Quantum Mechanics

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Lecture - 31
Tutorial - 5 (Part III)

We are now on to tutorial 5 and we will continue with few more problems based on commutative bracket and it will be interesting to do some more problems based on this. So in this set we have completed 3 problems. Let us go to the fourth problem.

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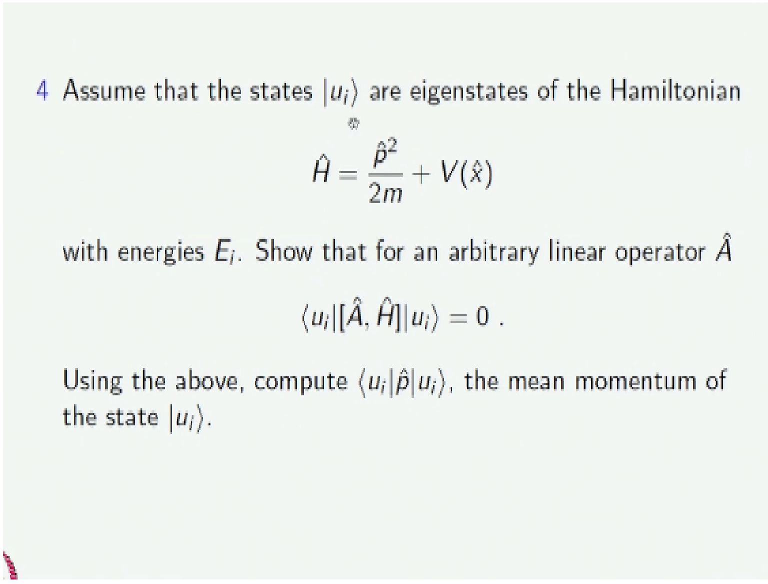

So in the fourth problem you have to assume that the states ui are eigen states of Hamiltonian. Hamiltonian is given to you. Hamiltonian operator is P square upon 2 m + Vx this is our regular way of denoting the Hamiltonian as the kinetic part plus the potential part. With Ei as the energies and you have to show that the arbitrary linear operator A which is when you take a commutator of A with the Hamiltonian and when you take the average you will obtain it to be 0.

So this relation you will show for an arbitrary operator A and then using this relation we have to compute what would be the mean momentum of this state ui. So let us start doing this fourth problem.

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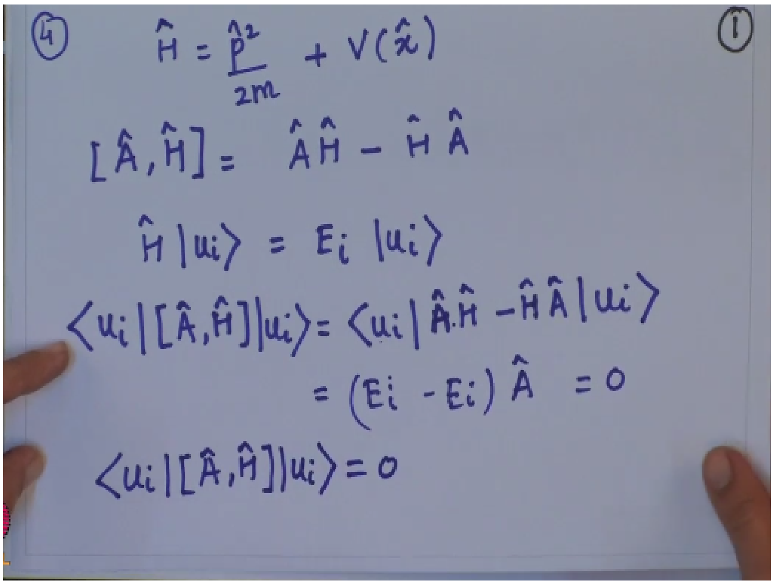

You are given a Hamiltonian operator as P square upon 2m. P is an operator, H Hamiltonian operator, so P square upon 2m + V of x. V is dependent on the position space and you have to obtain a commutator of an arbitrary operator A and H okay. So any arbitrary operator on H you have to obtain this. So let us get started with this. This would be nothing but AH-HA. Now you are also given that H is an operator when operated on the eigen state will give you a energy eigen value Ei ui, this is known.

Now let us compute ui commutator of A H ui, this would be ui, this commutator I have written here will be operator A, operator H - H A ui, now when H operates on ui it will give me an Ei and when H operates on this bra it will again give me an Ei. So you can see here you have a Ei – Ei term. So this one will operate on the left and give you Ei and this one will operate on the right and give you Ei and an operator A which is 0 right.

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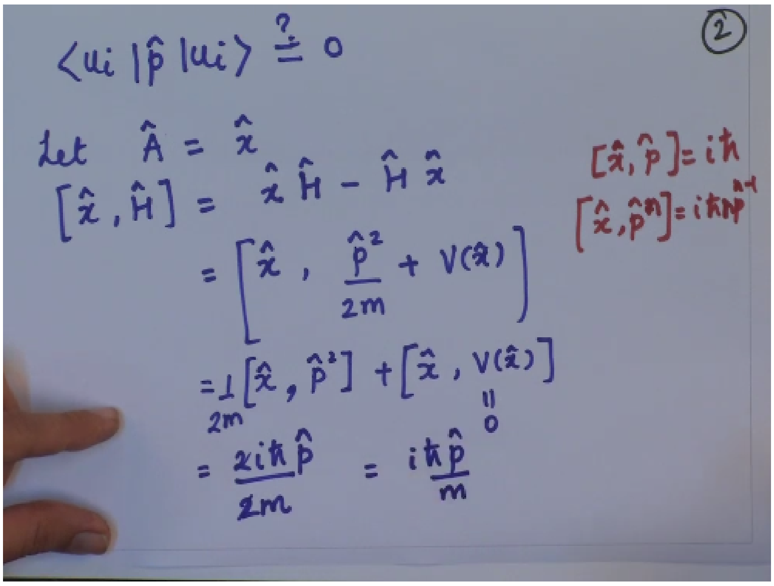

The next part what we will do is we have to calculate a quantity ui p on ui that is what okay. Now this quantity you will obtain from doing some algebra, you have to use the previous result to obtain this result. So let us assume, let us start, let operator A be some x let us try this okay. So what will be x on H, x on H would be x and H, commutator of x and H would be – Hx agreed.

Now what we will do is, we will try to get this operator in terms of, so let me write it here as H operator H we saw here was P square upon 2m + V of x okay. This is what it mean. Now you need not do this step okay directly and remember you know the commutator of x and p. Operator x and p is nothing but ih cross and x in the previous question, in question 2 you have seen what is x on p n.

It is ih cross np raise to n-1 okay. So this relation we have seen right. So here you have 1/2m I take outside I have this + x of V of x, commutator of x and V. Commutator of x and p will give me ih cross 2p okay. So 2ih cross guess upon 2m okay and this commutator of x and V assuming that V is linear we obtain this is to be 0. So this is ih cross p cap upon m. Let us continue with this, what we have obtained here.

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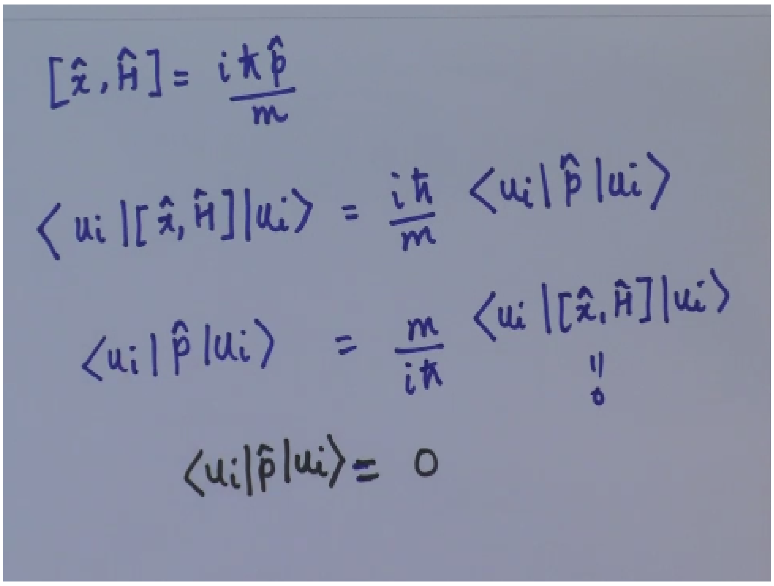

Let me rewrite the last step which we obtained ih cross p cap, p hat upon m, this is what we obtain okay. Now we have already calculated in the previous example ui x H let me take this on ui, this commutator between the ui in bra and ket okay. This will be ih cross upon m okay, being a constant I can keep this out ui p hat ui. This what I obtain by substituting for the commutator x and H and this left hand side we know so let me swap this.

So ui is p ui so this is the mean momentum of the state ui will be = m upon ih cross times ui x commutator of xn h. This we know is 0. So what we obtain in this result is the mean value of the momentum operator is 0 okay. It is very easy to show this. So with this example let us go to the fifth problem.

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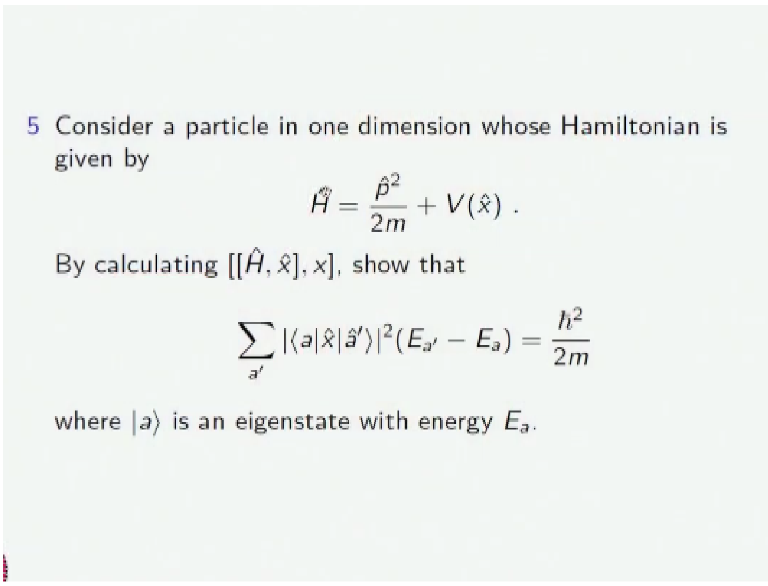

Fifth problem is you consider a particle in one dimension whose Hamiltonian is given by, in the previous example we had the Hamiltonian H is p square upon 2m + Vx, taking the same Hamiltonian we have to calculate commutator of Hx, commutator of H and x again commuted with x. So that we have to calculate and using that we have to show certain results. So let us get started with that.

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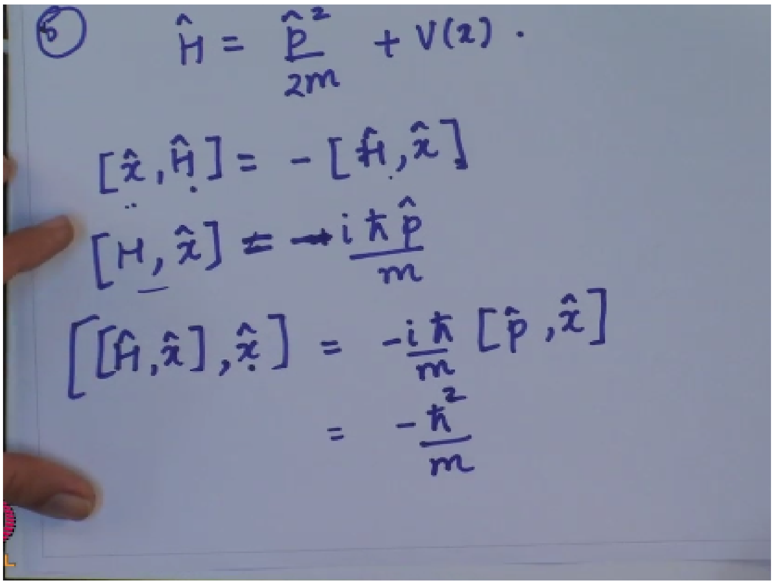

So in this operator, Hamiltonian operator is given to you as p square upon 2m + V of x, this is the same Hamiltonian which we had in the previous example. We have already calculated what is the commutator of x and H that will be nothing but commutative of H and x. So if I interchange this commutator operators then this sign would change okay. So I had in the previous example I had commutator of x and H was ih cross p/m.

So now it will become –ih cross p/m alright next is this is what we had x, H right. In the previous example we had okay sorry so we need to rewrite this as this will be +. So let me write this H, x would be nothing but –ih cross. So commutator of x and H is - of commutator of H and x. So this would give me this value right. Now what we do is we take the commutator of X and then H and x and then commute it with the operator x again.

So this would be nothing but ih cross upon m I can take the constant out and p cap x. Again we have commutator of x and p as ih cross. However, commutator of p and x would be –ih cross. So this would give me –h cross square upon m correct. So I have got this by operating by using the commutator result from the previous problem and the commutative result when you commute with the operator x then what we obtain is –h cross upon n.

Now you have to go a step further in which what I will do now is.

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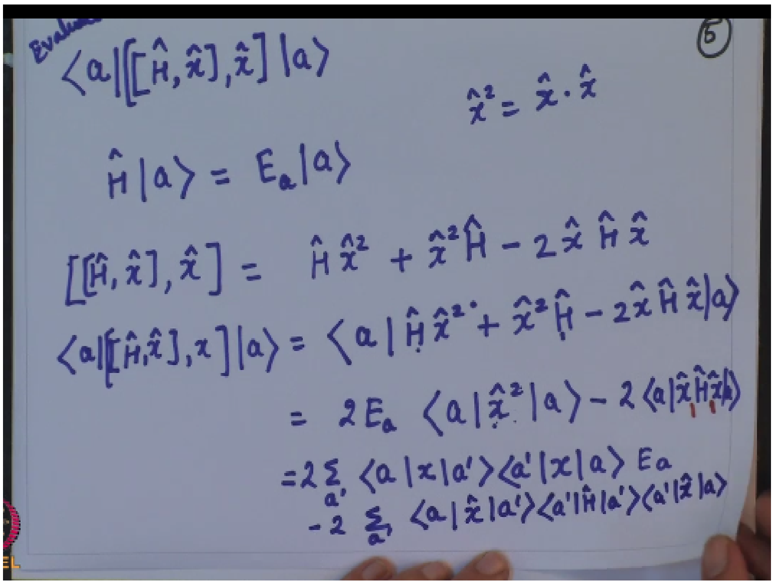

I will take a what we will evaluate here now is the eigen state ai or a I will place this commutator which I evaluated now between these eigen kets. So this we have to evaluate okay. So before evaluating this quantity let us do some simplification. So we know here that when the Hamiltonian operator is operated on the eigen ket we get the energy eigen value Ea and the ket, this is one thing.

Another thing is that when I write this commutator, now I am not solving I am writing in terms of the operators only without simplification. I am not using the commutator relation because I want to operate these operators on the ket, eigen ket. So let us take this H and x commutator commuted with the position vector or the position operator x will be H x square. When you simplify this commutator bracket you obtain this is what you obtain okay.

It is one more step which I have skipped. So you can simplify this in this manner. Now what do you have is you will take this operator and sandwich between the eigen states A, this quantity will then be, when I place this entire thing on the eigen ket a what do I obtain is the first term would give me a, I am placing this operator on between the eigen vectors or the eigen kets okay.

Now you can see here that when a the operator, Hamiltonian operator operated on the eigen ket will give me energy eigen value Ea. Similarly, this operate on this bra and will give me Ea. So I will have 2Ea correct, one contribution coming from this, another contribution coming from the second term will be a, this is the contribution coming from the first two terms.

Let me take the second term, second term has 2a xHxa. So this quantity we have to evaluate. Now what we do is we between, so this x operator square can be written as x times x. So between this x operators 2 operators I can insert a complete set of eigen ket such that I have a between x and x. So I have one x a prime, a prime x a. So this will be the first term which will give me this first expression.

And I have a Ea also, minus the second term again I insert a complete set of orthogonal ket such that I will have a. Now I will insert 2 of them. So here I will insert one and second one I will insert at this place, so what I obtain here is x cap a prime then I have a prime H hat a prime, a prime x a, this is what I have. Let us now simplify the previous step so what was the previous step.

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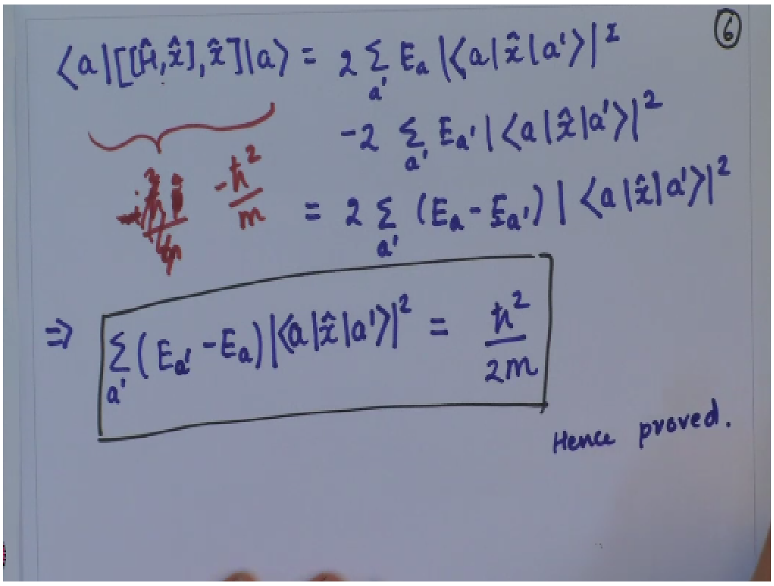

Let us simplify the previous step commutator of H and x commute with the operator x. This on simplification will result in 2Ea I can write this expression as a square, this is you can write this entire term as a square okay, it is like calculating them all, a x cap a prime square. This was the first term; second term is 2 summation a prime Ea prime again this would give me Ea prime this term okay.

So I am writing for this term and these 2 terms I write as square correct. This is I will just rewrite it or just simplify it to Ea - Ea prime times the square term correct. Now what we want is this term okay, only this term. So from this what do we get, we know the value of this term. We have calculated the value of this term, what was it –ih cross p cap upon m that is what we have evaluated.

It was –h cross upon m because commutator of H and X was – ih cross p cap upon m and the commutator of this quantity commutator and the operator was –h cross square upon m correct. Now so let me take this to my left. The quantity in the question that is E prime –Ea times the square term is = h cross square upon 2m that is what was asked to you to show. So hence shown okay.

So these were the set of 5 problems which we have done in tutorial 5 which gave us the idea or exposure to hands on the commutative bracket, the bra and ket notation and these examples actually were looks difficult, but very easy to solve. If you know the relations of the commutator bracket and if you read the question properly and understand it and by doing step-by-step you can easily evaluate these problems.

So next time, we will continue with the next tutorial 6 which will again we will have some challenging problems for you. So keep solving and keep watching the tutorials for if you have difficulties in solving by yourself.