if I say the magnetic quantum number for ml goes from - L to + L, because there are spin orbit coupling, the s also should have a spin quantum number and s; ms has to go from - s to + s that is it, that is the theory you have no choice, okay but as far as nonrelativistic quantum mechanics is concerned, this is the postulate for you but if you do Dirac equation in relativistic quantum mechanics, these things comes automatically.

(Refer Slide Time: 20:15)



This is what I was telling you that if you try to do the Heisenberg's equation and do dl z operator/ dz in your hydrogen atom or nonrelativistic Schrodinger equation, it has to be proportional to commutator of Hamiltonian with Lz, so it has to be 0 but this is non-zero in relativistic quantum mechanics because the H relativistic which is the Dirac Hamiltonian with Lz is nonzero, this you do not see in your Schrodinger equation.

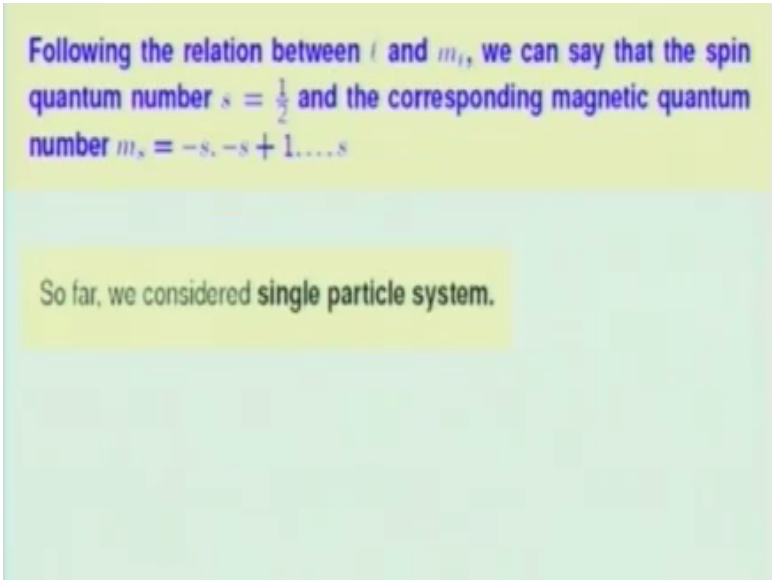
And the way to make for rotationally invariant angular momentum has to be conserved in relativistic theory, if you introduce another sz operator, this is also not 0 but you can show that the sum of Lz + sz okay, you can show that d/dt of Lz + sz which you will call it at the total angular momentum, this will be 0, okay, so this requires relativistic quantum mechanics and you can show the properties of Lz and sz are exactly how its commutator behaviour with its components.

How its commutator behaviour with other components in that space but these 2 are 2 different space, what do I mean by 2 different space, suppose, you want to compare bananas and apples, there is no way you can compare, amongst them you can compare, so it is like Lz with sz or sx, you just put it to be 0, they do not have any connection that is why you say it is a different space, any components.

I have written it for z and x but the L space and the S space are 2 different spaces, I am slowly dragging the fact that the spin quantum number is important but it is not derivable from non-relativistic Schrodinger equation but because of the experiment, we will put it by postulate that

there is a spin quantum number, s and ms, which is you know, you can take it as a Cartesian product or a tensor product and you can do spin operators on those days.

(Refer Slide Time: 22:32)



So, this is why the motivation which forces your s = 1/2 and the corresponding magnetic quantum number and we just looked at an electron in 1 over r potential here, right.