Quantum Mechanics
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Lecture - 24
Tutorial - 4 (Part II)

Let us start with the second part of the tutorial this part again we will have we will use the brand get notation to solve the energy eigenvalue problem to calculate the eigenvectors, energy eigenvalue and things like that. So, just a quick recap.

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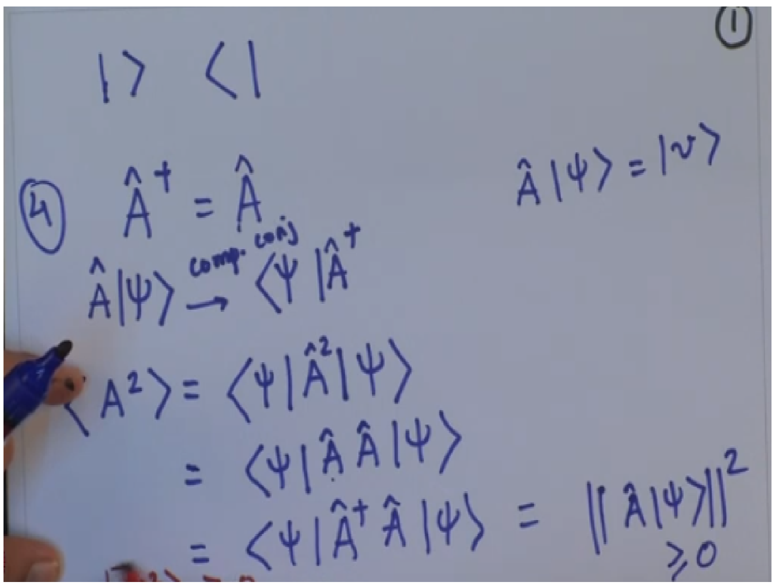

This is again notation, and this is a prior notation, so you are already familiar with this annual angular bra-ket okay which are for bra-ket notation. And let us move on to problem number 4 we have already seen a in linear vector space we calculate the norm of a vector and use we will use this hint in problem 4 that norm of a vector is always > or = 0. So, for a Hermitian operator is given to you a Hermitian operators is operator when take the dagger A dagger is A.

So, you have to prove in this that A square is >0 expectation value of A square >=0 so how will I write this? Operator A on psi will be this in get notation now the bracket of this would be A dagger and when you take the complex conjugate of complex conjugate you will get a A dagger and bracket of psi. Let me write this A square will be written as psi A square psi correct. Now you have to use this and the hint I gave you before.

So A square I can write as A dagger A A square I can write as A A. Now we know that A dagger is A, so I can write this as simple this I can write as we have seen this right.SO, this you can write as nothing, but this will be a complex conjugate to this I can write this as norm of and norm of any vector is always > or = 0 this is one way to do another way would be simply you define this as some vector v and do the similar exercise okay.

Both the ways you can do it, so we have proved here this is important and you should remember that a square is >or = 0 so the norm of a vector is always >= 0 so we have this in the fifth problem a Hermitian operator again is given to you.

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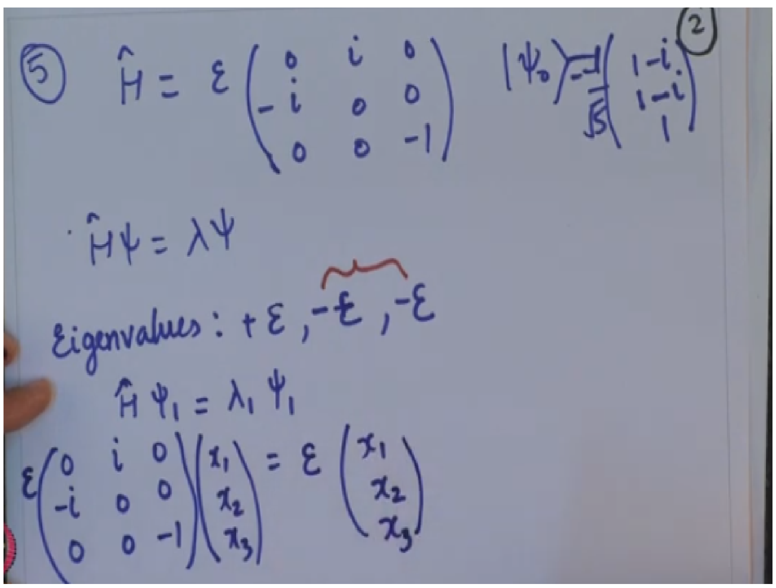

A physical state has a Hermitian operator H is epsilon I 0 0 0 0 -1 this is the Hermitian operator at initial state is given to that some initial state 1/root 5 okay 1-i 1-i and 1 okay and you are given seeds of questions on this so first question is what is the what are the possible results of the measurement of the system in state size 0.

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4 If $\hat{A}$ is hermitean, show that $\langle A^2 \rangle \geq 0$.

5 A physical system has a Hamiltonian $\hat{H}$ and an initial state $|\psi_0\rangle$ given by

$$\hat{H} = \mathcal{E} \begin{pmatrix} 0 & i & 0 \\ -i & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \; ; \; |\psi_0\rangle = \frac{1}{\sqrt{5}} \begin{pmatrix} 1-i \\ 1-i \\ 1 \end{pmatrix} .$$

What are the possible result of measurement of energy of the system in the state $|\psi_0\rangle$ and with what probabilities each of the possible value may be obtained? Calculate the expectation value of the energy.

So, this is one part and what will be the probabilities of each of the possible values that may be obtained then you have to calculate the expectation of the energy that is the last part that is a simple part we have done problems on wherein we have calculated expectation values of the energy. So, to get started with we have seen in the previous part that how do we calculate the eigenvalues and eigenvectors?

If a Hermitian operator any operator is given to you can simply use the energy eigen value equation. H Psi=the lambda psi where lambda is the eigen value so now you have 3 cross 3 metrics you will use the same logic and the eigenvalues so by now you have mastered the way to calculate the energy eigen values and eigen vector this what I hope and by looking at the metrics you can guess +epsilon -epsilon and -epsilon these are the energy eigen values .

So, you can see from that epsilon -epsilon -epsilon are the energy eigen values so what would this mean? That there are 2 eigenvalues which have -epsilon . So, you have a degenerate state let us go to this operator H is operated on the eigenvectors psi 1 which gives me the eigenvalue lambda 1 and it is not difficult again you will calculate this I give you one example and the rest you can do it by yourself.

Again let me call s 1 as x1 x2 x3 and I have a epsilon here and a epsilon here okay.

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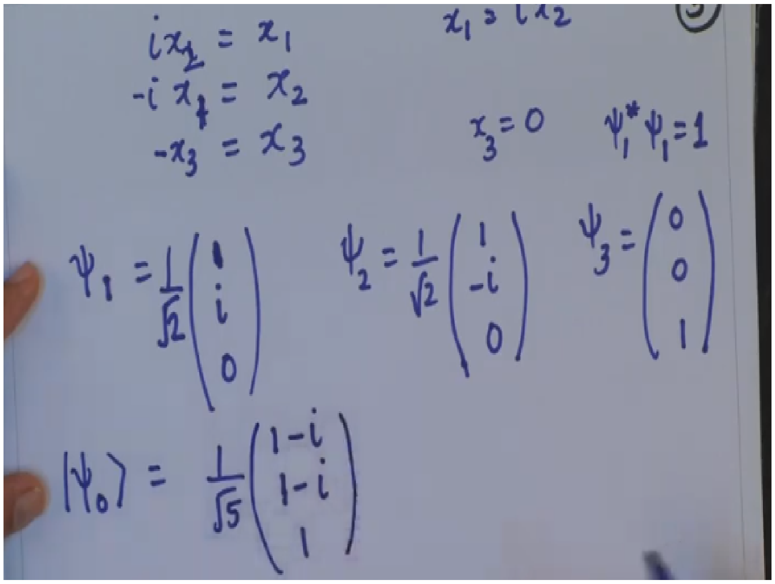

You have here x1 times I =x1 -I x2=x2 and -x3=x3 you have to solve these 3 and get psi so from here you can guess that x3 = 0 okay this is the only possible answer x1 is sorry you have x2 here x1 here so your x1 is 1x2 so your psi 1 will be 1i0 okay. Again I have used here psi star psi psi1 tar psi 1 is 1 to calculate the normalization factor and the value of x okay so the normalization factor and the value of x you can get from here.

But this you can similarly calculate psi 2 and psi 3 it is a simple task you have here 1-1 0 and psi 3 will be 0 0 1 I hope you will not have any difficulty in calculating psi1 psi2 and psi 3 you have to simply follow the steps wherein you will use the energy eigen value equation you have the operator H you will operate it on the vector eigen vector and using this you will get three equations you will solve them.

And you get the wave function the eigenvectors psi 1 psi 2 and psi 3. It is not a difficult task. Now the initial state size 0 was given to us so eigenvector sorry the initial vector was given to us as 1/ root 5 times 1-i 1-i and 1 okay break psi out in case sin out in terms of psi psi 2 and psi 3.

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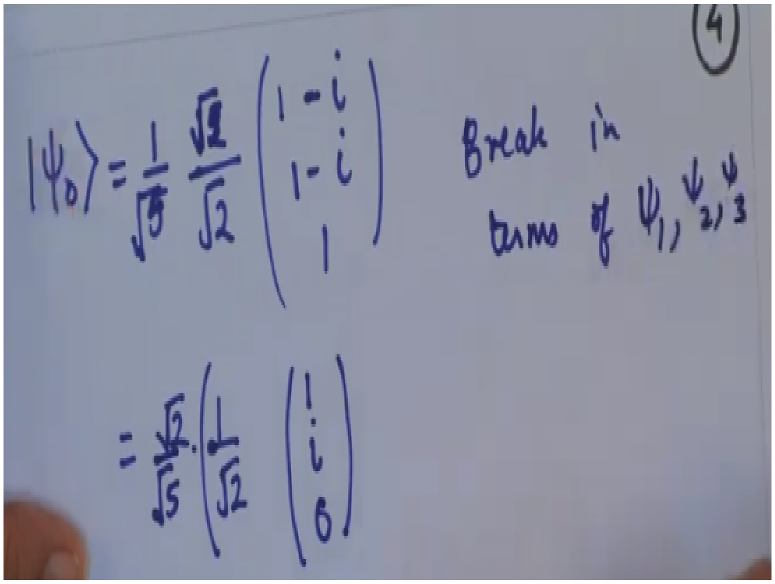

Anyways we have to calculate sin out, so I can write this as root 5 so I can write this as 1 root 5 sorry root 2/root 2 you can write this right and I have 1-i 1-i 1 okay, so this can be rewritten I can rewrite this as 1/root 5* root 2 okay then I can write this as 1 i 0 this is my one way would be to break this in terms of psi 1 psi 2 psi 3 another way would be and see what you get.

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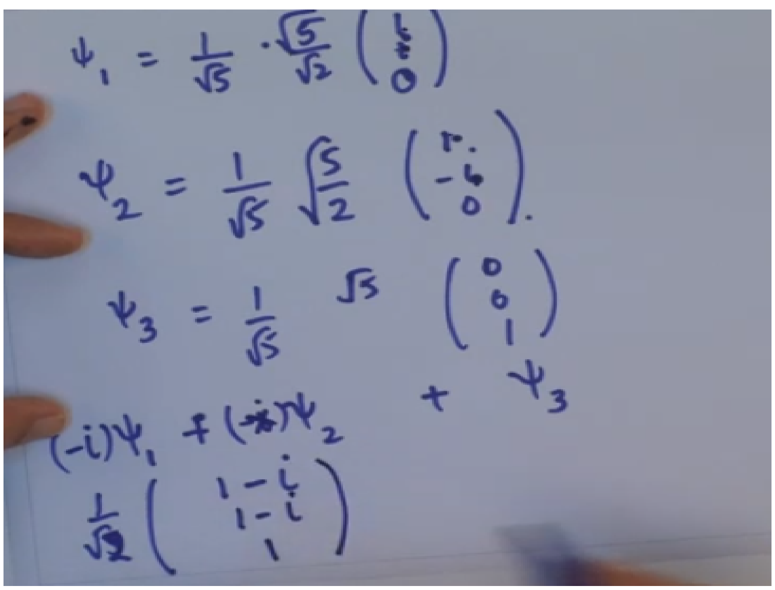

Psi 1 I can write it as 1/root 5/root 2 root 5/root 2 okay 1 i 0 okay and psi 2 as 1/root 5 again root 5/2 1-i 0 and psi 3 as 1/root 5*root 5 0 0 1 okay now when I add psi 1+psi 2+psi 3 what do I go get 1/root 5 okay psi 1,psi 2 psi 3 when I am adding I will have so basically this is 1/root 2 only I will have here 1 +1 I do not want 1+1 so I want 1-i so I multiply this by -i okay so I have 1-i and this is 0 so 1-i next one is i*-i so rather I will have to multiply this by -i.

So, let me multiply this by -i this is nothing so first term will give me -i+1 so 1-i second term will give me 1-I third term will give me .

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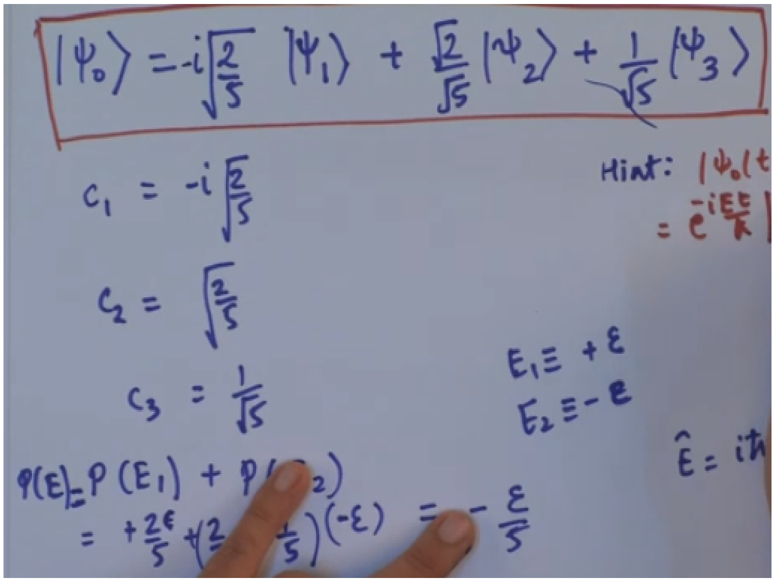

1 psi 1 . So, my psi 1 was nothing but 1 and i. So, you will have a psi1+I have 2/root 2 root 2/root 5 psi 2+1/root 5 psi 3 okay, so this is what I have okay now let us check the factors so 2/root 5 psi 1 so what was psi 1 psi 1 we have obtained here as 1 and i okay psi 1 is 1 and i. So, you will have to multiply this by-i, so I have a -i and the rest goes okay so we have expressed sin out hat is the initial state in terms of the eigen states okay.

So, this was a simple exercise it is a lengthy exercise but a simple exercise, so you can see the co efficient c1 is -i root 2/5 c2 is root2/5 c3 is 1/root 5 okay why did I write these coefficients these coefficients will correspond to the possible you can calculate the probabilities using this c1 c2 and c3. So, what will be the probability of finding the particle? okay remember this set of eigen vectors I have obtained in this manner you may get a different set of eigenvectors.

Wherein you may have different coefficient c1 c2 c3 maybe different in your case it may be swapped so what I am getting as c1 you may get as c2 and vice versa and this is not a unique solution. This is not a unique vectors, so you can have different sets of eigen vectors. So, when you try it yourself just check whether you get a different set of eigenvectors. So, the c1 c2 c3 is what you obtain now in order to obtain the probability on E1+probability of E 2.

So, P of E let ,e represent P of E as the probability of finding the particle in either state 1 or state 2 that is epsilon 1 or epsilon 2. E1 will be corresponding to eigen value+epsilon E2 will correspond to finding the particle with energy-epsilon. So, we have a degenerate state for E2 . So this would be simply square this, so you have -2/5 so when I take the complex conjugate, I have +2/5 and square of c2 will be again 2/5 and this will be again 1/5.

So, I have to multiply this by epsilon and I will multiply this by – epsilon so I have 3/epsilon, so I have -epsilon/5 is what I get the probability of the possibilities of finding the particle in this and this are the state. So, finding the expectation value is again a simple tasks wherein you will write the time dependent equation use the time dependent with function and then calculate the energy eigen and remember that energy operator is written as i h cross2/2t.

This is the energy operator. So, using this you can get the expectation value of energy so the expectation value of energy you will write let me give you a hint. Hint would be for expectation value you will write the wave function sin out of T you will have E raise to i t upon h cross upon and psi of sin out of t so sin out okay you will write the expectation value. So, here you will put the Hermitian operator.

And when you operate it on the state you will get either e1 or e2 either epsilon or-epsilon and then you can easily calculate the energy eigen value. And hope that this will make simplify these simplified steps would help you to understand how to calculate and you need to do it at least once by yourself to understand this properly and in this problem in the end you can see here that one thing which we have seen here is that the energy eigen values.

What we obtain one of them is degenerate and the other one is not degenerate. So, you can understand that when you find the particle in one state with energy epsilon and the other energy epsilon - epsilon there are two indistinguishable state which are degenerate . So, when you make such measurement it is difficult to distinguish the two states, so we will come to more number of problems on these measurements.

And then in the pipeline you have more problems on bra-ket notation wherein you will how you will learn how to write the conjugate basis vector and problems which will have involved more on energy eigenvalues energy eigenvectors. So, this part should be thorough and to be clear before you go to the advanced topics like the Schrodinger picture and Heisenberg picture. So, we are trying to slowly evolved from using the bra-ket notation.

And go to the evolution picture of Heisenberg so in the next tutorial we will continue with more number of problems.