Quantum Mechanics
Prof. P. Ramadevi
Department of Physics
Indian Institute of Technology- Bombay

Lecture - 21
Postulates of Quantum Mechanics - I

From the context of Quantum Physics let me explain and motivate you and how you can see Fourier transform so the way you do if you have a de Broglie way.

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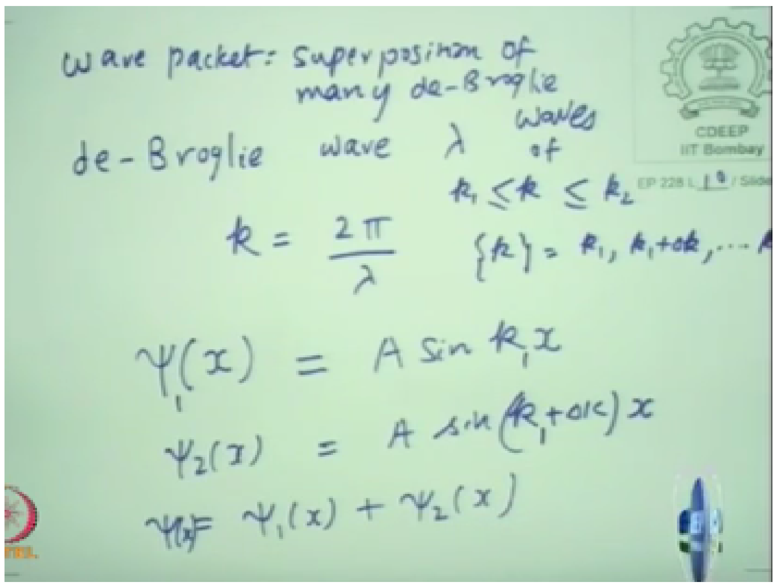

Wave of wave length lambda which means you can write wave number as 2 Pi/ lambda then you could write the wave function for this de Broglie wave as A sin K x. This I am sure all of you have seen yes or no it is like a electromagnetic wave with a specific wave length you could write this if you have time dependence you can put as K x – omega T let me just take Psi of X and you can do super position of two ways.

You can have Psi as K1 and you have Psi 2 of x as A sorry some other amplitude may be your same amplitude sin K2 let me take K2 as K1 + delta K X so what will this super position be it will give you a new super post wave which you can write it as Psi as Psi 1 of x + Psi 2 of x which is super post state can do that so when you have only 1 de Broglie wave it has a definite way length.

If you start adding 1 more wave then you will start seeing that there will be some kind of a super post state but not definite wave length but with some width which is proportional to your delta T we have done this as 1 of the assignment problems. If you super post many of them what is that call super post wave function or the super post wave I have just done 2 of them 2 de Broglie wave with definite wave lengths.

But if I have many of them let us take K1+2 Delta K 1 + 2 3 delta K and so on what are you supposed to get wave packet right wave packet is superposition of many de Broglie waves you agree of different let us take off K line between K1 to K2 in steps of delta K K1+ K1+ delta K so the list of Ks you can have is K1 K1+ delta K dot dot dot you go on till K2 so that is the wave packet super position of all possible de Broglie waves.

And this wave packet when we start doing what happens you lose a specific wave length for the wave packet de Broglie wave has a specific wave length a single de Broglie wave has a specific wave length wave number or wave length but if you start super posing many de Broglie waves the information about the wave length is gone all agree but what happens wave packet is very close to particle which means you will start seeing the amplitude to be maximum in real if I put it pictorially for you.

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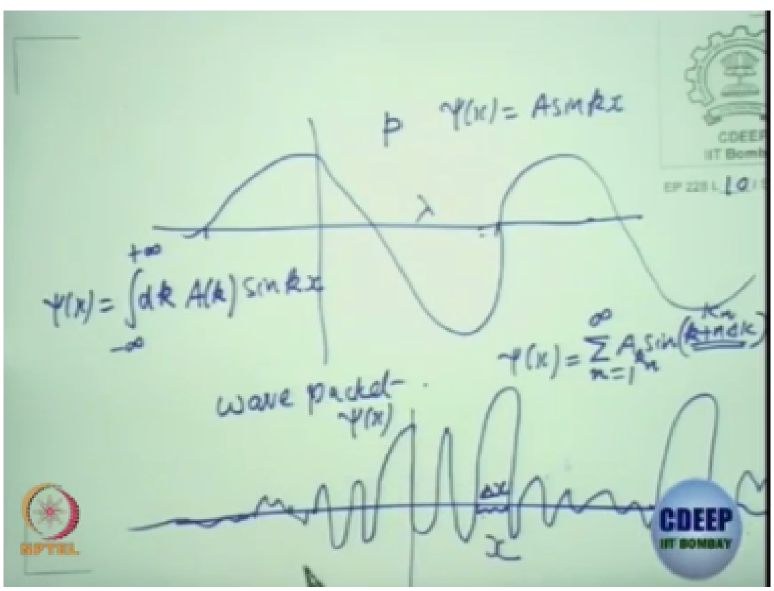

So de Broglie wave will be like this keeps going let me not stop it keeps going with a definite wave length a single de Broglie wave but how does the super pose wave going to be it is going to have some kind of a peak and then the amplitude dampens down right something like this. I am not good at drawing something again it goes or it may just damp down also not really drawing to scale.

But you can do this but then the wave length for such a this is what we call it as a super position of all possible de Broglie waves which will give you a wave packet suppose it damps down everywhere there are peaks only in this region okay let us say it damps down everywhere then this is the region in which you have maximum amplitude for the super post Psi of X verses X so what does it tell you.

The probability of finding the particle is maximum only in the small region which means you are able to localize it is not here when I want to find where is the particle it can be anywhere so that there is a periodicity at periodicity will go and it will damp down when you take a super post wave so you get a localization here for a wave packet whereas here you get a specific wave length.

Here you do not get a specific wave length you all agree so specific wave length means what by de Broglie hypothesis specific p so which means momentum is well defined but you cannot localize the particle here if it is a single de Broglie wave but if it is a super position of all possible waves then the amplitude is maximum in a small short region you can make it even more shorter by taking all possible wave lengths.

From wave numbers from - infinity to + infinity so you can take that if you do that it can get more and more this region which I call it as a delta X this region could get more and more shorter and you can get actually say this particle is localize here but its momentum is not its position is well defined but momentum is not defined correct so this is the main theme in what you can also be super position as Fourier transform.

So, for a single wave I wrote Psi of x as A sin K x if I want to write the wave packet and let us take K to be going to some - infinity to + infinity in the steps which are very small and delta k is very small formally you could write it as summation over n A sin K + n delta K you can take 1 step from 1 to infinity or something to do this but if I want to take delta k to be really very small what can I do.

I can put an integration this summation is not the discrete steps of the wave number K shifted by K+ of you have finite 2 de Broglie waves to super pose with K1 and K2 you can do the summation but if I say that if I want to take all the de Broglie waves with a continuum wave number then what is the next 1 you will do you have to put it to be a integration you all agree with that so how will you do the integration.

So, Psi of x formally I can write as it integral over d k then you can have even this amplitude can be a function of Kn and if you want to keep a subscript of this whole thing I could have called it as Kn so you could have an amplitude which is depending on K and you can have a sin Kx going from some limit if you want you can go from - infinity to + infinity so this what we have been doing.

But we are doing little bit more we want to keep the sin K x formally may not be the general we will have a cos K x so I said we could try and rewrite this as e to the power of ikx and AK so what is AK to be interpreted here suppose I replace this as e to the power ikx.

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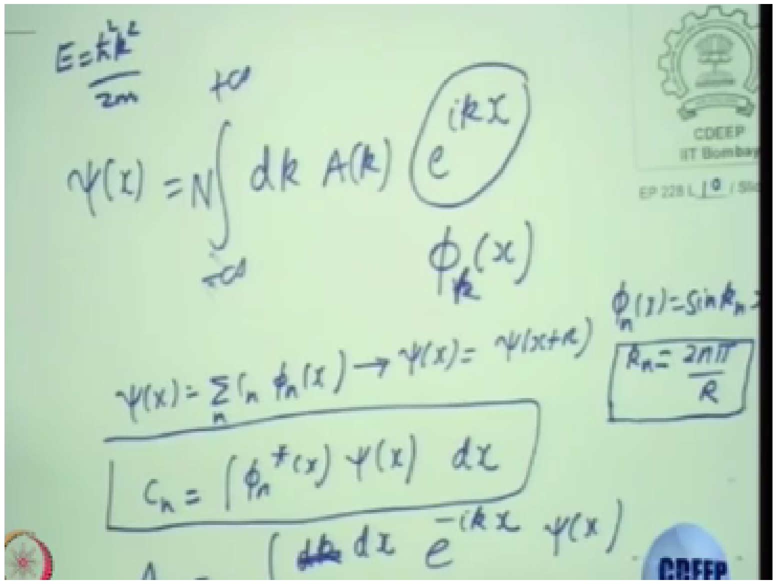

So, let me rewrite it again here sum normalisation we not need to worry about normalisation this is nothing but your familiar Fourier transform where Fourier transform A of K is Fourier transform of Psi of x in the electromagnetic waves you know what is super position you do look at beads and you do look at stationeries waves all these things I am sure you have done it but it is not very different from de Broglie wave is a matter wave.

Only thing is because it is a matter wave we always think that we will be able to locate the particle so to locate a particle and to look at a particle velocity which is a wave packet velocity we need to find out the omega by dk so these things look at your wave packet and the wave packet if you have super position of all possible waves and suppose you denote this wave by an exponential like ikx.

This is nothing but your Fourier transform what will be the inverse Fourier transform the next question you can ask? So, this one I told you in the function space basis function these are like Phi of X Phi k of X if it was a discrete sum suppose you did a discrete sum how will you find out what is A of K when we had Psi of X as a summation over n Cn Phi n of X what did you do C n was you integrate both sides Phi n star of X Psi of X integration is over dx.

This is what you did now what will you do here you have to take the Phi case star of x on this side and that will give you the co-efficient A of K so what will of A of K be A of k will be integral over d k sorry it would not be dk dx this is what you are going to re do in a continuum the integral over dx Phi k star of x is what? e to the power of –ikx how –ikx comes also you can see this is the inverse Fourier transform times Psi of X take the free particle.

The free particle will have energy to be h cross squared k squared over to 2m which is p squared over to m you can replace your k as 2me by x cross this K and you can start doing this that problem where it is involving Fourier transform go from the k space to the x space. Now you can go back and look at it nothing but it is a using a concept of Fourier transform which is the inverse of Fourier transform which you can also visually see it as wave packet.

Which is the super position of (()) (14.05) this is why the continuum comes this discretising which we do is only when there is a periodicity so this thing which I wrote here this will have a condition like Psi of X Psi of x +R in continuum you do not have that condition so that is why you will start seeing Psi n of x which you can write it as sin kn of x so you can write Phi n of x as sin kn of x.

For example x = x+r it should be the same will force Kn to be 2n Pi/ R you agree so this is why the periodicity condition discretises you momentum or the wave vector so if you put a particle on a circle of perimeter let us say 2 Pi R when you finish 2 Pi R and when you come back it is a same point you cannot have two different wave functions at the same point it is x + 2Pi r and it is better be same so that condition single valueiness of some periodic functions.

Will allow your wave vector to be discretized and if it is a free particle going from – infinity to + infinity there is no periodicity on k so that will force you to do this integrate you should know when to use discrete when there is periodic function or you put particle constraint on some region which some boundary conditions like this otherwise you will have k to be a continuum m and k is a continuum means you have to put an integration.

And there is no meaning in talking about summation so that is about Fourier transform inverse Fourier transform so let me get on to todays lecture where I want to formally introduce Postulates of quantum mechanics.

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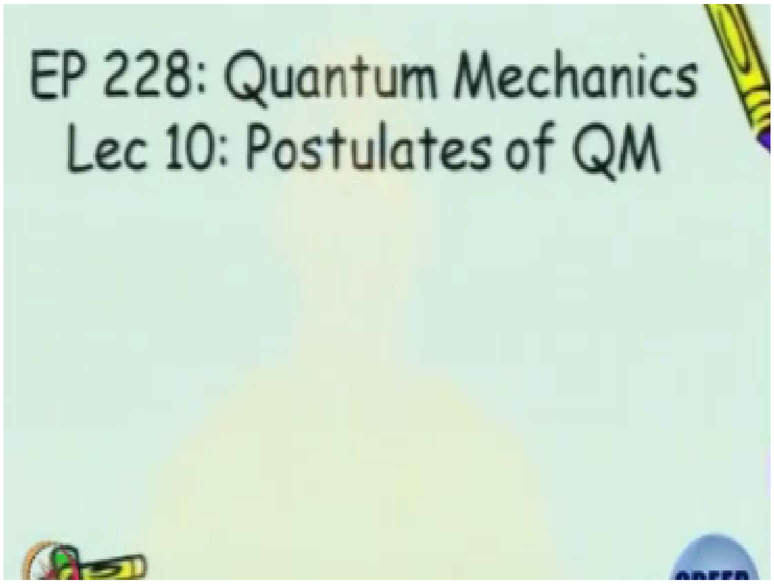

So, before I go what all we have done square integral functions.

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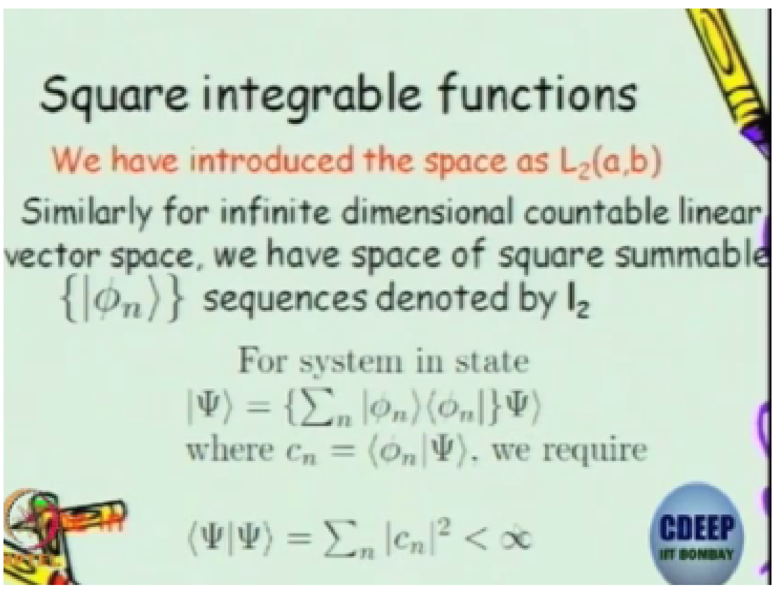

Which we denote it as L2 of a , b this we have done then I have also introduced for you square summable sequence collection of square summable sequence and that we denote it by little l2 a collection of this set Phi n which 1 is set is the sequence and that we denote it as little l okay and we will also seen that in such a l2 space we can write an arbitrary state by inserting this paranthesis which is a unit operatory.

It is just a unit operator operating on that state but you can reinterpret it as looking at this inner product of the state as a probability amplitude which we denote it as Cn and then the state should have a finite norm any arbitrary state forces that these Cn co-efficient should satisfy this property this is the condition for square summable sequences okay so slowly we need to get many times you will hear this word that the vectors space is always called as a Hilbert space.

So, just to spend few minutes on what is a definition of the Hilbert space and all the quantum mechanics problems assumes that we work in a special vector space linear vector space which is called as a Hilbert space. So, first of all the square summable sequence is a Cauchy sequence.

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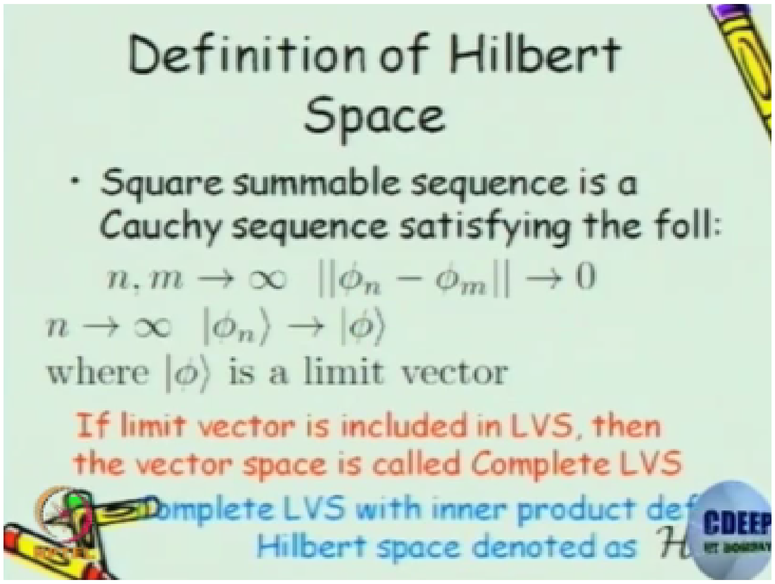

If it obeys certain properties you need to take n and m very large and if you take difference between these 2 vectors and take the norm of these two vectors it should tend to 0 similarly in that sequence when n is very large goes to sum finite values and then we call this state Phi to which it goes that is a limit way many times we are going to assume that whatever linear space vectors we are looking at is actually a vector space which has its limit vector.

And we call such a vector space at least technically it is called as a complete linear vectors space so these are some of the definition which goes in to the formal definition but many of the times we say we work in Hilbert space but I thought at least as quantum mechanics of course you should know that when do you call it as a complete vector of space if the limit vector is included in the linear vector space it is called as a complete vector space.

When do you call it as a Hilbert space not all linear vector space you should have an inner product to define that can be spaces where you do not have an inner product like really you do not need a linear product if you show me the equation you find solutions or any arbitrary equations you find a set of solution super position is also a solution but you cannot find what is dual solution to them?

So, some of them may not even attribute thinking about your products if you have the inner product also define then this complete vector space is called as a Hilbert space and in most of the text books and most of the literature they put this curly edge so essentially doing linear vector space but some of these things are assumed that your linear vector space with this limiting vector in that sequence finite summable sequence is also included.

And you have an inner product definition in that vector space that is why this whole thing together is what we called it as a Hilbert space this is for the definition why we called it as a Hilbert space okay we also briefly went through what is a operator for observables in the last lecture.

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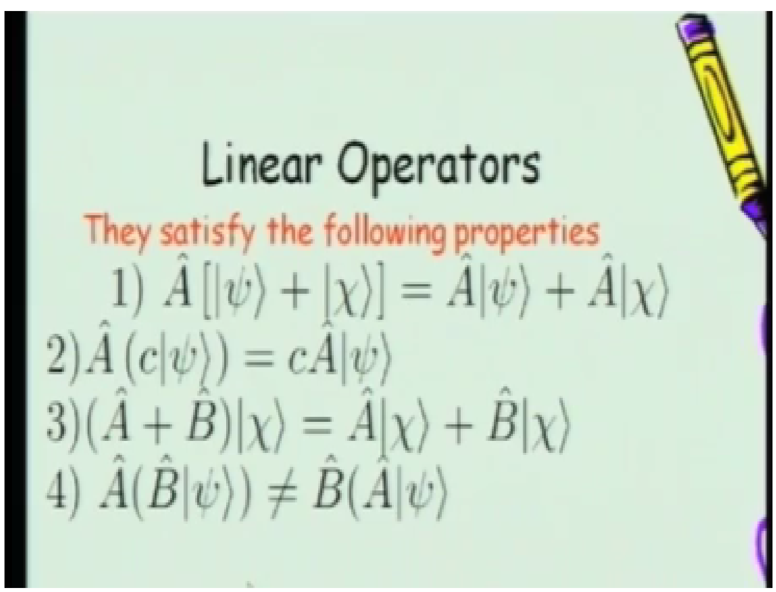

So I just thought you should also know what is the definition in this operators operate on vectors in the Hilbert space now and these operators are called linear operators if it satisfies that if you take a linear combination of 2 states and have a operator A then you can separate it out and so this is one of the properties the second property is that the complex scalar multiplying sign you can take the operator to operate on that state Psi.

And you can bring the complex number and you can also have sum of 2 operators on a arbitrary state can be written as operator on each state summed off and if you have 2 operators product earlier was the sum if you have a product when will this be = only when AB is same as BA so in general 2 operators AB may not be same as BA so in general AB operator on Psi will give you a different answer compare to BA operator on Psi.

So, the order matters end quantum mechanics all the operators when I write have to be very careful with the order.

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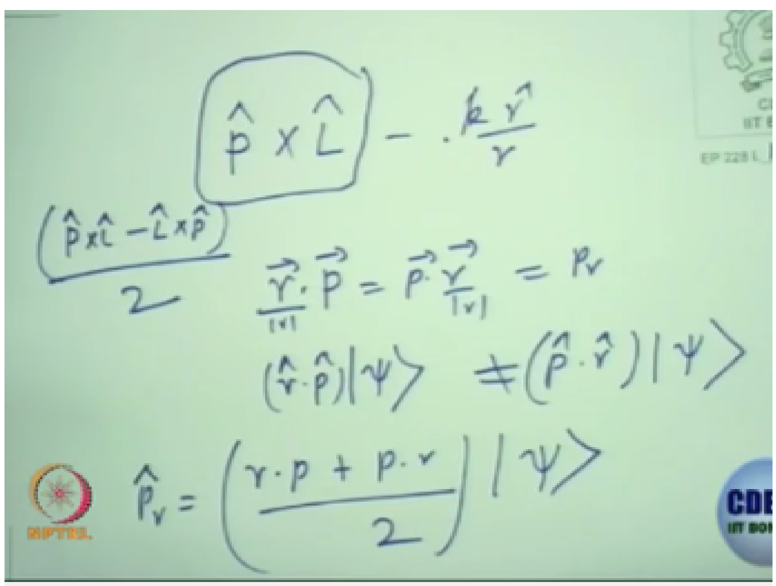

For example if you have a P cross L can have a operator like this can also have something like r.p classically we are allowed to do this r.p is same as p.r but quantum mechanically r.p operators operating on state Psi is it same do not take it as a unit vector some hard work is it same? It is not same as p.r on Psi what you have to do if I want to and what this one is called I have a unit vector r by mode r let me put it.

If I have a unit vector r vector dot reflect and if I have a unit vector what is this operator this is the radial component of your momentum so radial component of the momentum whether you write it as r.p this one or p.r it does not matter in classical mechanic but in quantum mechanics we have to be careful how do you make it careful I do not want this distinction on PR but here I see the order matters.

I have to do it very cleverly and how do I do that try to take a symmetric combination for this case you can take r operator p + p.r / 2 so this is also your Pr in classical mechanics by positive and not negative if I put a negative what will happen becomes 0 in classical mechanics what about here classical mechanics how many of you seen Lagrangian vector that has the P cross l that is Lagrangian vector component.

Actually there is a 1 over r that is Kr vector/r this is a rangelence vector how will you do the quantum mechanic combination for this piece will you take a + sign not a – sign this is same as - of L cross P classical mechanics how do you do it quantum mechanics operator P operator should be this you have to symmetrise for a A.B operator but if there is a A cross B operator what it will be you need to take you agree.

So, this is what we need to also remember whenever we do quantum mechanics you cannot just take your observable whatever differential operator but if there are more operators you need to make sure you want to make contact with some physical reality should I have to symmetrise should I have to anti - symmetrise so that is why I was saying general the 2 operators in classical mechanism you could have said r.p is same as p.r like that but I cannot do that.

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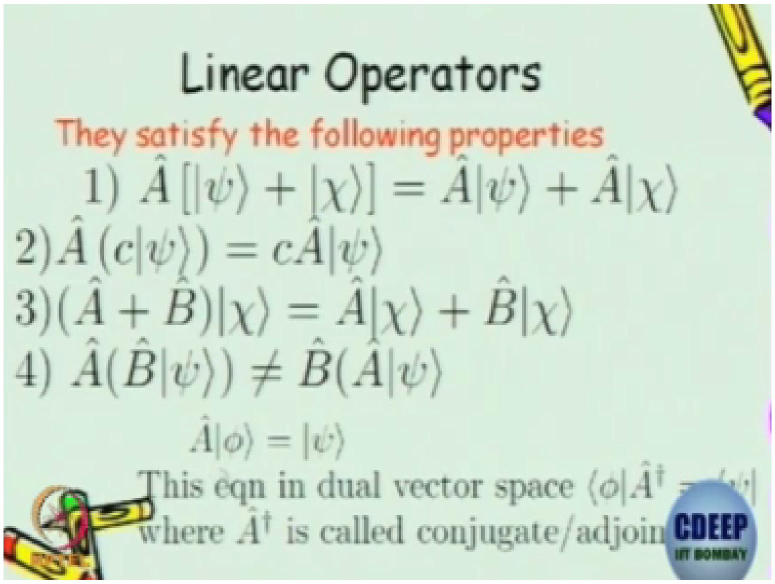

Okay this also we saw in the last lecture if you have an A operator operating on a arbitrary state Phi gives you a new state Psi then in the dual vector space you need to replace it by the complex conjugate and transpose if it is a matrix representation or we called this as a adjoint operator A dagger and that gives you the new state so a dagger is what we call it as a conjugate of the adjoint of the A clear kind of summarizing whatever you have seen in the last lecture.