Quantum Mechanics
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Lecture – 23
Tutorial - 04 (Part I)

Let us now move on to tutorial 4 which consists of again 5 problems that will be divided into 2 parts and just a recap of our previous journey in which we have discussed problem based on particle in a box. Then we had seen problems like how to calculate the energy eigen value energy eigen function etc and these problems will again we will have such problems wherein you have to calculate an energy eigen value and energy eigen function of an Hermitian operator.

So, from a wave function formalism let us now move on to bra –ket notation and in that you will see that we have already seen actually the lean in the linear vector space. We have seen the how to calculate norm how to calculate the sum inequalities we have learned like what shows inequality triangle inequality and now we slowly move on to the bra –ket notation and let us get started with the problems.

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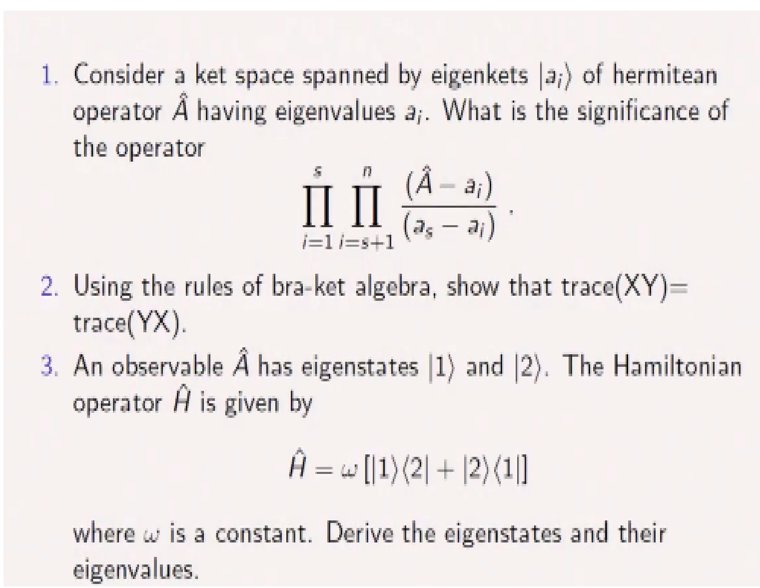

So the 1 st problem is you will see in after the results that this particular operator which is given to you A cap – i a upon as - ai is a special operator after solving this you will come to know what I am talking about so the 1 st problem you are consider a ket space span by a eigen ket ai is given and of Hermitian operates A so operator A is Hermitian we know what is our Hermitian operator A dagger is A is what is a mission operator.

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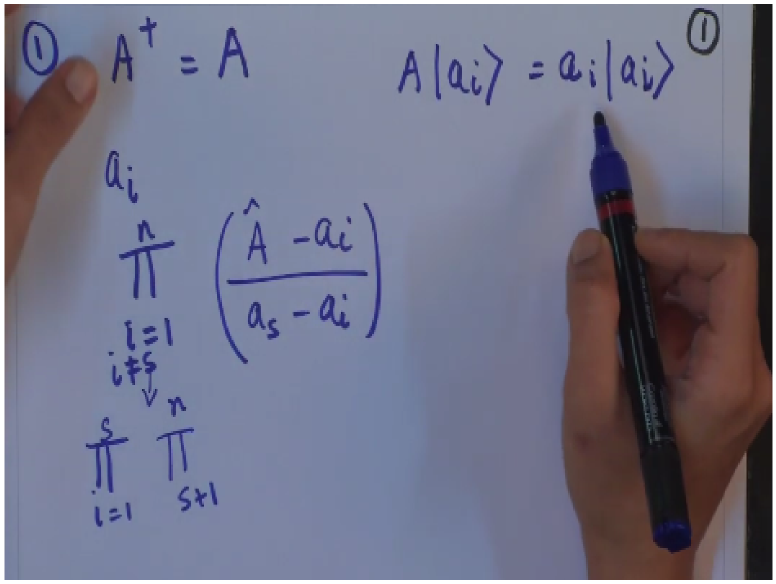

Remember this? We have seen it before but just to stress on I am writing here and we will be using this property if required the next thing is that this Hermitian operator has eigen value ai so ai is the eigen value of this operator so you have to tell what is the significance of this given operator i going from 1 to n A - ai / as - ai so this is an operator given to you and in the problem, this product notation is so let me write this year where i is not = s.

So, this is what the question means so in the question you have seen i is = 1 to s then s +1 to n okay so i not = s it is between 1 and n excluding the value of s that is what is given to you so now let us write this operator you will operate this operator on a ket that is eigen ket ai given to you so eigen ket ai is operated.

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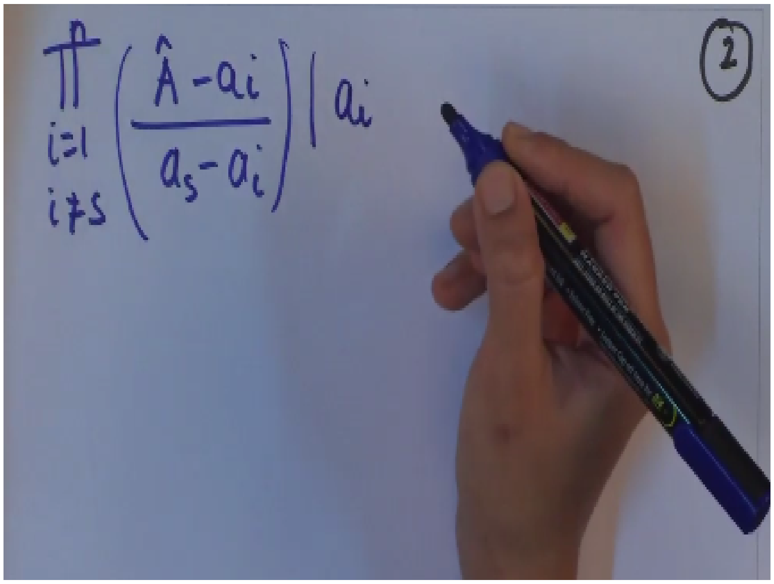

On so you have i is 1 to n let me write this notation A – ai / as - ai operated on the operator ai and here you must know that ai is the eigen value of this operator. So, let me write here when A is operated on the eigen get ai you get eigen value ai this is the eigen operator or the eigen vector sorry this is the operator and this is the eigen value so now when you operate this.

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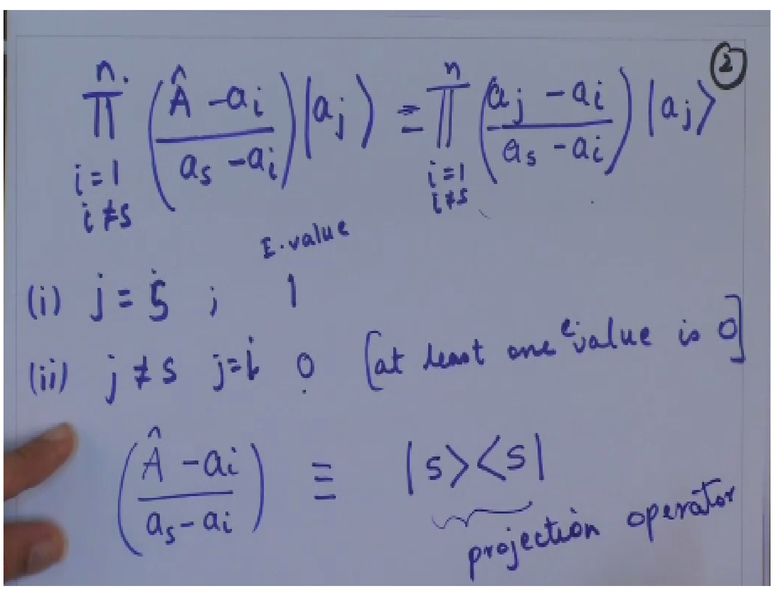

I = 1 to n and i is not = s so A - ai / as – ai this is the operator now I will operate a eigen ket aj on this operator so when ai is operated on a you get ai and now when I operate aj on this what do i get is aj – ai /as –ai this is what you get when aj is operated on this now you can see that from this. Can you make any conclusion? No so you have to see case by case so in case 1 okay there will be 2 cases.

So case 1 when j = i sorry when j = s so from this what do you infer and of course this sum of the product is there and this is there and now when j = i what do you obtain when j = i you get the right and side becomes 1 the eigen value becomes 1 so eigen value is 1 and when j is not = s this is not = but i goes from 1 to n and if it picks a value when j becomes = i at some point the numerator becomes 0 so at least 1 value is 0 at least 1 eigen value is 0.

That means for j not = s the eigen value becomes 0 this operator is simply written as this can be written as s okay so this is nothing but your projection operator okay remember projection operator is when s are same so you can really write this when you operate this operator on j.

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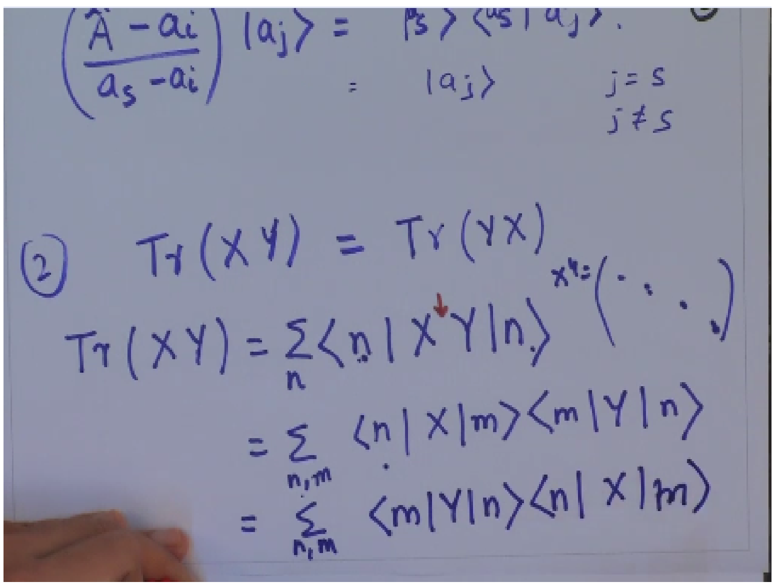

Operator a - ai is – aj this when operated on aj would give now in terms of this s operators when you write or the j th operator will obtain os in terms when i s = a let me write this as rather so when s =j you will have just aj so when this is 1 you have when j = s and when j is not = s this 1 becomes 0 so now let us go to the second problem this is an exercise for you to get hands on bra-ket notation so bra-ket notation.

You have to use to calculate that trace of x and y is = trace of y* x so this is what we have to prove trace of x times y = trace of y times x this is what we have to prove so in this we will use bra-ket notation so what is trace specifically sum of the diagonal element so now this trace in bra

–ket notation can be written as xyn so when you are multiplying and when you write this operates.

And do product and once you get the product you add up the diagonal element so you will add up this will be sum over n so when you sum up the n th element so this is your x times y and you will sum up the diagonal elements you will do this sum when we do this sum then n= n that means bra and part of it is same so 11 22 33 nn and so on you will add the elements so now this is n and in this I add I put a projection operator.

A complete eigen ket m let us assume that m this is y okay this is write this as summation over n and m and you can write this in the bra –ket notation like this so in the next step what would I do is I can actually flip this because this is a multiplication so till will not matter if I just flip these dummy n dices and this is now the projection operator which is 1.

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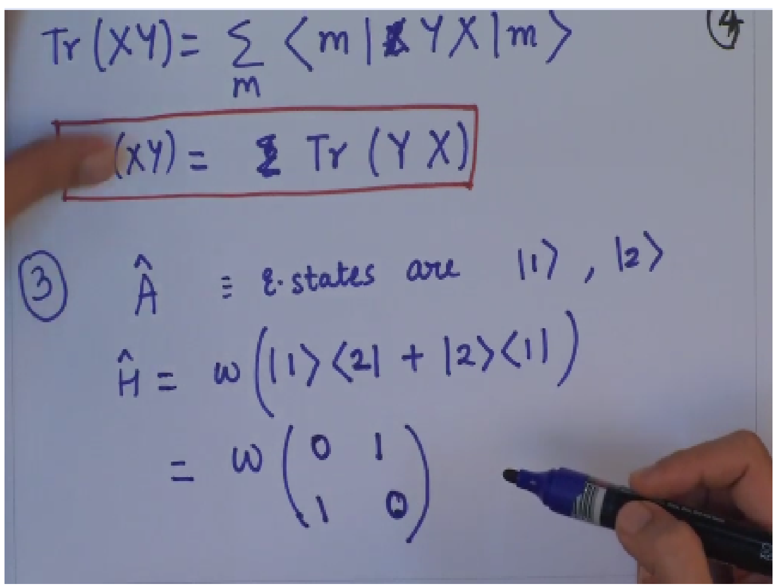

I can write this as m trace of x and y will become now y times x m so in the previous you have seen that you have m y and x so now this will be nothing but this trace I can rewrite again as trace of y times x so this is very simple exercise and it is expected that you in order to get hands on bra – ket notation you must do such simple exercises and even in coming problems we will see that using bra – ket notation it is very simple to solve the problems.

Let us go to a next simple problem which is a is given to you that an absorbable a has eigen states 1 and 2 absorbable A is given to you which have eigen state to its 1 and 2 and to the Hamiltonian operator so the eigen states of this operator are 1 and 2 and the Hamiltonian operator is given by something like this is given to you. Now this can be written as bra-ket notation can be rewritten as the matrix form.

Where omega is a constant and you have to derive the eigen state and their eigen values. okay so here omega so you have 1 2 so when you are writing a matrix this will be 1 1 component and this will be 2 2 component. 1 2 will be here and 1 and 2 1 will be here 1 and 1 1 component and 2 2 component are not given so this is the way you will write. So, you can see in the previous tutorial also we have seen such problems.

So, going further to the next part of this problem where you have to calculate the eigen state and the eigen value you can see from this Hamiltonian operator and guess the eigen values. It is very simple.

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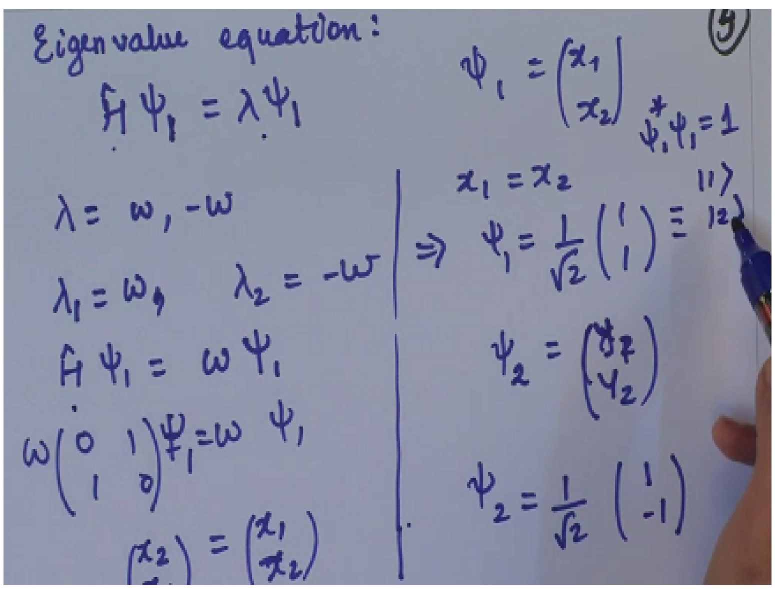

So, you remember the eigen value equation so those who all do not know how to calculate let me demonstrate again. This is the eigen value equation operator H on the eigen vector on a vector Psi 1 is = lambda is the eigen value and Psi 1 is again the eigen vector so once you have to calculate once you calculate the eigen values you can easily calculate the eigen function so from this I can say that the eigen values are omega and - omega.

It is very simple you have to just substitute for the Hamiltonian operator and calculate the corresponding eigen then what do we have is for if I call lambda 1 is omega 1 then I will have operator H Psi 1 = lambda 1 is omega and lambda 2 is – omega so H operator on Psi 1 is omega Psi 1 then you know now what is operator H omega Psi 1 then you can simply know from here so when I do this i have a Psi 1 here.

Let me write Psi 1 as sum x1x2 then what do i get here is x2 x1 = x1x2 then let me do it here only x1 = x2 this would imply that you have Psi 1 is 1/ root 2 1 1 it is very simple to see by Psi is nothing but 1 1 so how do i get the values of x here again i will use normalisation conditions so Psi star Psi is 1 then you find out the normalisation factor and then you can click this now again.

This is for so Psi 2 let me Psi 2 x2 sum y1 and y2 so for that same way substitute for Psi 2 your lambda 2 is – omega. So, you will have x2 as - x1 x1 as – x2 so if your x1 is 1 then your x2 is -1 so your Psi 2 will be 1/ root 2 . This is the way you can right now again I have to rewrite this in terms of 1 and 2 operator eigen vector I can write in terms of the eigen state 1 and 2 how can i do that.

This is again simple step wherein you will write this. Let me demonstrate for the 2 nd 1 you can easily calculate.

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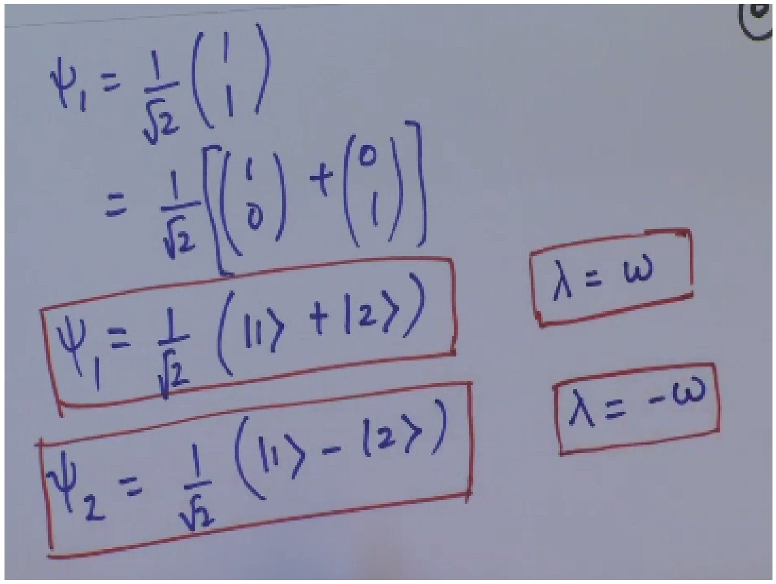

Psi 1 we have got this i can rewrite it as 1 0 + 0 1 correct so now this is 1 + 2 same way Psi 2 you will get something like this so now you have express the eigen states in terms of the states 1 and 2 so you have Psi 1 as this Psi 2 as this and this correspond to eigen value omega this corresponds to eigen value – omega so it is a simple way of doing> if you have not done such examples where in you have to calculate the energy eigen values or eigen vectors.

This exercise is a must for you this is a simple example and that will help you to solve more difficult or different problems so you must try at least one such problems and in the next part of the tutorial we will have two more problems let us meet in the second part of the tutorial.