Quantum Mechanics
Dr. Jai More
Department of Physics
Indian Institute of Technology – Bombay

Lecture - 30
Tutorial - 5 (Part II)

The fifth tutorial, so fifth tutorial has interesting problem which is problem 3. We have been seeing how to find out the eigenvalues and eigenvectors in a way which we discussed in the first part of fifth tutorial, you write the eigenvalue equation, find out the eigenvalues, then you find out the eigenvectors and then now let us try to do some smart work.

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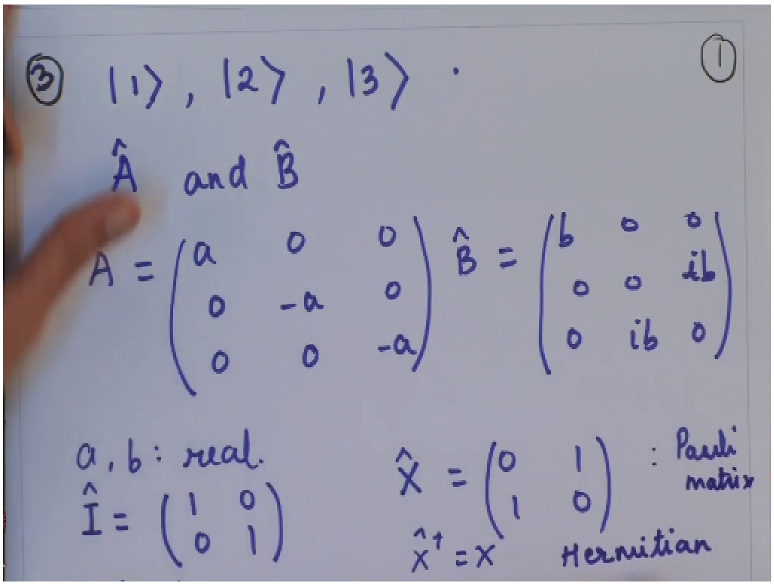

Consider a 3-dimensional ket space with simultaneous, in the question we have 3-dimensional ket space with vectors or the kets which are orthonormal, get 1, 2 and 3 are orthonormal and the form as a base ket of operator A okay and B okay, and operator A is represented as a 0 0, 0 –a 0, 0 0 –a, this is operator A. In operator B what do we have is b 0 0, 0 0 ib, 0 ib 0.

A and B are real. First operator you can see, you can simply use eigenvalue equation you need not use eigenvalue equation. So diagonal matrix. So you can look from here and say what are the eigenvalues. So eigenvalues will be a –a –a, so operator A is degenerate with eigenvalues, 2 eigenvalues as –a okay. So let us see operator B. Operator B you cannot do it quickly as we did for A.

So let us write this in some form okay. So the third A question is to find out the eigenvalues and eigenvectors of B. A you can just verbally see it is very simple. So B I can write, let me define something I, we always represent I as identity, identity operator right, and let me call this X as some 0 1 1 0 okay. This is an identity operator I have swapped these okay, swapped these operator and I get an operator X.

So this is an operator, so this looks familiar, you have seen such operators. They; is one of the Pauli's matrix and now I will try and write operator, these operator B in terms of identity and X operator. Let us see whether it is easy to write and recognize the eigenvalues and eigenvectors.

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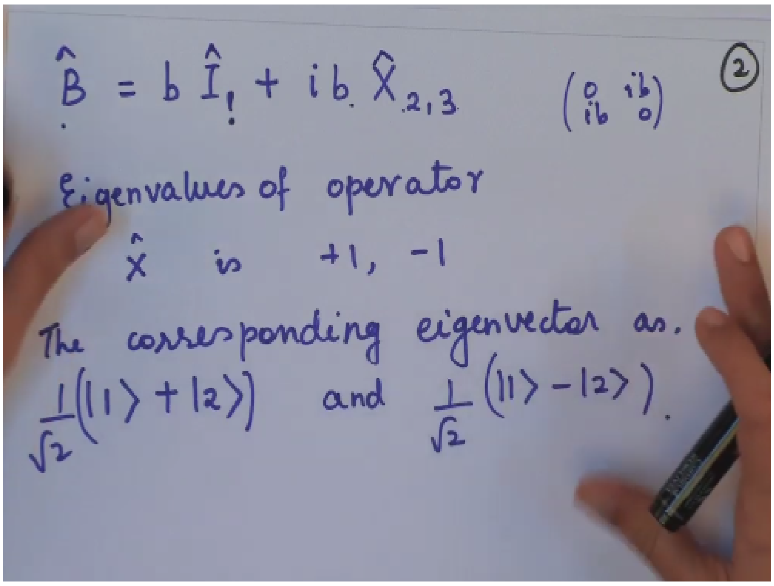

So operator B, I can rewrite as operator B can be rewritten as b. This is b times I1 + ib X2 X3, okay, so this was the operator X 0 1 1 0, this is the identity operator I which is 1 0 0 1 okay and you can see from here that X is a Hermitian operator. If you take the X dagger to be = X. So X is Hermitian. So eigenvalues of this Hermitian operator will be, this is a Pauli matrix. Eigenvalue of Hermitian operator will be real.

And this operator is also called a swap operator okay. So eigenvalue of this operator would be simply 1 and -1. So let me write the eigenvalues of so eigenvalues of operator X would be +1 -1 and you can write the corresponding eigenvector as you can write this corresponding eigenvector as 1 2 1/root 2 ket. In terms of ket 1 and 2 okay. So for operator X the eigenvalues are +1 -1 and the corresponding eigenvectors are given by 1 and ket 2 it is very simple.

Now, we have written down B in terms of X and I. Let me explain this to you. So B is the coefficient which is real. I1 correspond to the first component I11 component and ib 2 3 that means the second so this part is 0 ib ib 0 okay and the rest of the terms are 0. So I can just write in terms of this representation of the vector, the operator in terms of these operators. So operator b in terms of identity operator and the poly operator, poly matrix okay.

So this we have done using this information and what we have obtained here can we calculate the eigenvalues and eigenvectors of B.

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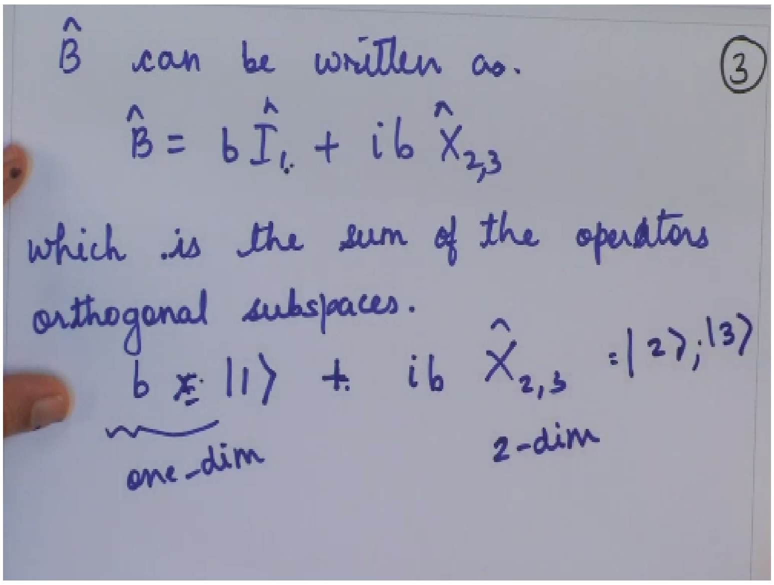

What we have here is that operator B can be written as I just rewrite this as before B is bI1 + ib X 23. This can be written as B as identity + the new operator and now we have that which forms which is the sum of the subspace, this will be the sum of the operators which are orthogonal subspaces. This will give me one dimensional subspace which is spanned by the eigen ket 1 and a 2 dimensional subspace this is 2 3. So this is a 2 dimensional subspace which is spanned by 2 and 3 ket 2 and 3 okay.

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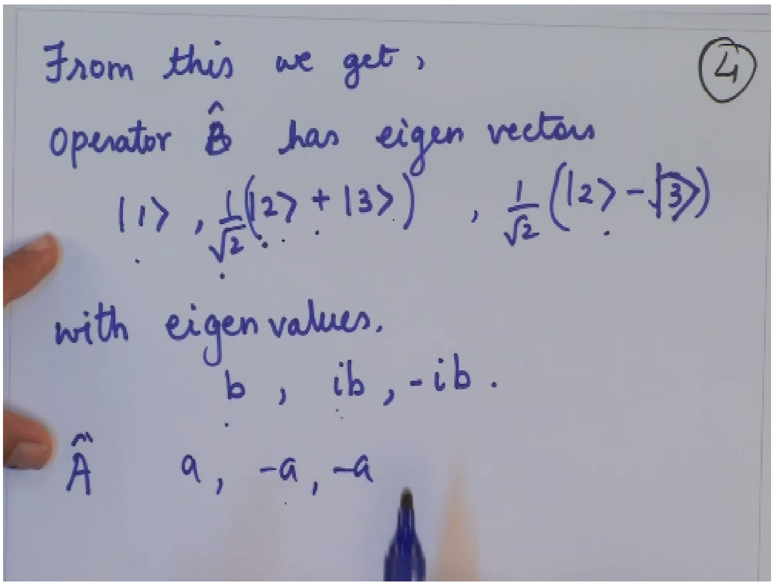

Eigenvectors of operator B is nothing but 1, 1/root 2 ket 2 plus ket 3 1/root 2 ket 3 2 – ket 3 with eigenvalues okay. From this we can guess the eigenvalues also b and ib and ib. Because we have seen ib and –ib, because we have seen here on the second page the eigenvalues of this operator is +1 and -1. This is an identity operator. So it will have eigenvalue B so and the corresponding eigenvalue of this operator.

We have written down this and the eigenvalue, the eigenvector of operator x is this quantity and eigenvector of identity operator will be ket 1. So just precisely you can see that operator B has eigenvectors 1, 2 + 3, ket 2 + ket 3/root 2 ket 2 – ket 3 by root 2 and the corresponding eigenvalues are b, i and –ib. So you can see that these are nondegenerate okay; however, the operator A has eigenvalues a –a –a, so this is a degenerate state or eigenvalue –a.

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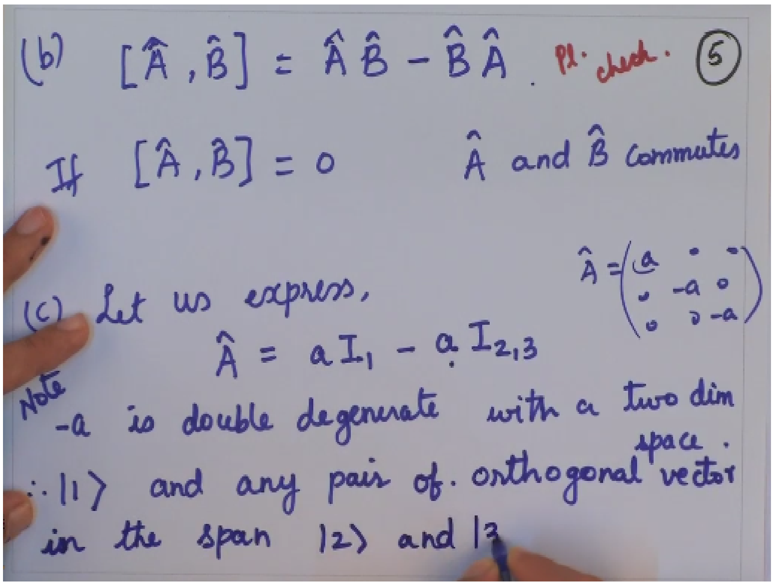

So commutator would be multiply matrix A by B - matrix B by A. So if you get A B is 0 then A and B operator A and B commutes okay. So this you have to check please check this okay. It will be a simple exercise. So I will leave it to you all. Next is to find a new set of orthogonal ket. Now we have a set of orthogonal kets 1, 2 and 3 given to us. You have to find out a new set of orthogonal kets which are simultaneous ket of both A and B.

So this is possible only when this is true. So you have to find out the simultaneous eigen ket of A and B. This is possible when A and B commutes, then they would form a simultaneous eigen kit, so you got the hint. So let us express operator A as, operator A I can express it as aI1 okay where aI1 is one-dimensional identity operator - we have aI2 3 again this is an identity operator because what was A? Recall, A as a -a –a, 0 0 0 0 0. So this will give me aI1 – aI2,3.

So this will give me aI1, one dimensional identity operator and a –a 23 okay. So this is what we have then what we do is – A, just note this that - a is double degenerate okay. So – a, eigenvalue –a is degenerate, it is double degenerate and what do we have here is - a is with a 2-dimensional space, it is double degenerate with 2-dimensional space. So therefore what do we say that 1 and any pair okay.

What do I mean by any pair, there are 2 kets, ket 2 and ket 3, so one with any pair of the orthogonal vector in this span 2 and 3.

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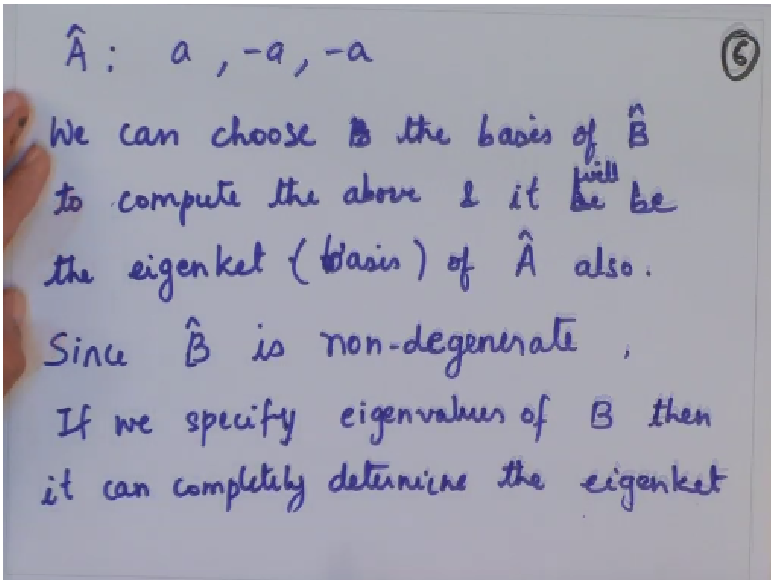

So A has eigenvalues a –a –a this is what we have and in the previous part I said that ket 1 and any pair of the orthogonal space vector is in the span of 2 and 3 will form the basis of A okay. So A has eigenvalues a –a and –a and we can choose the basis of vector B to compute the above and it will be the eigen ket or the basis of A also okay. This is possible only when A and B commutes.

So if A and B commute and if you choose a basis for B then automatically it will become basis for A also, since B is non degenerate it has 3 eigenvalues b, ib and –ib. So it has different eigenvalues and the specification of B, if we specify the eigenvalues of B then it can completely determine the eigen ket. So we will stop here and we will continue with the next 2 problem in the next part.