Quantum Mechanics

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Lecture - 32
Schrodinger and Heisenberg Pictures - I

Today the theme is to look at just like in classical mechanics you have active viewpoint and passive viewpoint right. What is active viewpoint and passively point? Coordinate axis is kept fixed and you look at operations on the vectors or you try to look at the operations of the coordinate axis so those are the active and passive. I sure you have all seen this right. So similarly even in quantum mechanics you can have 2 pictures.

One is called as the Schrodinger picture, the other one is called as the Heisenberg picture okay. So we will try to, the physics is the same whether you work in Schrodinger picture or Heisenberg picture, but you should not do half of it in Heisenberg, half of it then you will not get, the same thing with the active and passive viewpoints right. So we will get to this, before I get on to this I thought I should complete what we said in the last lecture about incompatible observables okay.

Not observers so incompatible observables are 2 linear operators, permission operators whose commutator bracket will be 0 or nonzero? nonzero right, so this is what we saw that in that specific case you wanted to show that the standard deviation of operator A multiplied by standard deviation of operator B.

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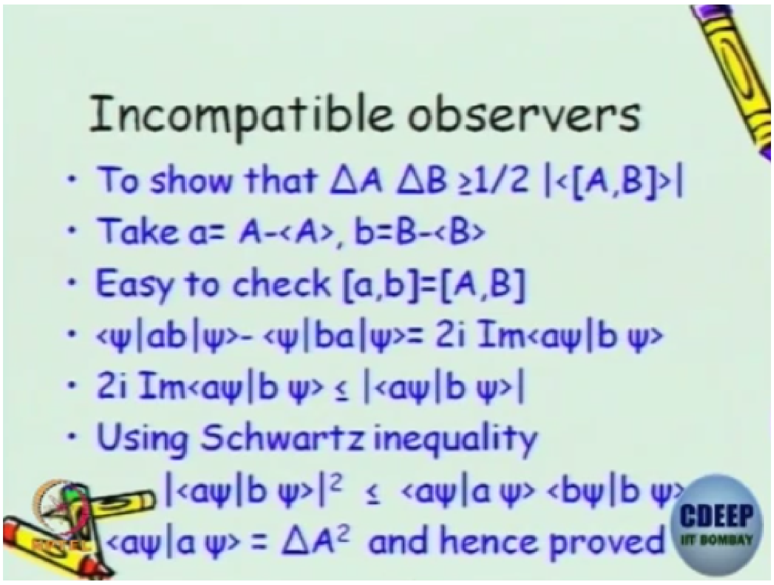

This should be greater than or equal to expectation value of the commutator bracket. If they are compatible, then it will be 0. So you can simultaneously measure the 2 compatible observables. There is no disturbance, there is no uncertainty. Only when there are incompatible then you have this problem that the 2 standard deviations or 2 uncertainties the product is having a bound which is determined by the commutator bracket.

And this is the question you need to prove this okay. So let me just briefly recap on the slide, the solution for others who have not tried it, but you can go back and rework things I am not putting all the steps but essential steps are put in there okay. So first you try to write a small little a to be the operator A - the expectation value of the A operator. So this is basically taking you deviation from the mean value.

Similarly, little b in the same way, what happens to the commutator bracket of a with b, a with b commutator bracket what will happen to it? It will be same as little a with little b why? expectation value is some kind of a number, so it does not really play a role okay. So expectation value is a number, so you can try to check that little a little b commutator is same as the observable operator A commutator with B.

Then you can also use this fact that if you take the matrix element of the little a b operator between some arbitrary say psi, you interchange that, that is nothing but the commutator bracket evaluated as an expectation value right and the step you can verify okay, you can try to write a with psi as some phi, b with psi as some kai and then this will be the star of it. So if you take an inner product of 2 states – the star of the inner product of those 2 states, the difference will always be proportional to the imaginary value okay.

Will you try this, just check it out, take b with psi as some new state, a with psi as some other state and you can try and show that this use the inner product property. What is the inner product property?

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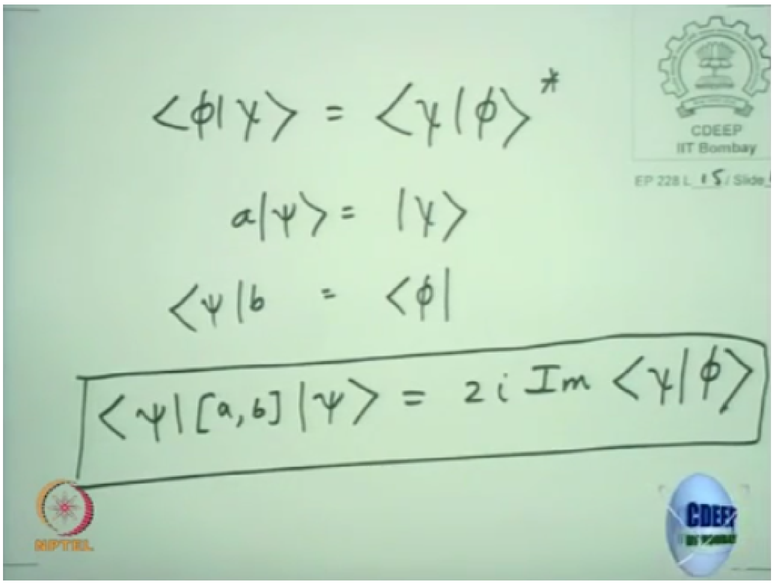

They are Hermitian operators; they are observables okay. So use these 2 properties and you can show that psi with commutator a, b is twice i imaginary part of okay. I will leave it to you to check this okay. Now you can add further imaginary part is always less than magnitude or the so you can take that modulus and then use your Schwartz inequality. What is Schwartz inequality, the dot product if you take then you can show that that modulus squared is less than the norms of the individual states is that right.

So now you can plug back this into this and this a psi with a psi is nothing but delta A squared, this also you can check. So once I put this in so this is delta A squared, this is delta B squared and you go back here to the step and you can show that and the commutator of a, b is commutator of capital A, capital B, hence you can prove delta A delta B okay. So I have just given you the (()) (07:14) steps.

You go back and fill it the gaps and you will be able to prove this okay. So I do not think there is a mistake in this half factor. This overall 2 factor because this imaginary part is what you are going to say and then you have to put in the 2i to get quick. Any questions? So the theme of this equation is to show that if you have incompatible observables there will be uncertainty in precisely measuring both the observables simultaneous.

Because the commutator bracket is nonzero. So that is why the commutator bracket of x with p is not equal to 0, it is nonzero that is why you have uncertainty. If you try to measure precisely the position of the particle you lose information about the momentum or equivalently the wavelength of the wave and vice versa. So any 2 operators whose commutator is not zero then there will be uncertainty in the measurement.

You cannot simultaneously measure both of them precise. So that is the verbal way of saying the mathematical equation which I have written. So this 2i should be removed here, this imaginary part is less than or equal to this and then you substitute it here and then you will start.

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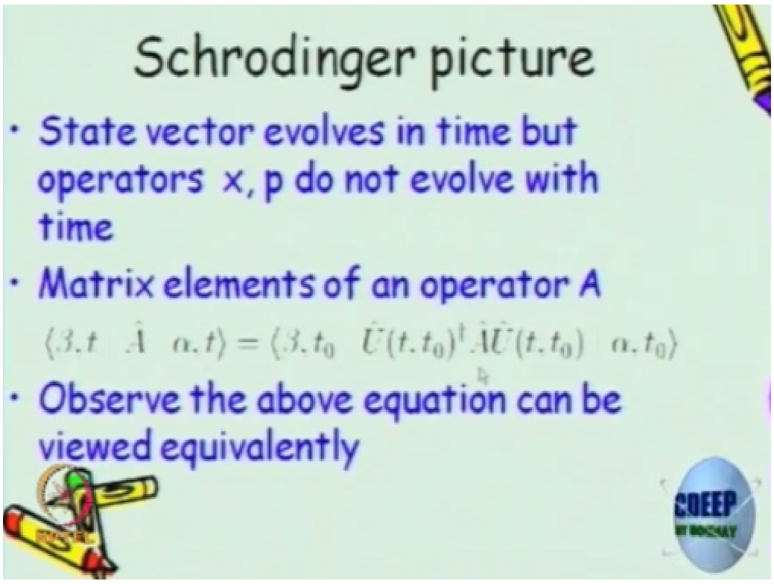

So I was just telling you that there are 2 viewpoints but the physics has to remain the same, one of the convenient viewpoint which we follow in quantum mechanics is the Schrodinger picture okay. So what is Schrodinger picture, this I have already said in the beginning, the state vector evolves in time okay. So the way we take, so the way we try to write if you remember.

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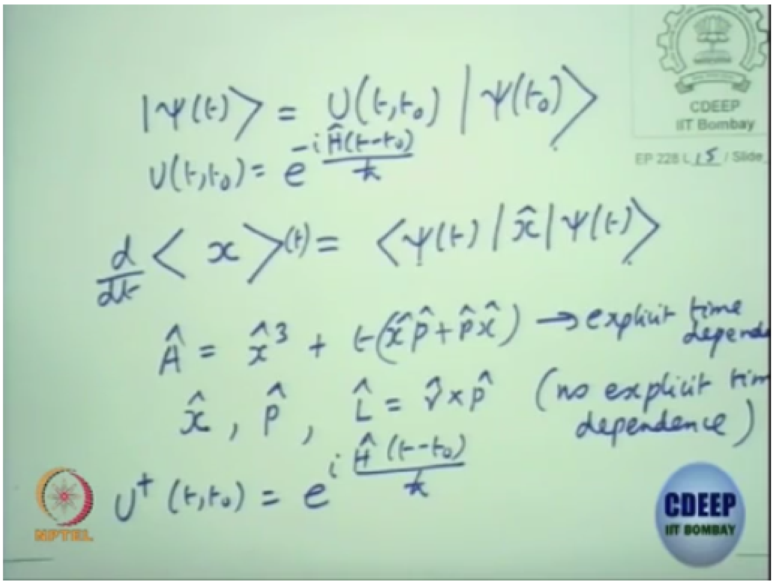

Psi if t we wrote a unitary operator t, t0 on psi of t0. So we took the state vectors to evolve in time, but when we want to find expectation value of x for a system prepared in the state psi, we try to write this as psi of t, x psi of t. So the x operator does not have any explicit time dependence like you could have some operator like a operator as x cube + t times xp squared you know could have some operators like this.

So summation, so this has a explicit time dependence. We are not looking at such operators, when I say x, x does not have explicit time dependence, p, angular momentum L which is r cross p, no explicit time dependence, okay but because the state vectors are evolving in time you could have, could do time derivative of the expectation value of x which in principle could be a function of time right.

So those are the kinds of things which we do in Schrodinger picture where we look at these operators which are independent of. There is no time dependency there, the time dependence is put on the state. So the time dependence shows up on the state vector, but the operator is independent of time okay and they are looking at simple operators which do not have any explicit time dependence or this kind of operators.

This is also an allowed operator, but this has an explicit time dependence, we look at operators of this type which do not have an explicit time dependence. State vectors evolve in time but operators x and p do not evolve with time, as operators they do not, as matrix elements of those operators, that matrix element itself could be having an evolution in time due to the statement.

So state such operators, which corresponds to observables, if there is no exquisite time dependence. We could take this could be a position operator or there could be a momentum operator or angular momentum operator state vectors you could have market elements between 2 different state vectors at the same time, equivalently you can use the Schrodinger evolution equation which is the unitary operator acting on the initial state at initial time t0.

Similarly, on the bra vector it is going to be the U dagger. So this matrix element on the left hand side explicitly can be written as the matrix element between the initial states, take the initial time to be t0 and with this unitary operator on both sides sandwiching the A operator. What is this unitary operator? You have done this. This is e to the –ih t-t0/h cross. So U dagger will be the inverse of it okay.

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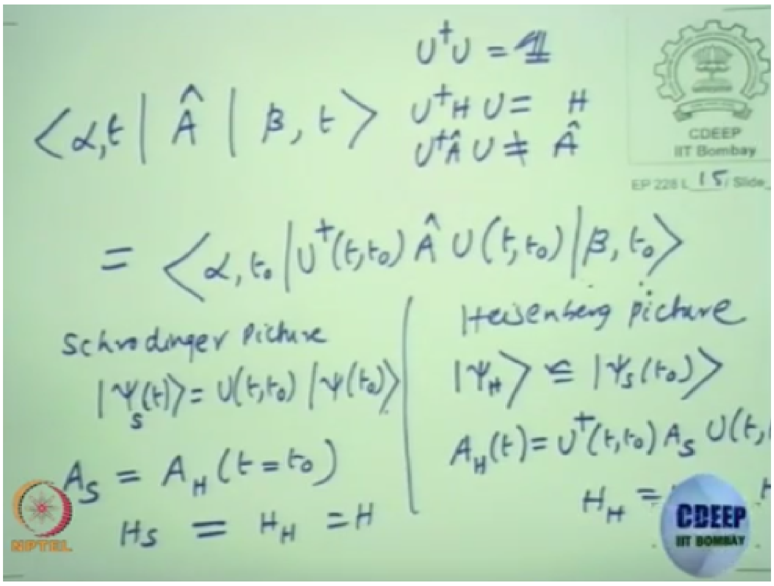

So the matrix element alpha of t, you can write alpha, t or alpha of t both are equivalent and then some observable which has no explicit time dependence, beta of t. You can write this as alpha of t0, U dagger, t, t0, a operator U of t, t0 beta of t0. This is kind of ringing some bell, something it conveys this equation. What is it conveying? You can try to freeze the state, change the operator and make sure that the matrix element whether I work with the state evolved or the operator evolved with the state fixed is the same okay.

So this is what it tells, either you evolve the state and evaluate the matrix element or you fix the state and modify, put this to be time dependent. This is what I was trying to say that there are 2 viewpoints. One was the Schrodinger picture, which was this left hand side, which you have done many times, but can be tried to freeze the states to be an initial state and put whatever is the modification on the operator.

So that is what is being done. So this kind of gives us an indication that you can equivalently view the above equation in a different picture.

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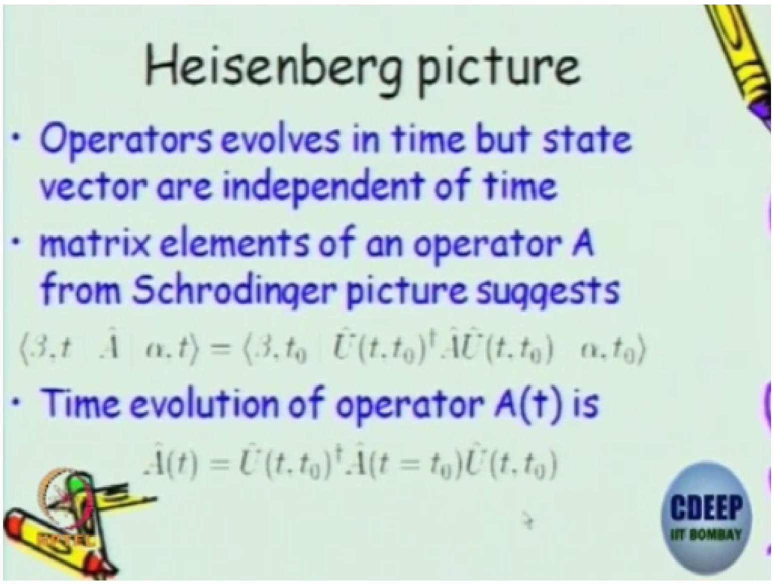

That different picture is called as the Heisenberg picture okay. What is this picture now I have already spelt it out, it is the converse, make the state vector to be frozen to some specific time t0, do not allow it to evolve in time, but make all your linear operators corresponding to observables which are Hermitian operators to evolve in time, but what is the form it is not arbitrary, it is dictated by the matrix element which we saw in the earlier transparency?

So what is that form? The form is suggested by looking at this equation that there is a time evolution of operator A of t which should be the A operator at the specific time t = t0 and you do the time evolution in this way okay. So this is very important that in the Heisenberg picture we take the states to be frozen at t0 and we take all your operators at t0 and then multiply U dagger, pre multiply U dagger and post multiply U to write the time dependent operator.

To be very precise I should put a subscript H here okay. So let me write it here. So Schrodinger picture, Heisenberg picture. So you will have psi of t okay and if you want you can put an s here, just to remember that it is a Schrodinger state, equivalently I could write here psi H without putting any time dependence, it is nothing but psi s at t0. You freeze the state in the Heisenberg picture that it is fixed at t0.

So any operator here which I write AS, AH of t will be U dagger t, t0 AS U t, t0. This AS can also be treated to be equal to operator is frozen here, there is no time evolution of the operator here, but there is a state evolution of the states here in the higher Schrodinger picture equivalently in the Heisenberg picture states do not evolve in time but operators evolve in time, what happens to the Hamiltonian.

HS here and HH here, are they different or they same, why? This involves exponential of the Hamiltonian operator, they will commute, U dagger U is the identity right. U dagger U is identity; U dagger H U is also but U dagger some other operator A this is not equal to A in general. So because of this property U dagger HU = you can show that this is same as, you can write formally as H.

Hamiltonian is same in both the pictures, but other operators in Schrodinger picture do not evolve in time whereas they evolve in time dictated by the time evolution operator and this is done in such a way that the matrix element evaluated in the Schrodinger picture is going to be same as matrix element evaluated in the Heisenberg picture.

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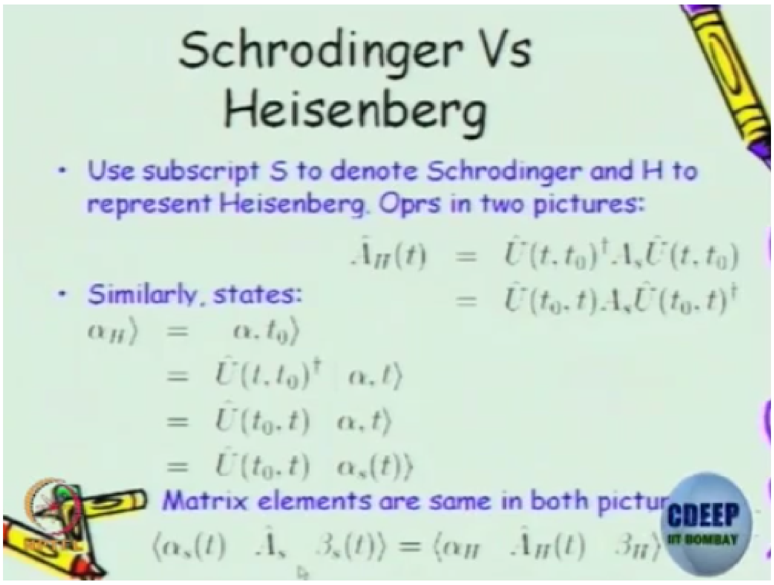

So this is just to summarize here. Use subscript S to denote Schrodinger, H to denote Heisenberg, so the operator in the Schrodinger picture is A subscript S and you can write the Heisenberg operator which is time dependent as the unitary evolution. The dagger operation from t to t0 or t0 to t can be equivalently written as without the dagger, but interchanging the initial and the final time right.

Instead of going from t0 to t, you can go from t to t0 which is an inverse step. These are things which you should play around with the unitary operators.

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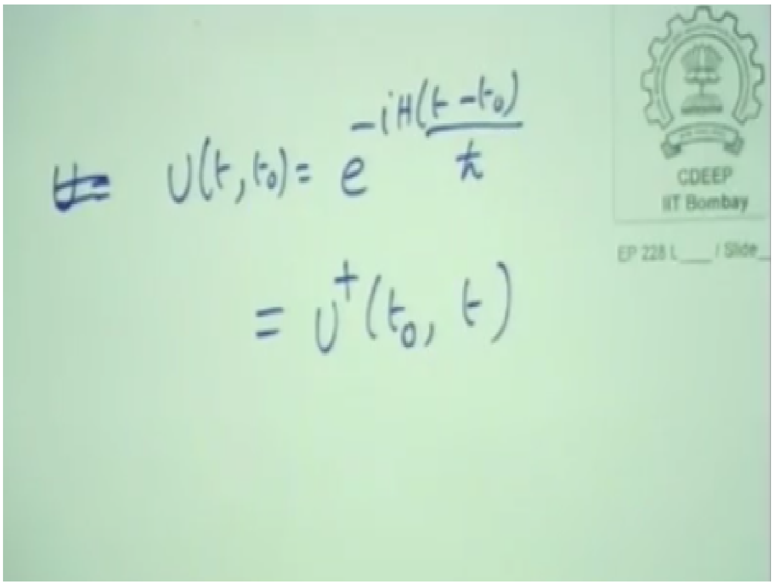

You can write U of t, t0, you can also write this as U dagger of t0, t right, both are same. The once which I have written here is for the operators similarly the states you can write the Heisenberg state to be frozen at T0 which is the Schrodinger state, equivalently write this in terms of alpha S of t and you can use this unitary operator from t0 to t to relate any state at time t in the Schrodinger picture to the Heisenberg frozen state.

So play around with these unitary operators U dagger t, t0 is nothing but u t0, t. As I said alpha of t the state evolution is what I call it as the Schrodinger picture states and you can relate the Heisenberg picture states which is independent of time evolution. This is U dagger, there will be a U, U U dagger is identity, so you will get alpha H to be alpha of t0. These are all various ways in which you can write the same state, is this clear.

So this is what is always insisted whether you work in viewpoints which are passive or active, the physics cannot change so the matrix elements are the ones which contains the physics. So the matrix elements in both the pictures should remain the same. So either you work with Heisenberg picture, where we evolve the operators as functions of time but you do not evolve the states and do the calculations equivalently.

You could evolve the states and keep the operators independent of time and do the matrix elements there, both will be the same. Sometimes it becomes convenient to choose which picture will be much more convenient okay. So just like if when we did this earlier basis, I was trying to say that choose the basis where it becomes the eigen value basis for some operators.

Similarly, here sometimes the convenience will be working Heisenberg picture will be convenient, sometimes Schrodinger picture will be convenient and you can decide depending on the given quantum mechanics problem okay, but both are equivalent, you can work either in Schrodinger picture or you can work in Heisenberg picture.

So both are equivalent in the sense that if you try to matrix element of a operator in the Schrodinger picture which is time independent where the states are time evolving this is same as freezing the states and working the time evolution of the operators which we call it as the Heisenberg operator.