Quantum Mechanics
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Lecture - 36
Tutorial - 06 (Part I)

So let us start with harmonic oscillator problem. In problem 1 of this tutorial you are asked you are given a potential energy term in terms of the Cartesian coordinate X and Y.

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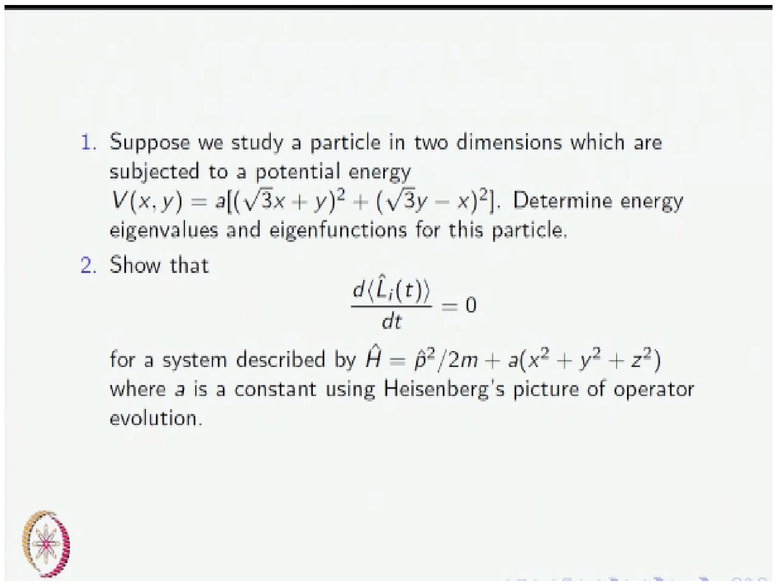

And you are asked to determine the energy eigenvalue and the corresponding eigenfunction for the particle which is in subjected to a potential which is a two-dimension potential. So in order to solve this question, we will try to simplify the potential given to us.

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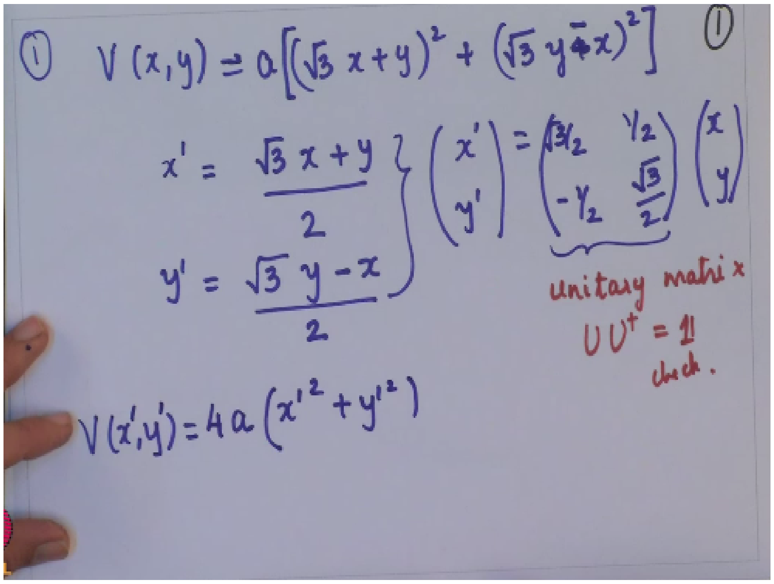

For example, here problem 1 potential given to us is function of x and y hence it is two-dimension potential and you have the potential is of the form 3x+y the whole square + this is a square bracket square root of 3y+x the whole square, okay. For harmonic oscillator just try to recollect the form of potential while solving this problem. So here let us redefine these coordinates as X prime as let me write it as root 3x+y/2 and Y prime is square root of 3y there is a - I think, yes there is a –x/2, okay so there is a square over here in this expression.

So I am; when I am rewriting this expression I will have a X prime square + Y prime square. Now from this, this is actually a transformation; when I write this in matrix notation, in matrix notation I will have x y; this will be 3/2 square root of 3/2; this will be 1/2 then I have -1/2 and square root of 3/2. So this is a unitary matrix, you can see and you can also check, you can check that you U U dagger is 1, you can check this, Okay. So this is a unitary matrix.

And we can now write the potential as a times 4 and X prime square + Y prime square. This will be the expression for the potential in new coordinate X prime and Y prime.

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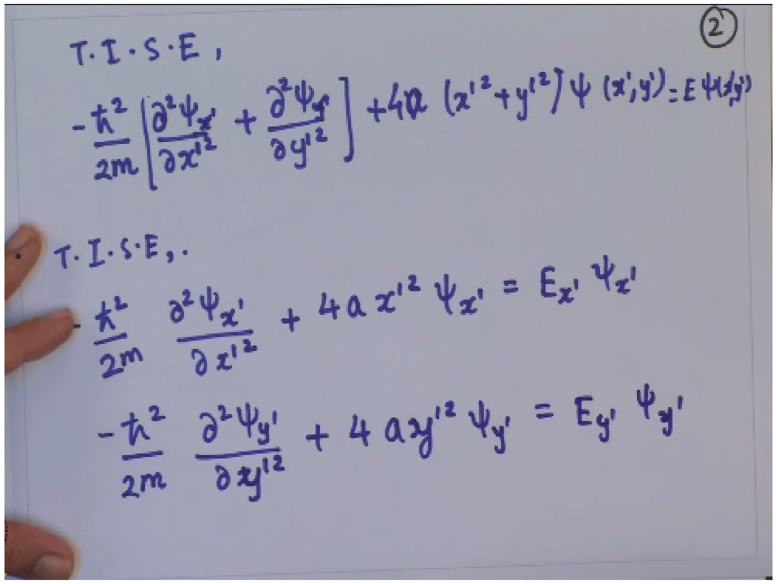

Now what we go to the next step what we do is we will write the time dependent, Time Independent Schrödinger equation. I am using a shorthand notation, Time Independent Schrodinger equation for 2 particle subjected to a potential that is two dimensional potential is d square Psi/dX prime square + d square Psi/dY prime square. Now remember I can always skip this subscripts and superscripts but since I am writing it for the first time let me put these subscripts. So – h cross square upon 2m then I will have a potential term that is V.

Or let me write here as 4a x prime square + Y prime square Psi of x, X prime, Y prime, okay. In the latter step we will write separation of variable so that time we will write the subscripts that will be better that is = E Psi X prime Y prime, okay. Now if I want to do separation of variable to get this Time Independent Schrodinger equation what do I do is I will have minus h cross square upon 2m del square Psi X prime/del X prime square; this is the kinetic term + 4a x prime square Psi of X prime = to Ex prime Psi of X prime.

You can use separation of variables and write the equation Schrodinger equation for X prime coordinate and XY prime coordinate separately. So the second expression would be del square Psi Y prime/ 10 Y prime square + 4a y prime square Psi Y prime = Ey prime Psi of Y prime. So these two equations we have. Just recall the standard form of Schrodinger equation for a harmonic oscillator subjected to a potential 1/2 kx square, okay harmonic oscillator potential. Just recall that.

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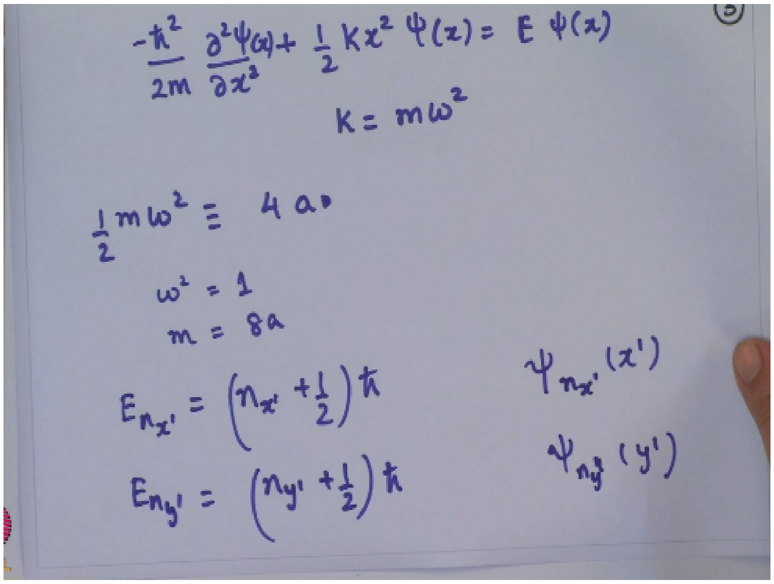

A expression or harmonic oscillator potential 1/2 kx square Psi of x = E Psi of x. And when you compare this term with the potential which we have what do we obtain, we find that all or k is nothing but m Omega square. So when you compare m, 1/2 m Omega square with our expression 4a x square; I will drop so 4a. So if I assume Omega square to be 1, then my m would be 8a or if I assume m to be 1 then Omega will take some value 2 or something square root of 8 or something.

So this would be the expression which we have obtained is very much similar to the harmonic oscillator potential with somewhere different value of m and Omega in terms of a. So the energy eigenvalue for X prime would turn out to be X prime + half h cross and that for Y prime would turn out to be in Y prime + half h cross. And our Omega here is 1. If we assume Omega to be one then this is the expression we obtain, okay.

And the wave function corresponding wave function X prime Psi of n prime Y prime these are the corresponding wave function for this particle. Here you must note here that;

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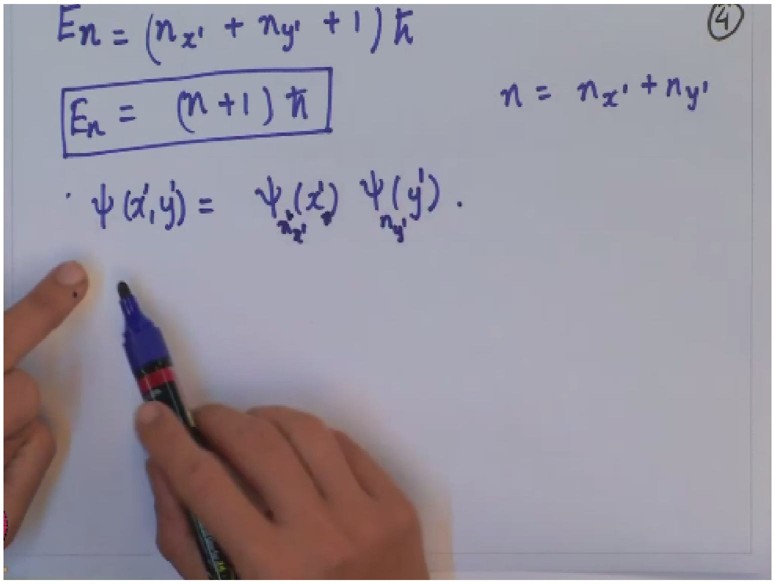

Your total E or total n is nX prime + nY prime + 1, okay. All En is, okay where I can write this as n+1 h cross where n is nothing but nX prime + nY prime. So here we have to find out for different values of nx and ny we can find or we may have rather a degeneracy. This is my En and the wave function Psi of x, y is; the total wave function is x, y and you can write the wave function as; so we have changed the coordinates in X prime Y prime, so this would be X prime Y prime with nX prime and nY prime, okay.

So the wave function is a function of X prime and Y prime, two coordinates because you have a two-dimensional wave function given to you and since the system is two-dimensional. And we can separate out the variables as the function of X prime only or Y prime only. And depending upon the value of nX prime and nY prime you will be able to determine whether you have a degenerate state or not.

So when you explicitly write the expression the value of n depending upon the states that can be determined. So for a system which is described by a Hamiltonian given by the Hamiltonian operator is given by p square upon 2m + the potential part which is a x square + y square + z square and we have to evaluate what will happen if we use the Heisenberg picture of operator evolution.

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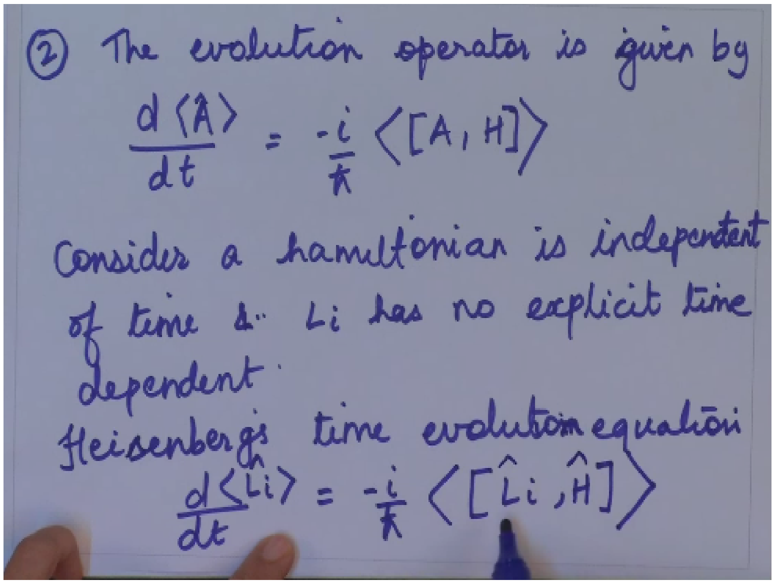

Evolution operator is given by; in general, we have seen this relation okay, when there is no explicit time dependence on the operator we simply write this relation which is also called as the Iron Fist theorem. And now we have to consider we are given a Hamiltonian which is such that; you we have seen the regular harmonic oscillator Hamiltonian particle in the Box Hamiltonian.

So a particular Hamiltonian is given to you, consider Hamiltonian given Hamiltonian is independent of time, okay. And with this assumption we proceed and that explicit dependence of time on Li is also not there that is to say that & Li has no explicit time dependence, okay. That is why we are not having the second term, there is no explicit time dependence on Li. And, so we now write the Heisenberg time evolution operator that is what I have written in general. Now I will write for the case of Li.

So Heisenberg time evolution operator or the equation is given by; for Li I can write it as that is one of the component of the momentum operator Li. So i can take value 1, 2 or 3; so i can take value X Y or Z. So this term you need to evaluate we need to evaluate the commutator of a Li with the Hamiltonian, okay.

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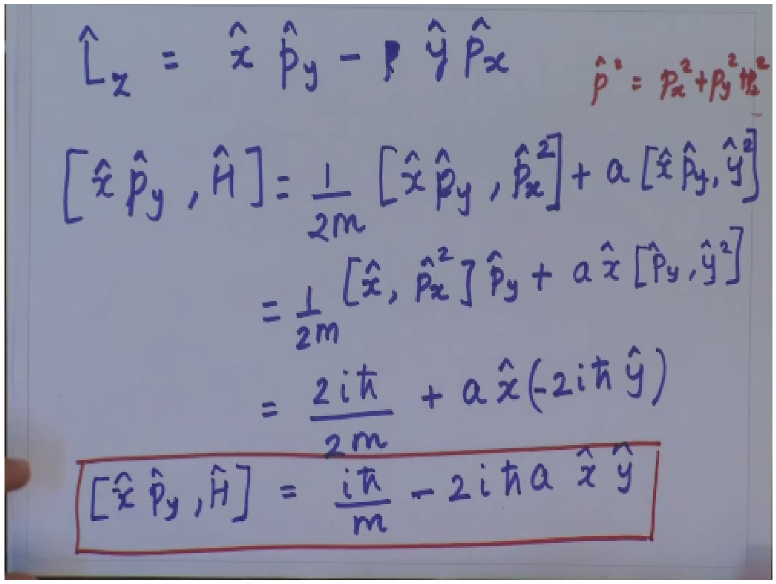

So in order to proceed say, let us consider Li to be the Z component. So for Lz we know the definition we can write it as p cap the x cap Py – y, Px, okay. We know this definition. So in short you have to find out commutator of x P cap with Hamiltonian and y with the Hamiltonian. So x Py will calculate first. Let us break the problem, okay. So x Py commuting with Hamiltonian; we have the form of Hamiltonian given to us as you will have 1/2m, okay.

We can take it outside and you have to recollect the commutation relations again which we have done before, okay so we will have Px, okay. Now we know that the Hamiltonian is p square/2m and P square I can write it as just recollect this part that P square can be written as Px square + Py square + Pz square, okay. So, we know that X will commute will not commute with Px. You will get a nonzero term. And X and Py; X and Pz will give me 0 terms.

So these terms would actually not contribute they will lead to 0, so I can drop these two terms at this stage, okay for the kinetic part; and for the potential term is a times again the same thing. P cap X cap Py cap, X Y Z. So we know that these terms would also similarly you have to see which one would contribute. So we have X hat square, Y hat square Z hat square. So only term that will contribute in this case will be Y hat square that is Y square term and this is a constant which I have taken out.

So now we use the commutation relation. So the first term would give me X cap Px times Py; and we know that commutator of Py Px is 0 so these, the second part of this would not show up, there is no contribution. And here we have a x cap; I have Py, y cap square. And again x and y would give me 0 commutation of x and y square they commute so that the second term will again not contribute. So what I am left with here is I h cross I will have I h cross, okay upon 2m.

So here I have to write Px square okay which I was missing so you have; okay be very careful there are so many commutators and you have to remember the relation and explicitly write the terms I am writing explicitly those terms which contribute and the terms which are 0 I have just dropped them. So a x, this would give me again; I will have a 2i h cross times y hat, okay. So on simplification I get i h cross upon m + I have, here I have to be very careful Y square Px will give me i h cross so I will have a - sign over here, okay.

So I will have a -2h cross upon. So here this is -2i h cross a x cap y cap. This is the result I get after simplification. This is x Py, H, okay. Similarly, by just inspection you can thing or by inspection you can guess the value of the commutator of u Px with the Hamiltonian.

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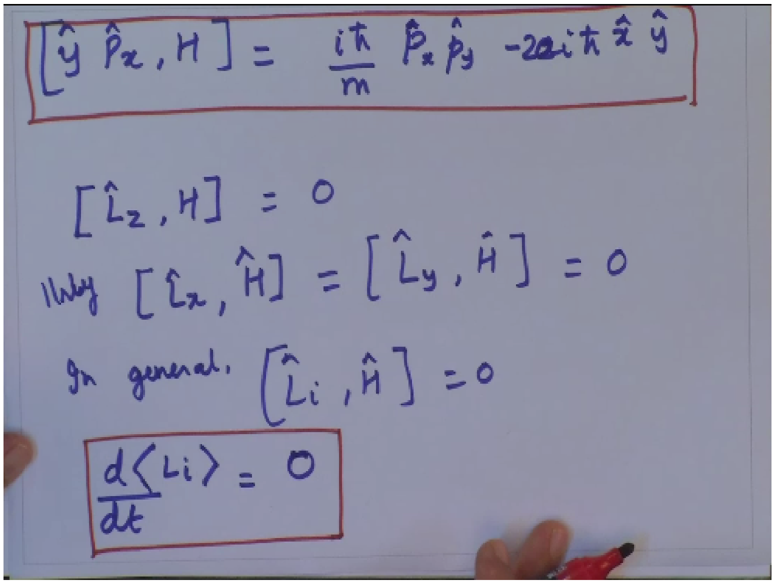

Y Px commutators with the Hamiltonian would give me I h cross upon m where I have Px Py, okay Px Py, okay. So commutator of Y Px with the Hamiltonians Py, okay we will get the similar contribution I h cross x cap y cap. So now with this you can find out Lz with the Hamiltonian operator you have to subtract this term with this; you can see they are equal and opposite. So this is the second term i h cross Px Py upon m-2ai h cross x and y.

So this gives me 0. Similarly, as an exercise you can evaluate Lx and H and Ly and the Hamiltonian operator, this would give us 0. So in general Li with the Hamiltonian operator is 0 where I can take value X Y & Z, okay. So from this we infer that Li when you differentiate with respect to T that is Li is independent of time or it is stationary with respect to time, okay. So this is the second problem we have done. The third problem I will just give simple hints.

It is a very simple problem; you have this exercise will be repeated for the hydrogen atom problem similar exercise. So this particular exercise I will give you hints and then you can work out. It is not a difficult task; you will also find this in many books.

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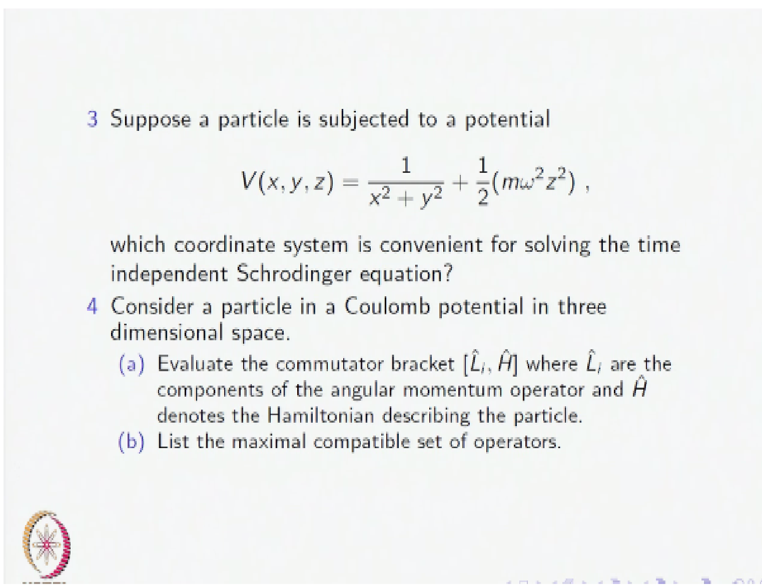

So 8-1 page, third problem. Particle is subjected to a potential which is given to you as 1/ x square + y square + 1/2 m Omega square z square okay, which is in the Cartesian coordinate system. So you have to write the potential in the convenient coordinate system and solve the Time Independent Schrodinger equation. So you can see from this expression of the potential that the convenient coordinate would be cylindrical coordinate.

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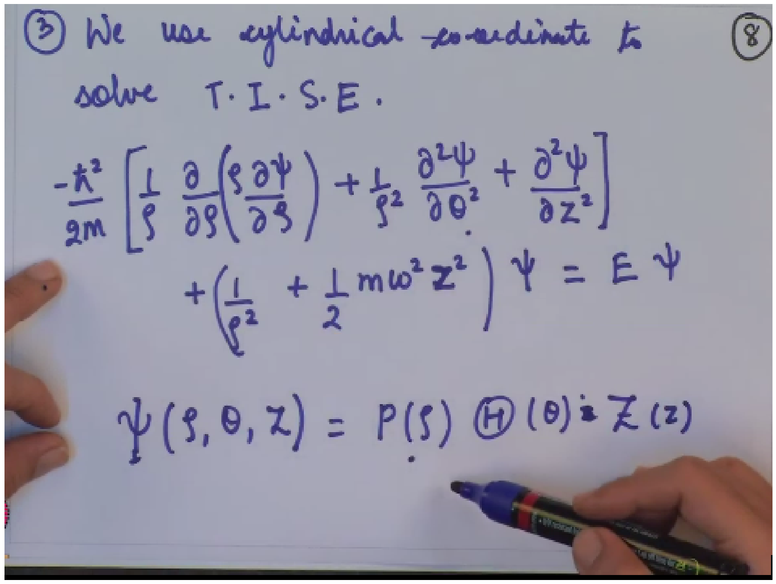

So we use to solve Time Independent Schrodinger equation, okay. So in polar coordinate we have to write the Time Independent Schrodinger equation P square h cross square upon 2m, so first let us write the kinetic part dou/dou Rho Rho this is Rho this is Theta and then we will have the z coordinate + the potential term; so this was the kinetic term. The potential term is 1/Rho square + 1/2 m Omega square x square or x square you have z square, right.

Psi of, so this will be Psi will be function of Rho theta and z, which is E Psi. So her so here you have the kinetic part in cylindrical coordinates then you have the potential part so your Rho as now X square + y square has now become Rho square and the z coordinate is as it is; so z is your length of the cylinder, so x and y is your Rho dimension. With this now what you will do is you will do a separation of variables and try to solve this problem.

So your wave function is a function of Rho theta and Z. So this you can rewrite it as P of Rho capital Theta of theta and Z of small z, okay. So this you will substitute in first equation which I have written for Time Independent Schrodinger equation and then by separation of variable you will do this in two steps, okay. So first it will be easier to separate out z and then you can see that Rho and theta are entangled; in the second path you can remove this entanglement.

So this hint I think is sufficient to rewrite this time Independent Schrodinger equation and then solve for the solve the Time Independent Schrodinger equation and obtain the corresponding equation for P theta and Z. So two more problems in this tutorial will be continued in the second part.