Quantum Mechanics
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Lecture - 22
Postulates of Quantum Mechanics - II

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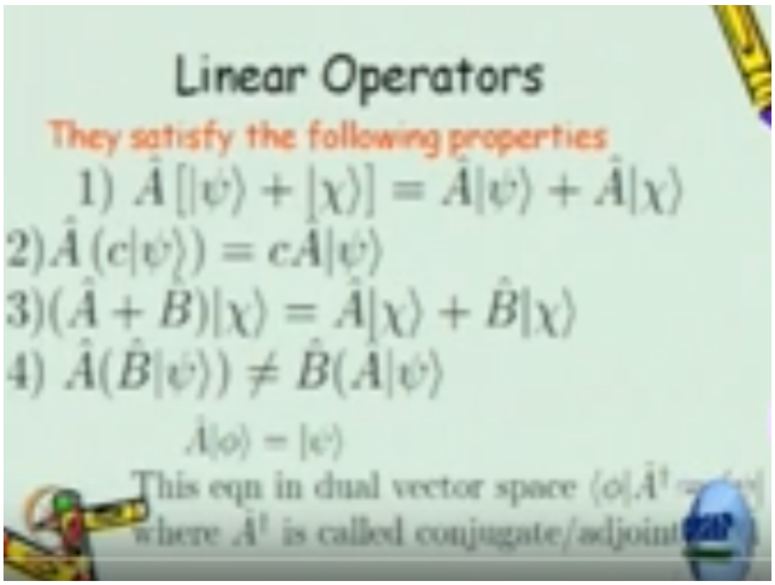

Clear kind of summarizing whatever you have seen in the last classes

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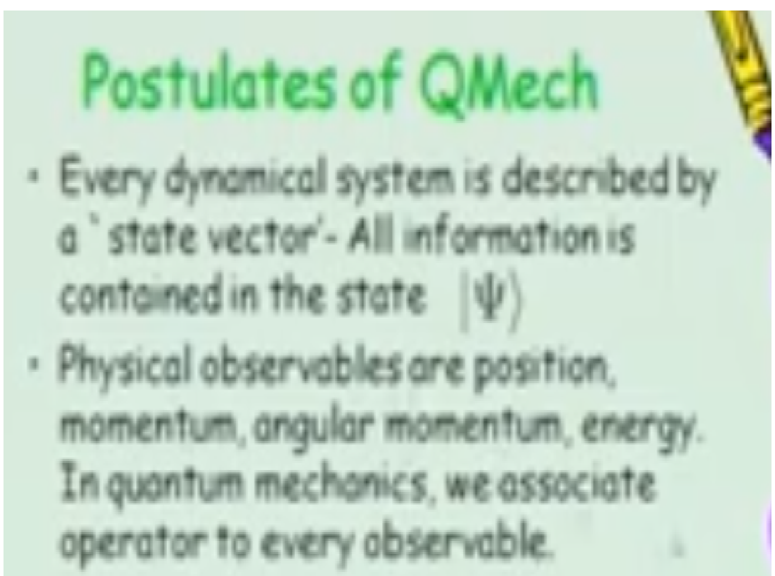

So getting with this all these definition in place what are the postulates of quantum mechanics. So this I have been repeating many times you a quantum mechanical system with some dynamics and you formally say all the information about the state of the system is going to be contained in this side so that is the first posture. You say that the system is prepared in the state side so all the physical observables.

Some of the simple ones which we are familiar position momentum, angular momentum energy. In quantum mechanics we associate operator to every observable.

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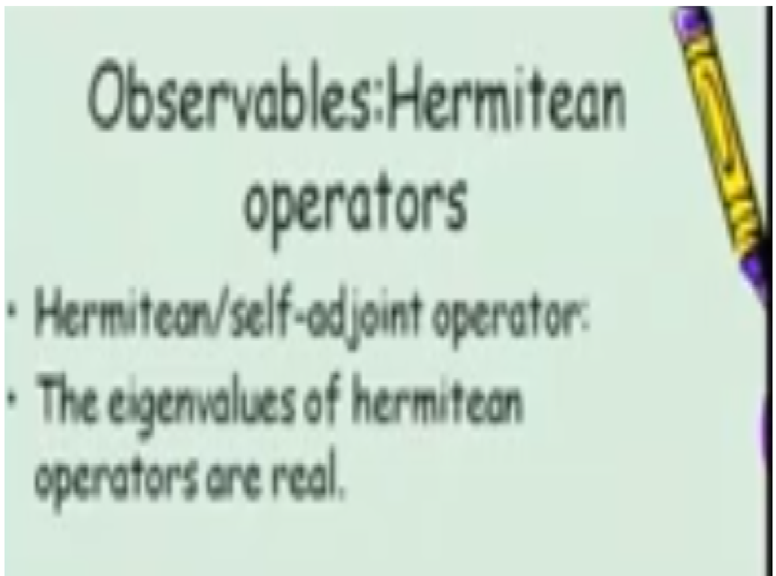

Which is as follow of the in this also we saw that all the adjoint operator should be = to the operator which is what we what we call it as Hermitian condition. The eigen values of Hermitian operators have to be real or equivalently any expectation value of any operator If it is real then that operator corresponds to the observables. Yes that is the first requirement. All the operators in quantum mechanics which corresponds to observables.

Must satisfy linear all the operators in fact whether it corresponds to observables or not take the to be linear operator. There are some operators which are anti linear operators also but that is something you need to think. What is an anti linear property you know at least for the observables which we are going to study is linear. Let me answer this question.

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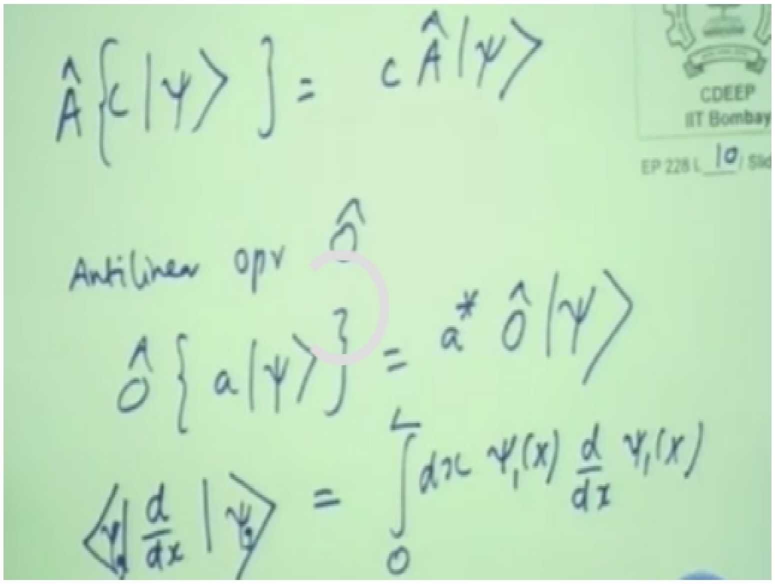

So, we had this A operator on c times psi we wrote this as c times A operator on psi. So, this is the linear operator if it is anti linear operator you have to complex conjugate co efficient. So, anti linear operator let me call it as o satisfies some point if we have time, we will talk about this there is a time reversable operator which as to satisfy this property we will come to it. So, at least for all the practical purpose.

Right now what we are going to take is the observables which we are studying like momentum, energy on these things satisfy the linearity property. So, you are looking at linear operators. There are some more subtleties here that all your this anti linear operator which time reversal if you do two time reversal it is identity, so it is only defined for discrete operation but whereas for continuous operation like translation or rotations the operator has to be a linear operator. So, you have seen this also

An eigen value equation I defined it for any operator for example A operator this lambda I is called the eigenvalue and psi I s are the eigen states of this operator. You have to get used to tis notation eigen value equation means which is an eigen value which is an eigen function and the eigen function will be a eigen function with respect to one operator. Sometimes there will be another operator which will also have the same eigen function 1 of x.

Then this is the same eigen function for both the operatos.so, all this possibilities exist it can be a different eigen function or it can be the same eigen function okay. So, this is eigen value equation and we also need linear operators to satisfy this property this is also I said whenever you take the initiate state this A psi will gave you a new state you can replace it as psi with A dagger and with psi and that is an inner product property which should have the star.

And then we also said that for observables whose expectation values have to be real you have to have this property will force that A= A dash. So, I asked you to find out d/dx expectation value on a state psi right and you have done this in the weight function formalism. Take the particle in the ground state or first excited state on this expectation value can be taken to be if this is ground state tis will be integral over dx 0 to L psi 1 of x d/dx psi 1 of x right.

Formally you can do this not even for a particle in a box. You can take In general a free particle you can take this from -infinity to +infinity. And say that the way function should vanish well defined it belongs to L2 of-infinity to +infinity the way function should vanish at + or -infinity and you can work this out. Please d0 this and you will see that will be same as negative of integral.

So, this is a star negative of integral dx-infinity to +infinity okay so you can have a d/dx tag psi 1 of x psi 1. So, you can show that the expectation value of d/dx is same as negative of expectation value of d/dx dagger. What does that mean it gives expectation values in general complex a+ib to be same as. Let us write this to be a+ib okay this one will tell me it is a-ib with the negative sign.

So, a=-a means it is going to be 0 so it is only b which will survive. B is only purely imaginary o if you take the expectation value of del/del x operator you can show that the expectation values are purely imaginary. So, they do not correspond to observables. Observables should have real eigen values and the expectation values which gives by Ehrenfest theorem related to your classical results practical results they have to there.

So, the only way you can make it real is you put a ih cross here and this will become a -ih cross sorry this is -ih cross and this will become a +ih cross. It will also come to why it is -ih cross el/del x for the momentum operator. I do not think you know how the derivation goes to show that the linear momentum as an operator has to be -ih cross del/del x. So, this + and this – will give you the same okay.

So, then there is no – sign here I will play around with a+ib sorry a-ib right because there is a dagger which means these two will cancel only ab. So, that is why the expectation values of observables have to be real let us just take a toy example how to do it for a d/dx operator. Fine for all observables which we study the eigenvalues have to be real, so those operators are called self-adjoined operators which are denoted by a=a dagger they are also called Hermitian operator.

There is a slight subtle difference between Hermitian and self-adjoined but for all practical purposes in this course we can take we can call Hermitian as self-adjoint operators we will not into that. There is a formal way in which you have to look at domain of an operator and so on we will not get into it. I introduced you the Fourier transform which kind of told that if you have a square summable sequence.

And a finite dimensional or a countable infinite dimensional all these things we have done. But when we did the Fourier transform by taking K to be a continuum there also it is actually a uncountable continuum linear vector space right.

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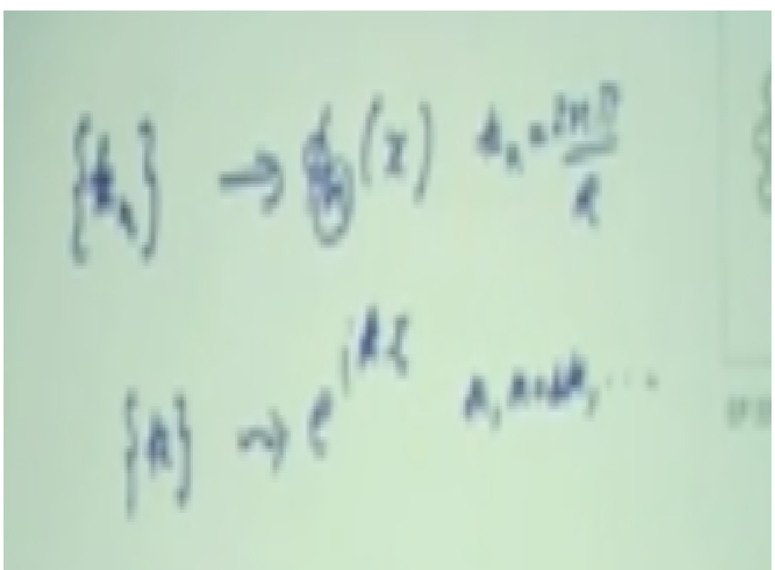

So, the set of kns which we have which corresponds to the phi n of x for periodic functions we could have a k which is similar to what we wrote c to the ikx as the basis functions right we did this. So, this one has a discretized index so kns is a set of allowed vectors and for that we have this corresponding functions phi n of x you may recall that right and then we also went into a wave packet where we used the basis function that needed the ikx and k was a continuum.

So, with periodicity we showed kn to be like 2n pi/r here this k is a there is no condition k will also go as k k+delta k so on it is dk is very small so this is a continuum, so this vector space is countable, but it can be infinite dimensional right a particle in a box stationery states the phi n of x which you write, and it can go up to there is no cut off it is infinite dimensional but countable. In the sense that it is summed up series right.

But if you look at a free particle these basis functions are also for a linear vector space. But here there is a continuum because there is a linear vector space to be an uncountable linear vector space. Slowly taking you into abstract notation by showing you some simple examples I hope you appreciate.

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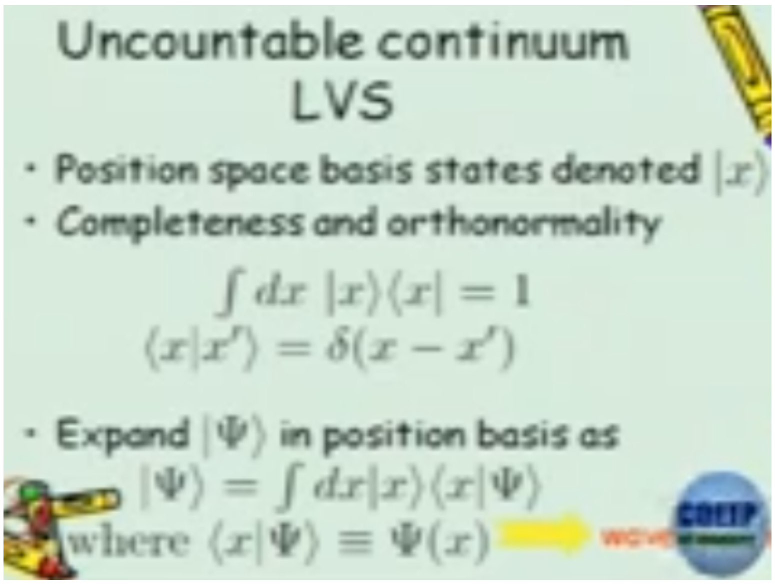

So, this is where we get into the uncountable continuum linear vector space. Some of the things which we can start doing as we can introduce basis states in the position space okay. So, x is a continuum it goes from -infinity to +infinity like the k. So, I will formally follow the derived notation write the basis states in position space by a ket x allowed. How will you write completeness in orthonormality in this continuum vector space.

Exactly like what you did in the discrete you have to only replace wherever summation over n was there by an integral over tx because x is the variable, so we have I the continuum linear vector space uncountable there is an outer product summed up all the positions, but the positions are continuum. So, there is no point in putting a summation you have to put an integration. Suppose you are in some kind of a checker box.

That you know the ball will only fall at l,2l ,3l then this position will become discretized. Then the integration can be removed, and you can put a summation. But I am trying to say that the particle can be anywhere from -infinity to +infinity. So, this is a well-defined position basis for that and following our completeness and orthogonality for a continuum vector space we can define the completeness relationship of this.

And orthogonality to be the derived delta function confining myself to one dimension. What will happen if I go to 2 dimension there will be two integration dx dy the state which I have written I could write x,y outer product ad we can do this. What happens here it will be a derived delta function in to dimension, I am taking you through this so that slowly we will see what exactly is wave function which you all studied which you were doing you could write a wave function.

Or you can define the direct formalism over ket and ket in the position basis ket in abstract basis and we can see how to get wave function which you study using this position basis. So, that is why I am trying to stress the position basis. Any arbitrary state you can expand in any basis that is what I was trying to tell you it is not required that you have to expand it in momentum basis alone. You can expand it in position basis you can expand in energy basis.

Like all your particle in a box then you expand it a super post wave it is taking the stationary state which are energy basis. The psi ns Hermitian operator on them have you a specific energy nth excited state energy. So, that is an energy basis so here can do the psi in a position basis why not any basis is good enough. So, if I do that, I can just insert the identity operator and write this what will we call this as this inner product.

It is the probability amplitude for an arbitrary state psi to be in position specific x. This probability the amplitude in the position basis is what you all study of wave function. What you have been seeing as wave functions are nothing but the probability amplitude for an arbitrary state psi to be in the position x is given by this inner product and that is what we call it as a wave function.

So, whenever you do wave function there you can go and replace it as an inner product and we can try to see whether everything is consistent, so this is what we call it as a wave function.

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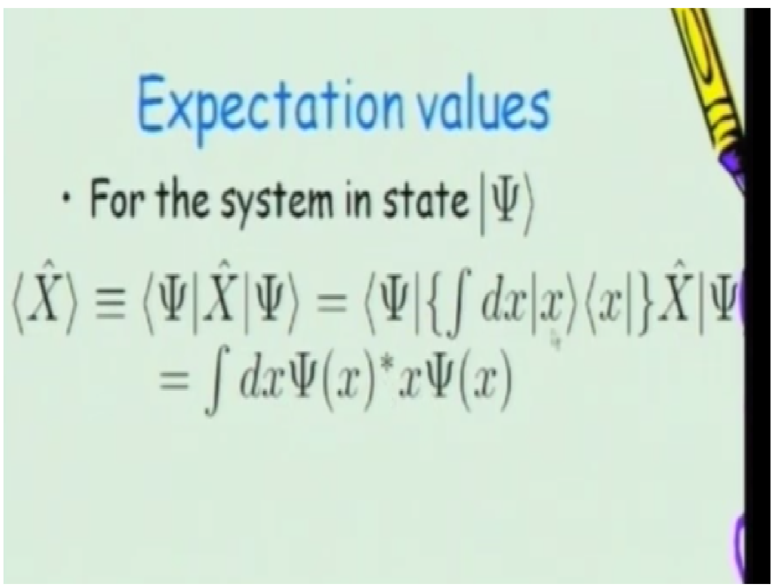

So, what is expectation value? This also I said for a system prepared in state psi the expectation value of some operator X for example let us take this to be the position operator in quantum mechanics in the direct notation I would have if this state is in psi then this is what I would have written but equivalently I can insert this parenthesis which is nothing but a identity operator which is what I have done here.

And once I put this identity operator you can see that this psi with this x is a complex conjugate of your wave function. And then the x operator on the x basis will give you an eigenvalue equation. X operator operates on this state x will give you an eigenvalue x times x. So, operator is gone you get an eigen value then what are we left with a psi an inner product x what is that? So, we are going to define an operator x on the position basis.

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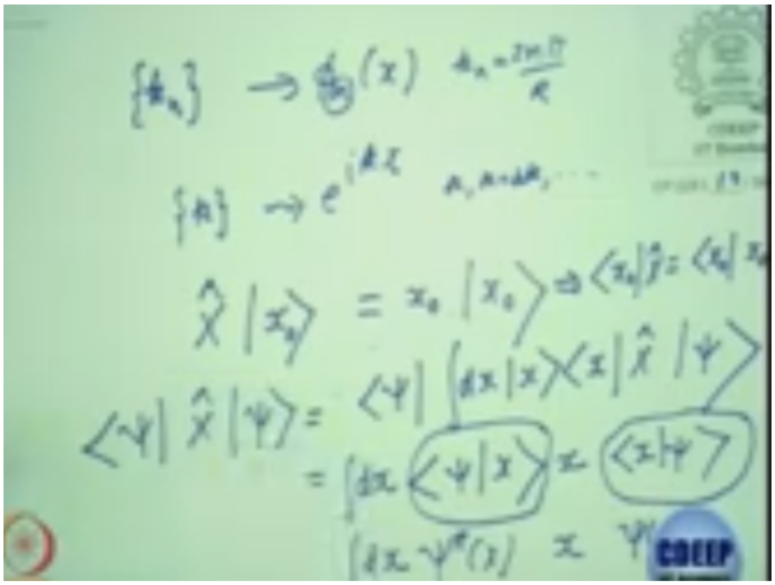

So, let us take some x0 specifically if I say it is a position basis the position operator on x0 state will give you. Suppose if I take a state with some momentum and I operate with some momentum operator I am supposed to pull out that number of that state, right? so this state is at a let us say this state at x0 and I have a position operator it hits that state what you will get a position eigenvalue it will give information about that state and xo.

So, this is an eigenvalue equation for the position operator with eigen value x0 and get x0 that is what I call it as a eigen function. So, the position basis means they are eigen states of the position operator okay. So, if we use this then whatever I wrote psi x operator psi formally I could have a psi and then integral/dx xx and then I have a x operator on psi. So, this integral/dx I can take it out not a problem.

This will be psi with x and you have a x operator operating on x that will give you eigen value x you are still puzzled I wrote x0 earlier yeah but all the observables the eigen values are real. All your position operators are all it is an adjoint operator I you want you can take it x star you can write but x star is same as x for all observables. Eigen values are always real, so you do not need to write star.

If it is some other operator which has no corresponds with observables, then technically you have to put a star there. Otherwise I do not need agreed so here I am trying to say that x0 with x operator is formally you would have written x0 with xo eigenvalue as a star but since this is a observables lambda =lambda star so this will be the eigen value will be real. So, we do not need to put a star.

Is it okay then what else is left that comes out with psi so this piece is what we call it as wave function psi of x. And if you call this as psi of x this one has to called as psi star of x and x integrated. You get back your earlier quantum mechanics where you did in the wave function formalism but here, I had the direct notation, but I inserted a position basis completeness identity operator in position basis and then went to the familiar wave function.

It is this notation is so neat that you can go back and forth between the wave function and the state vector formalism. I do not know this angular bracket is so really very elegant now we all universally follow it, so I want you to appreciate that you can also start using this by inserting unit operator in between any time for convenient. Like if I am looking at position operator, I will put a unit operator in the position basis.

But if I am looking at momentum operator, I could do unit operator can be in any basis I can also put a momentum test right you start seeing that start doing that momentum basis and the corresponding momentum operator on the momentum basis will give you the appropriate eigenvalues and to go from position basis to the momentum basis what will you do again you do this inserting unit operator right.

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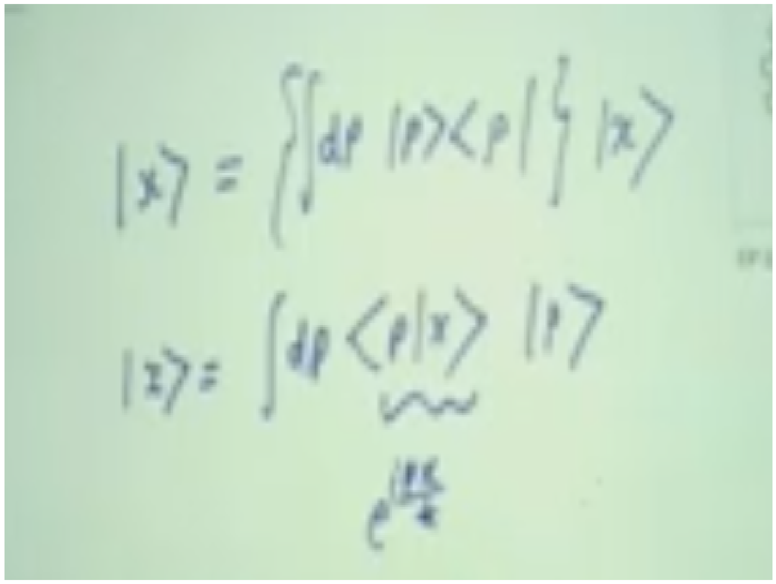

Let me formally write that you will have a position basis which I can write dp pp this is identity operator operating on x nothing wrong. So. if I take x then this is dp state p and then this inner product of p with x what is this inner product one has to find, and we will do this at some point and show that it is should be b then b and then the product or does this one and maybe at some point so that to I px/h cross.

So, this will turn out to be e to the I px/h cross we will prove this okay. In some senses going from one basis to another between the momentum basis and the energy basis sorry momentum basis and the position basis the Fourier transform shows up. But I have introduced for you that there is a unit operator in the continuum linear vector space uncountable linear vector space which is this is the completeness condition.

And the corresponding orthogonality condition in momentum space will be pp prime if you are in one dimensions you have px px prime then this will be a delta function of px-px prime. So, you need to remember that when you are in unaccountable continuum linear vector space you will have the derived delta function for ortho normality. We did this in the function space you go back and rewrite the function space is similar to what you do with the wave functions.

You will see all of them will be visible there. So, for an system in an abstract state psi we can work out this expectation value for a position operator. Either in this wave function formalism or here and both are equivalent, This is exactly=evaluating psi star x x psi of x which you have done in your first year course. So, once we have operators and expectation values, we would also like to see what is the uncertainty in the evaluation.

It is nothing similar to your standard deviation right so how do you define that? you find you define uncertainty in the position is find the expectation value of the square of the position operator subtract out the expectation value of x and then take the square. Like standard deviation standard deviation you have to do it on the expectation values. See you have done with the square root thank you yeah there should be a square root, or you can write square delta x.

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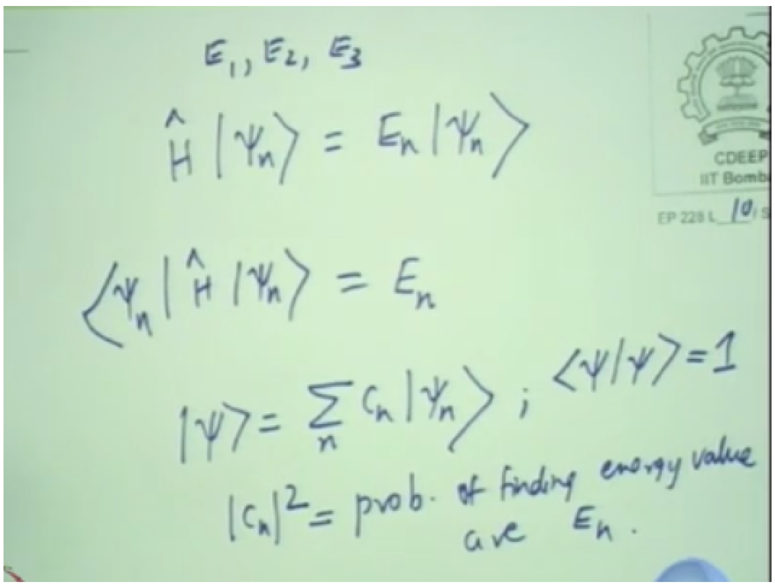

Okay I want to spend some time on this measurement issues if you want to measure position if you what to measure energy you know given a state to you how do you know what is the energy of that state for example if it is a stationery state you operate a Hermitian operator, or which is equivalent to finding an expectation value of Hermitian by Ehrenfest theorem it should be=to the energy or average energy.

But if the state is an eigen state then it would have actually give you the exact energy. So, Hermitian suppose its operating upon some psi n is en times psi n and you state is in our psi n this is the expectation value of psi n what will this be gives you En. So, if state prepared to be a perfect eigen state if you try to measure the energy of that state or try to find the average energy of the system prepared in that state both will be same giving you some energy eigen value.

But wat exactly you are doing in expectation values for a super postulates. Suppose I have a super position cn on psi n what exactly are you doing here. When you try to do a measurement what happens system collapse to one particular which state, we do not know any of these. Suppose if I tell you the system has these stationery states possible, I know some value n it will collapse right.

But how will I determine cn you take the same identically prepared system. Let us take 100 of them in each of that identically prepared system. Simultaneously you do the measurement and find suppose take the ten rows each ow is doing measurement on our identical system you same as the energy somebody says E2 as energy and so on. If it infinite I can find out how many of you got e1 how many of you got E2 and I can take the average.

So that mode cn square cn squared is the probability of finding energy value as En. Okay since we also take it to be normalized the state psi is normalized what does it mean?=1. So, this is what for a specific energy when you have infinitely identically prepared system when you try to do a simultaneous measurement each system you will have different energies. Somebody will get something but if you take that.

And see tabulate how many of these systems got e1 how many then you can actually find mod cn squared. Okay so this is what we get information from a state vector and we find this mod cn squared by doing this on an identically prepared system. So, measurement when we do where I take the state one person when he does the measurement with that table, he is doing a measurement he does not know whether he will get E1 or E2 or E3 or something.

But it will definitely be in that set. If the system gas stationery states with energy E1 E2 E3 when he does some measurement, he will not get 1.1 times E1 okay or he will not he will get E1 or E2 you cannot get something intermediate in between that he will not measure he will measure only one of the stationery state energy eigen value which stationery state eigen value you will measure nobody knows he will either get E1 E2 or E3 and so on.

But he will not get somewhere in between you will not get something like E1+.01 or something you would not get that that is for sure but which one he will get he himself will not know that is where the probability gets into the picture. Tat is very intense nothing you can do. so, measurement by an observer on a state psi will make it go into one of the stationery state this is what we call it as collapse of the wave function.

And you will have an energy which you cannot predict what it will be. But it will be one almost that set of the stationery eigen value so if you take for measurement take n identically prepared system.

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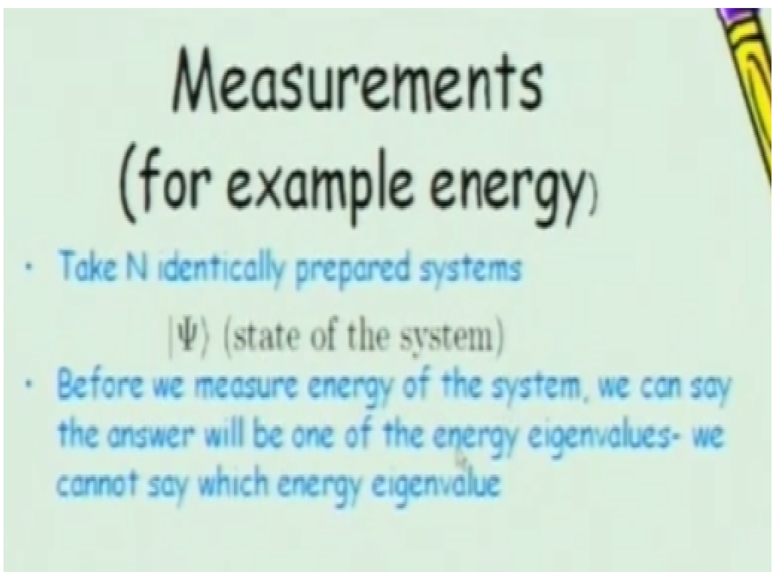

Psi is the state of the system before we measure the energy of the system, we can only say it will be one of the set of a loud energy eigen value you cannot say which energy eigen value. So, once you do a measurement the state collapse to one of the energy eigen values energy eigen states.

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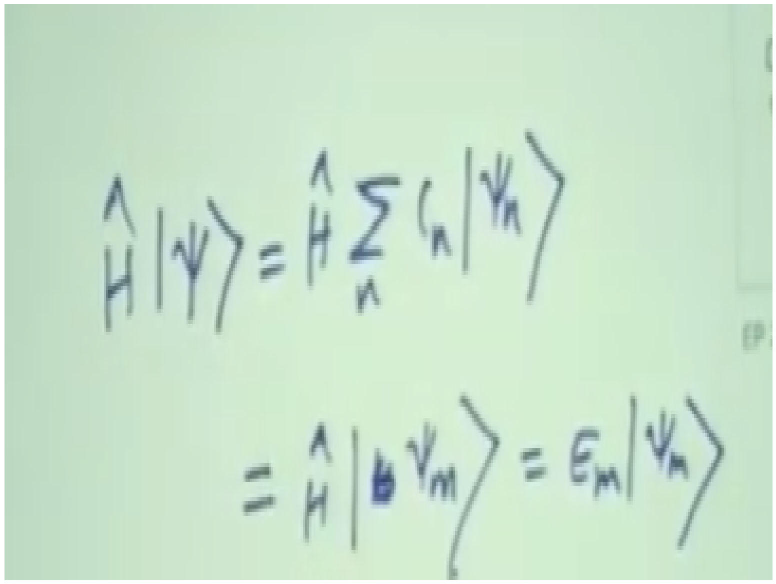

So, when you do this measurement when I do this Hermitian operator on psi which is Hermitian operator on summation /n cn psi n what I will get is I will get some specific em state so psi m state this is what you will get and after tis when we start measuring Hermitian on this it will give energy Em on psi. It will remain in that stationery state after that an observer who is trying to measure an arbitrary state will see that the state psi will collapse.

To some stationery state which stationery state he does not know when he is doing the measurement. It is not a specific m one of the set of this. After he has it will remain in that state throughout it will remain as psi m again if he does the measurement on it, he will not get a new energy he will get the same energy is that clear. So, the measurement on a system leads to a collapse of psi to psi m a specific eigen state and it will have an energy which is measured.

After that if we again repeat to measure the energy again it will remain in that state, then you can have a predictive answer takes the super prostate and does the measurement only measure he does not know what is the energy even before measurement he cannot guess but he gets it to collapse to a specific stationery state which is an energy eigen state. After it collapse if we try to measure again and again.

He will get the same energy value so as long as the system is undisturbed. What do I mean by that I cannot make the stationery state to undergo some other transformation I do that then it may again go to a super postulate As long as I do not disturb the system, I keep it after collapse it will remain in that collapsed state which is one of the eigen states of that operator. Okay let me stop here.