Quantum Mechanics
Prof. P. Ramadevi
Department of Physics
Indian Institute of Technology – Bombay

Lecture - 33
Schrodinger and Heisenberg Pictures - II

Recall classical mechanics, when you had equations of motion, I am sure you would have seen this.

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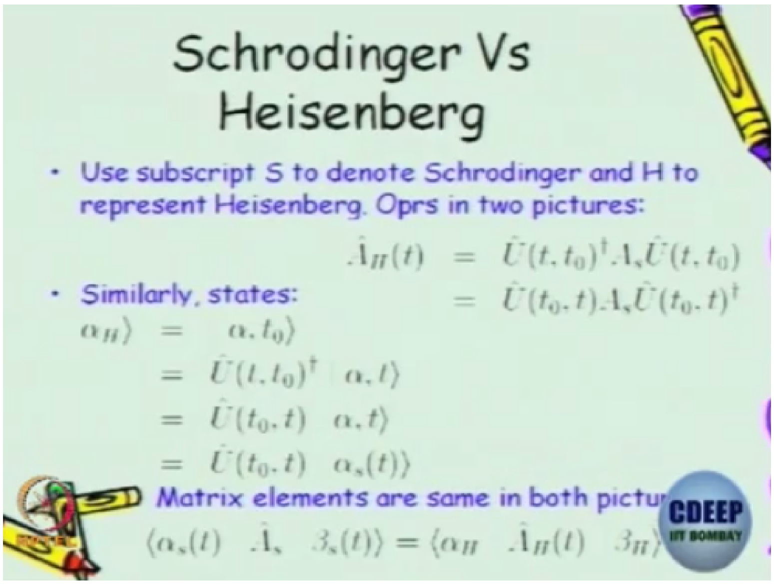

In classical mechanics,

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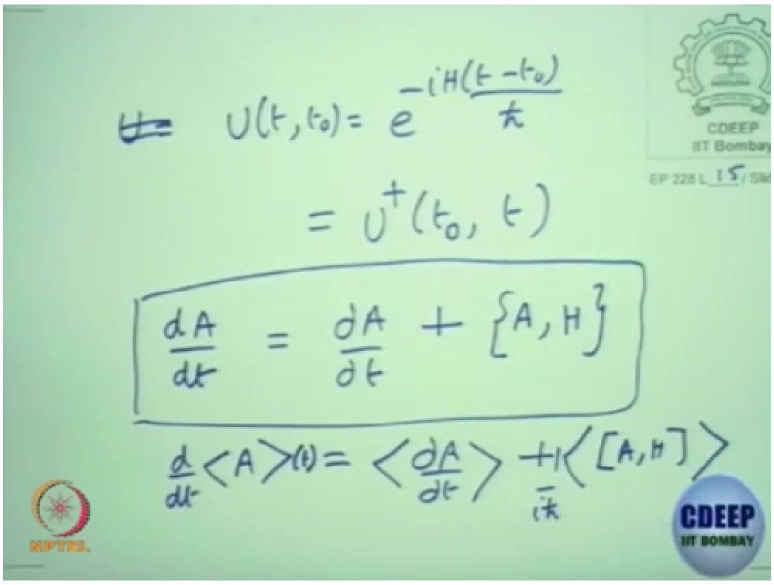

dA/dt is del A/del t if it has explicit time dependence, if you look at operators which do not have explicit time-dependent I can forget about this + horizon bracket of A with, you have done this in classical mechanics? What is the expectation in quantum mechanics? Ehrenfest theorem tells us d/dt of matrix elements or even expectation values should be expectation value of del A/del T + horizon bracket becomes commutator bracket with a 1/ih cross okay.

So this will become a commutator bracket matrix element that will be probably a 1/ih cross here. this is what is my kind of a guess, I am not proven now. We would like to prove so that we are consistent with Ehrenfest theorem that the familiar equations of motion which you see in classical mechanics should be reproducible for the matrix elements in quantum mechanics. So this matrix element as I said this A is the function of t.

How you get A to be a function of t, in Schrodinger picture is the states the function of time. In Heisenberg picture operators are functions of time. So when I do the d/dt I will either do d/dt on the state in Schrodinger picture or I will do d/dt on the operator in the Heisenberg and I need to derive this okay. So that is what I am going to do next. This is just to show that everything is tightly consistent.

Whatever you get in classical mechanics when you go to quantum mechanics this is your expectation are we consist.

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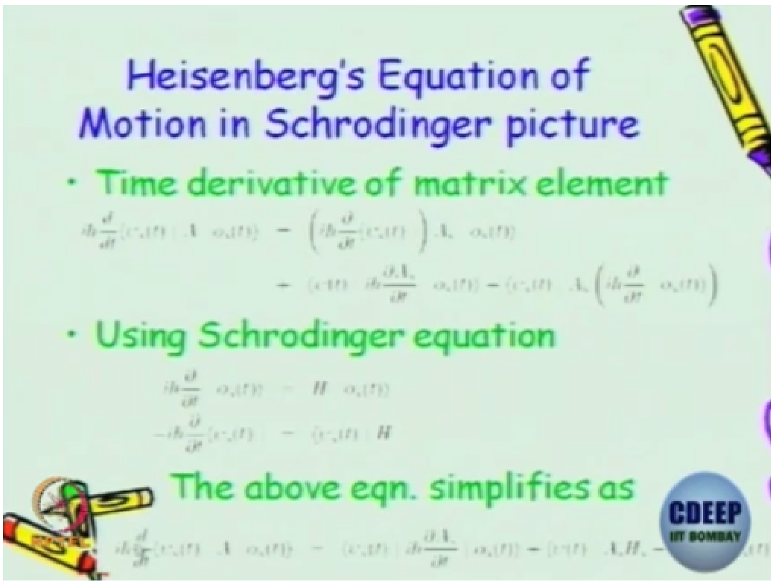

So I have stressed the fact that either you work in Schrodinger picture or Heisenberg picture both will give you the same physics. Now let us look at Heisenberg equations of motion in Schrodinger picture. So you take the time derivative of a matrix element d/dt of this. So in

Schrodinger picture as I said the states evolve in time. So we will try to put del/del t on the state.

A is not affected but Ax could get affected if it has an explicit time dependence. If you do not have an explicit time dependence this term you can put it to be 0. If you have t times XP + PX or something, then there is an explicit time dependence. So this term in general is possible if there is an explicit time dependence and then the last, the ket state okay, then what next, what will you do next? Use your postulate for the equation for the time evolution of the state. So del/del t of psi of s is H on the same ket right.

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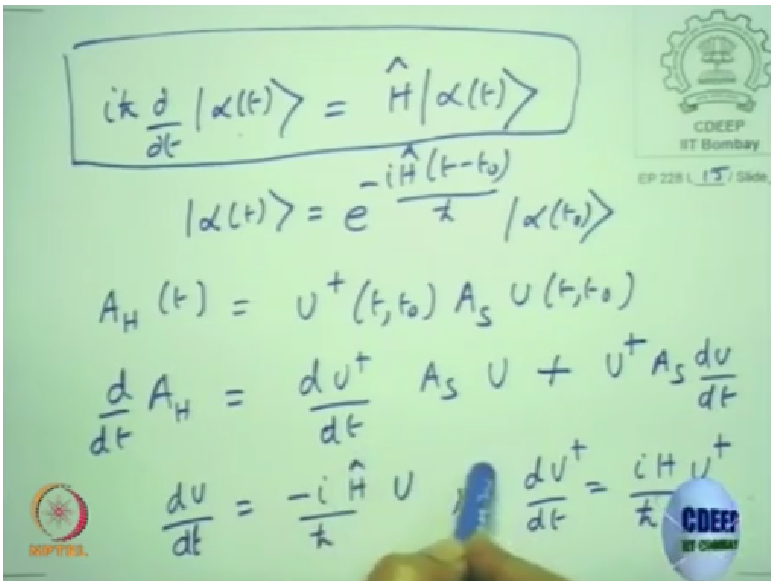

What is this right, this is from this place only we derived the unitary evolution? This is the Schrodinger equation or the state evolution of the state, time evolution of the state we have this equation which ultimately gives us for time independent potentials you could show that this is nothing but this we did right. So we need to substitute this, we need to substitute this in your simplified matrix element and see what we get.

So substitute for this Hamiltonian (()) (05:08) so this will also give you, so when you do it on the bra what happens, will there be a negative sign or no, ih cross del/del t on 5S if it is H on 5S. If you take the dual. So dual will give you the bra-ket, the bra, the dual vector, ih cross will become –ih cross del/del t is that okay and Hamiltonian H dagger is H. So you need to substitute this and this in the first term and the last term okay, substitute that.

What do you get, slowly it is coming right, you get a h, you get a –h and the order matters HA is not same as AH so you have to be careful and will we get a commutator? This is the minus sign because of this, simplify this equation all of you, what will you get after you simplify? So the time derivative of this matrix element of an A operator turns out to be the matrix element off if there is an explicit time dependence you have a partial derivative of the operator plus the first term would have given you a –HS AS, this term will give you ASHS.

What is this? This is nothing but the commutator bracket. So we have tried to prove what you have seen in classical mechanics in Schrodinger picture that it is the matrix elements which obeys exactly the same equation with the Poisson bracket replaced by the commutator bracket/ih cross this is what I was saying. This is your expectation and we have verified, but as a check we should also redo the same thing in the Heisenberg picture also.

We did this in the Schrodinger picture but we should also redo this in the Heisenberg picture to make sure that the same physics equation is got even in the Heisenberg picture.

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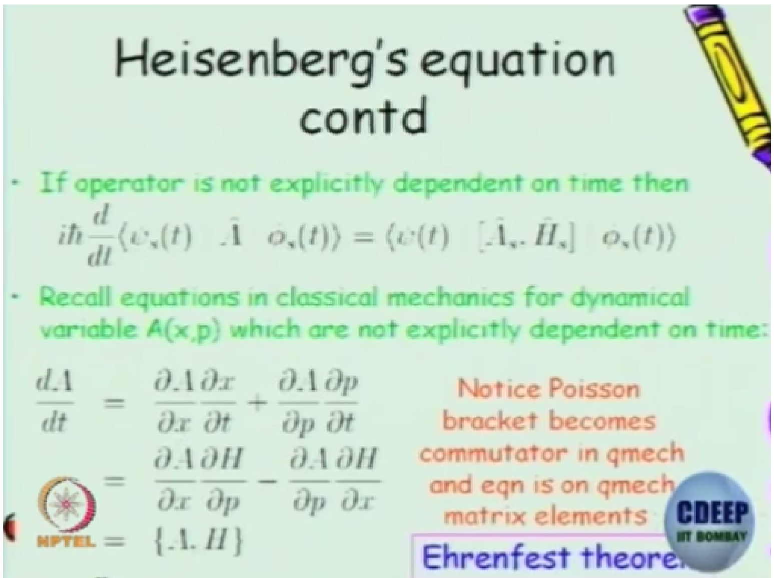

So let us check that the operators which we are going to consider do not have explicit dependence on time which means del of AH/del t is set to 0 okay. There is no explicit time dependence. The partial derivative of the same operator with respect to time we will set it to 0. You have to show we have already seen in the Heisenberg picture that the matrix element, time derivative is explicitly the commutator bracket, this we have seen in the last slide.

And this is what I said that if you try to recall how a dynamical variable which is not explicitly dependent on timing when you do it in classical mechanics you will write dA/dt as del A/del x * del x/del t + del a/del t * del p/del t. There is no time dependence, explicit time dependence here. All the observables or all the dynamical variables in classical mechanics is dependent on face pace coordinates.

Face pace is x and p, you could have a time also, but if you had that time you will have a partial derivative of A operator with respect to time, that I did not put because I am just taking A to be only a dependent on x and p. When I say it does not have explicit time dependence this is what I mean. It has an implicit time dependence coming from the time evolution of this face space.

There is no explicit time dependence and use this Hamilton's equation to rewrite things and then you see that it is nothing but a Poisson bracket. This is just to recall and we see the similar thing happening here for the matrix element which is what is called as the Ehrenfest theorem expectation. So notice Poisson bracket becomes commutator bracket in quantum mechanics and equation is on quantum mechanics matrix elements.

So this is nothing but your Ehrenfest theorem. So far we are still in Schrodinger picture. We have not gone into the Heisenberg picture. This equation which if you see the classical mechanics they call it a Heisenberg's equation of motion. So that is the, this equation is called Heisenberg's equation and this we have verified in the Schrodinger picture, the classical mechanics equation becomes in Schrodinger picture on the matrix.

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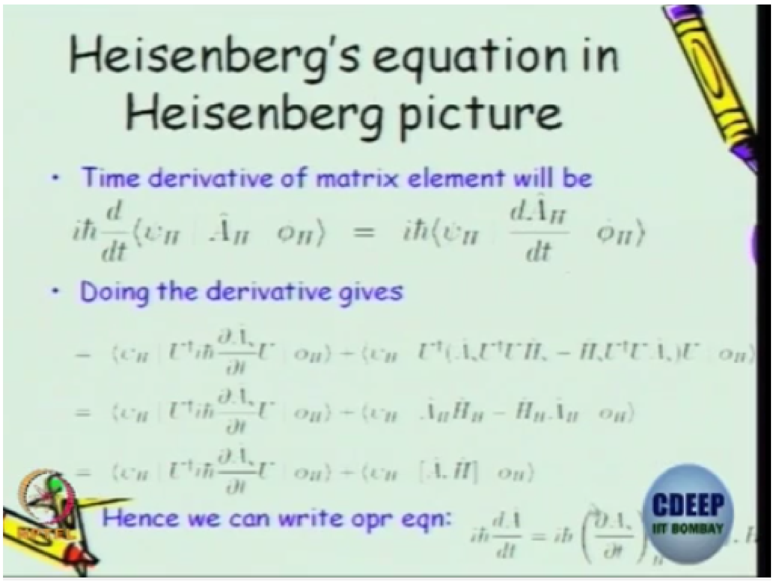

So now we will do the same equations in Heisenberg picture. In Heisenberg picture and we take d/dt on this matrix element interestingly the stage do not evolve in time. So what evolves in time is operator. So that is why this matrix element is explicitly dAH/dt. How do we do this. We know AH the explicit form, AH is nothing but U dagger As U right. You can first do, there is an explicit time dependence you will have a del AS/del t.

Otherwise you have to have D U dagger by Dt. So if you try to do that what will happen? There will be a HS coming up from there right, shall we do that. Let us do it together and then maybe then we can flash this light. So AH of t is U dagger t/t0 AS U of t, t0, d/dt of AH is dU dagger/dt AS U + U dagger. I am just suppressing all this t, t0 okay. U dagger AS dU/dt right.

What is dU/dt, -i/h cross Hamiltonian times U, d U dagger/dt will be +i/h cross U dagger. Here in this particular case U is the function of Hamiltonian. Whether I put this side or that side should not really matter. This is very crucial yes, quantum mechanics is very crucial you have to keep, but where I put this is very important. I cannot put this H; I have to substitute back here.

I have to keep that because you cannot cross the AS okay. So you cannot cross the AS. So can we simplify that substituting this here.

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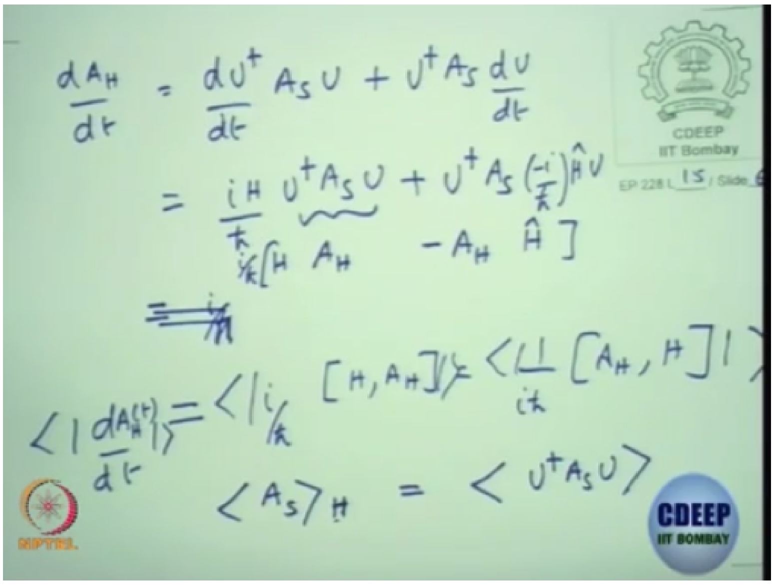

Is that right. Do a couple of things here, tell me what you can do, i/h cross you can write this as, what is this AH, similarly I can swap this H and U and write this as AH H with the minus sign and here you will have, so what happens, both are equivalent, the beauty in Heisenberg picture is since the operators are functions of time I do not need to too much worry about matrix elements here.

I can focus on because the states do not evolve in time. This equation will be exactly same as what you see with Poisson bracket in your classical mechanics in the Heisenberg picture, but you have to say that you are in the Heisenberg picture so that everybody knows that the state vectors do not evolve in that. Putting the matrix element, you can put it, but those are frozen states.

They do not evolve in time. Ehrenfest theorem is still not violated, but we have tried to rewrite in a different viewpoint which is the Heisenberg picture where the road operators as functions of time and the equations are exactly similar to what you see in classical mechanics as an operator equation with Poisson bracket replaced by the commutator bracket over ih cross, okay so do the derivatives that is what I did now for you.

You can go back and sit with this and do this and you can show that there will be if there is no explicit time (()) (16:12) then you can try to forget about this first term. If there is an explicit time dependence then this is an operator, this term is the operator in the Heisenberg picture right. What is the operator? del AS/del t operator with U dagger U is what I will call it as a Heisenberg picture operator.

So I ignored this term for simplicity saying that we look at observables which I have no explicit time dependents. Even if you have an explicit time dependence you can write and operate equation for the time derivative of an operator as the commutator bracket and you can have the Heisenberg picture operator where del AS/del t is the Schrodinger picture operator. Heisenberg picture operator means you will insert a U dagger in front and U on the.

So any operator which I give you suppose I write AS with subscript H, this should be read as, this is the meaning okay. The subscript H means I have to write the Heisenberg operator and I can write it like this. So wherever you get this you should replace it. So that is the theme of these 2 pictures and we will have some problems on it on these 2 pictures how to relate one to the other.

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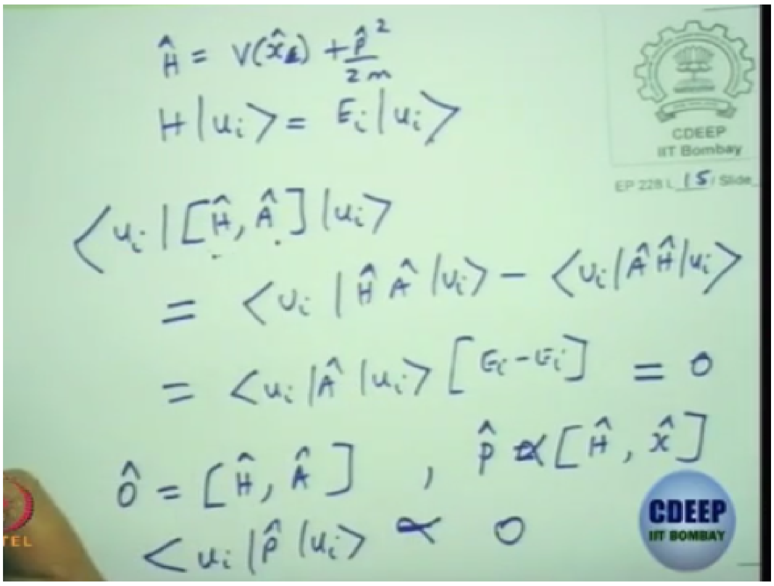

So the thing is if you want to look at stationary states, Hamiltonian on let us say stationary states which are Ei or Ui, you can always show that any linear operator, the matrix element what will this be? it is Ui Hamiltonian operator Ui – Ui A operator Hamiltonian on Ui. This will be Ei-Ei right. Why start that Hamiltonian. So this is going to be 0 right. So the commutator bracket of this Hamiltonian with A on the stationary states of the Hamiltonian they are always 0.

So many times these things will help us to try and look at time derivative of the operator very nicely some of these things. The other thing which I was trying to say is. If you can write some other operator as commutator of H with A, okay, so that was the question in their assignment show that your momentum operator. What was the Hamiltonian given somebody? V of x momentum operator can be written as up to some factors, let me not worry about it, it is proportional.

Once I write some other observable as the commutator of the Hamiltonian with something that helps me to (()) (20:33) say what is this 0 by this argument. Take a harmonic oscillator for example. Harmonic oscillator if I asked you to find what is the average momentum what will you do? expectation value of momentum, what will you do? In the wave function formalism, what do you?

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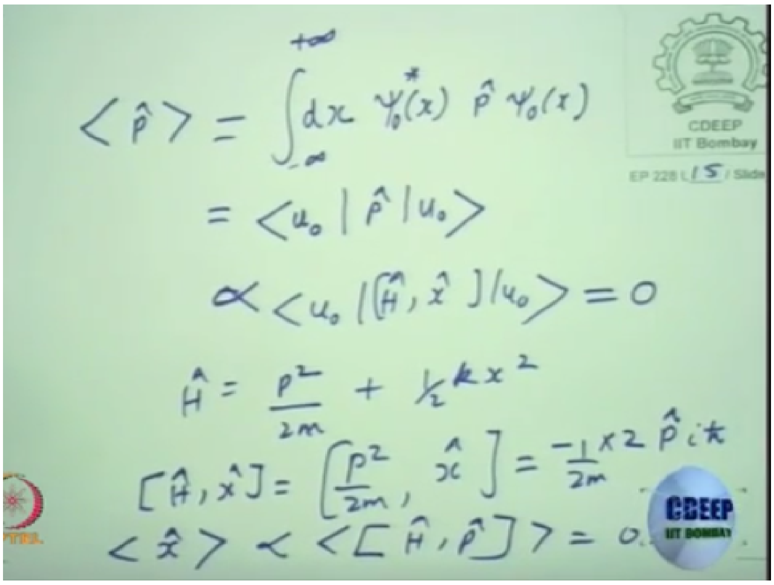

Let us say the particle is in the ground state, what would you have done? Dx psi 0 of x p operator psi 0 of x, what you would have done. Equivalently this can be rewritten as the Z0 U0 p operator U0. This explicitly if you do using this Hermite functions you can show that this expectation value is 0 right, but now instead of doing it this way I have tried to say that this is same as U0 Hamiltonian for the harmonic oscillator is up to some factors.

I am not worried about those factors. P squared/2m + 1/2 half kx squared. Let put m to be 1 and k to be 1 then you can always write Hamiltonian with X, let us do Hamiltonian with xS, which one will contribute amongst these 2 terms, p squared/2m alone will contribute and what will this p? 1/2m * 2 * p operator. So essentially I have shown that 2ih cross yes, thank you yeah, excellent.

So essentially what I am trying to say is instead of doing this long winded calculation I can exploit whether this observables can be written as the commutator bracket of Hamiltonian with something else which can be any linear operator. This we have shown it to be 0, if I can write my momentum as the commutator it is 0. Same argument instead of working with this you could have worked with, if I want to find what is the expectation value of x operator you can show this to be proportional to someone, this is also 0.

I am just trying to say the power of using just Dirac notation, the commutator bracket are you know it is completely invisible, but lot of things are happening. Okay, so let me stop here.