Quantum Mechanics
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Lecture - 18
Tutorial 3 - Part II

So in tutorial 3, we have seen 2 problems and now we are going to continue with the remaining problems of this tutorial. So we will start with the third problem now. So third problem is an exercise where in you have to express the operator.

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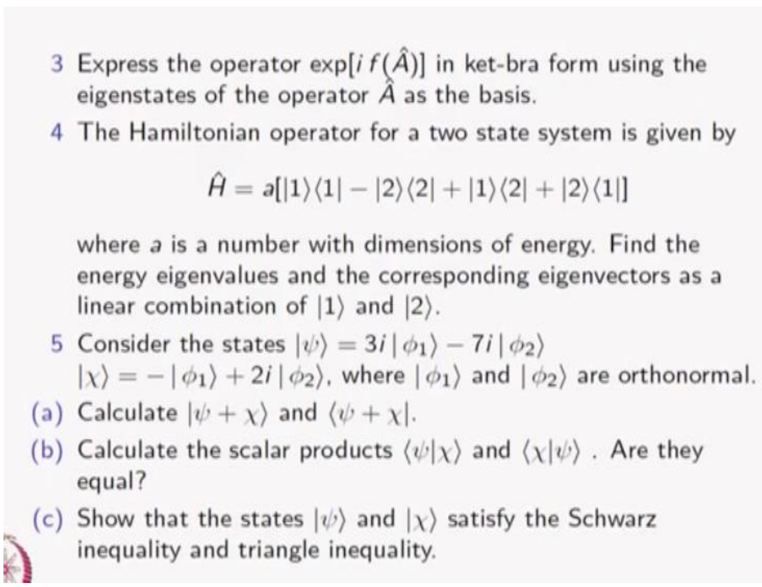

Which is given in terms of the exponential $e^{if(\hat{A})}$ in brand ket notation forming, form using the eigenstate of the operator $\hat{A}$ on that, on the basis. So this problem is very simple. I can give you a hint of this problem and you can do it. So first of all we assume problem 3. In this problem what we do is we assume.

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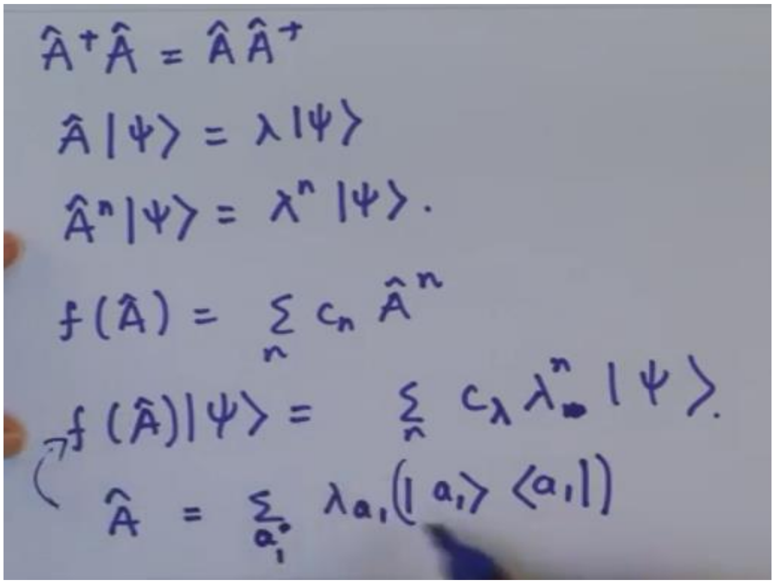

We assume that this operator is normal that is $\hat{A}^{\dagger}\hat{A} = \hat{A}\hat{A}^{\dagger}$, and when you operate this operator on a wave function, it will give you some eigenvalue λ on ψ. So if this operator is operated n number of times, what one obtain is $\hat{A}^{n}|\psi >= \lambda^{n}|\psi >$, okay. So now this operator can be written in terms of a series, power series. So f($\hat{A}$) can be written as $\sum_{n} c_n \hat{A}^{n}$. So n times we have operated.

So we can write this operator in terms of power series and this will give you the corresponding terms of the function of f($\hat{A}$). So now if this operator is operated on the wave function f($\hat{A}$) is operated on the wave function, one would obtain the eigenvalues. So we will write c λ in terms of c_n, so oh sorry, this is what we obtain okay. So in terms, so this function in terms of the eigenvalues. So these would correspond to the eigenvalues.

So now this operator if I want to write in terms of some operator in terms of the eigenstate as the basis. So we can write this operator as some A prime, let me call $\hat{A}'$ as the eigen in terms of the projection operator. So let me call $\hat{A}'$ or $\hat{a}_1$ as some state. So $\hat{a}_1$ but when you project out, when you do the apply the projection operator, we obtain the eigenvalue of operator $\hat{A}$ and similarly for $\hat{A}^{n}$, we can write in terms of n.

So the same thing which I just now demonstrated here and then this can be easily expressed in terms of, so f($\hat{A}$).

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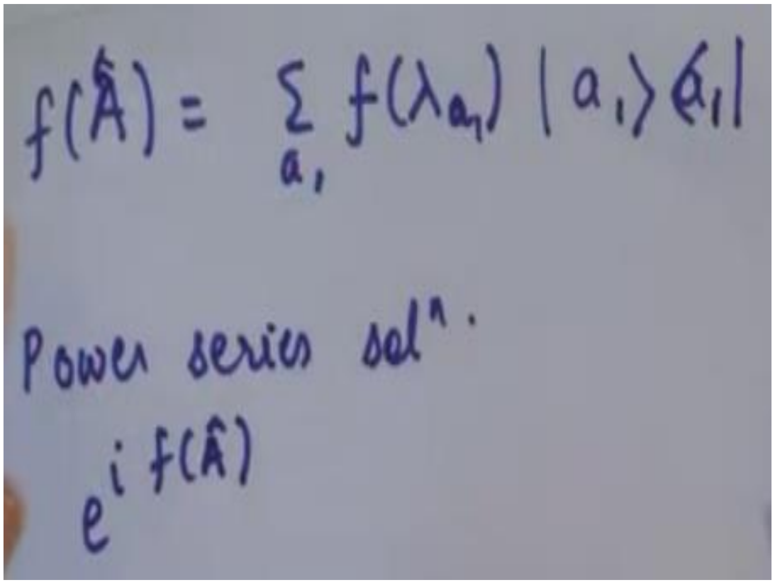

You can see now directly that f($\hat{A}$) can be expressed in terms of the function of the eigenvalue, function of the operator is expressed in terms of the function of the eigenvalue and the projection operator of the spaces okay. Similarly, 1 can go and write it in terms of the power series solution. This can be written as power series solution okay. That is you take this and as an exponent of that is $e^{if(\hat{A}')}$, can be expressed in terms of these basis.

So simply you can rewrite in terms of the eigenvectors in terms of this basis. So the operator on this basis and now we go to the next problem. So in this problem a Hamiltonian operator is given to you for 2 systems and you already have an exposure to what does this mean?

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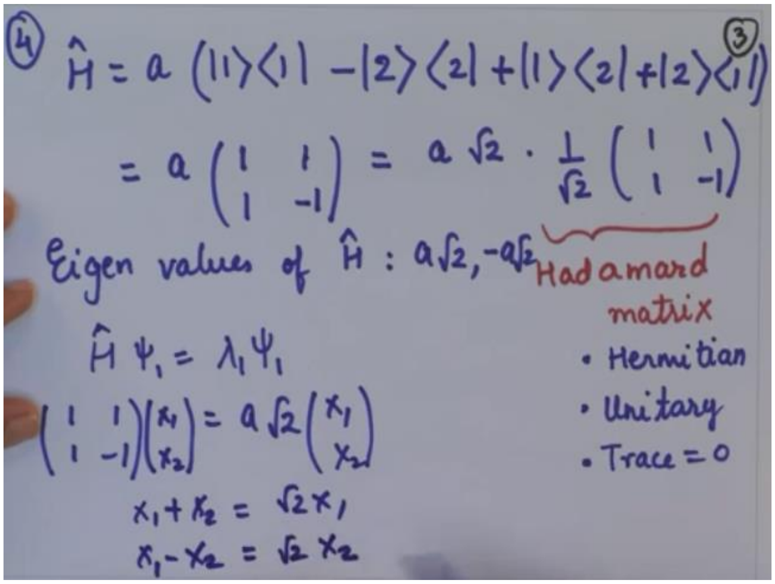

If you have, if the Hamiltonian is written in terms of these eigenvectors, I think you all know now by now that this would mean that you can write this operator Hamiltonian operator as |1>< 1| means this when I am writing in terms of matrix. This is a bra-ket notation. So here I am writing in terms of the matrix. So this would give me |2>< 2| is -1. This is +1 +1. So simply I can write it like this okay. Now this if I rewrite as 1/√2 $\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$ okay.

So this is nothing but your, this is nothing but I think you have already come across this Hadamard matrix okay. So now we know that what are the properties of this matrix. First it is Hermitian, then Unitary, it is a Unitary matrix. It is Hermitian and you can see from you it has a 0 trace. So trace is 0 okay. So these are the 3 properties of Hadamard matrix and now you can simply you.

Since you know the eigenvalues of this matrix, which is 1 -1, you can obtain easily the eigenvalues of operator H would simply be what it will be for 1 -1 is for the Hadamard matrix. So it will be a $\sqrt{2}$ and -a $\sqrt{2}$. So these are the 2 eigenvalues. So using these eigenvalues you can easily calculate the eigenvector. One of the common way a trivial way to calculate it is by substituting $H|\psi_1 >= \lambda_1|\psi_1 >$, where λ_1 is the eigenvalue of this operator.

So we have 2 eigenvalues λ_1 is a$\sqrt{2}$. Suppose I take λ_1 is a$\sqrt{2}$ and λ_2 is - a$\sqrt{2}$ okay. So just one example I will just demonstrate. So H is nothing but $\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$ and let me call this column vector as some $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, okay which I need to calculate this is a$\sqrt{2}$ $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ okay and now it is very simple. You just have to evaluate this. So $x_1 + x_2$will be $\sqrt{2}x_1$, $x_1 - x_2$ will be $\sqrt{2}\, x_2$.

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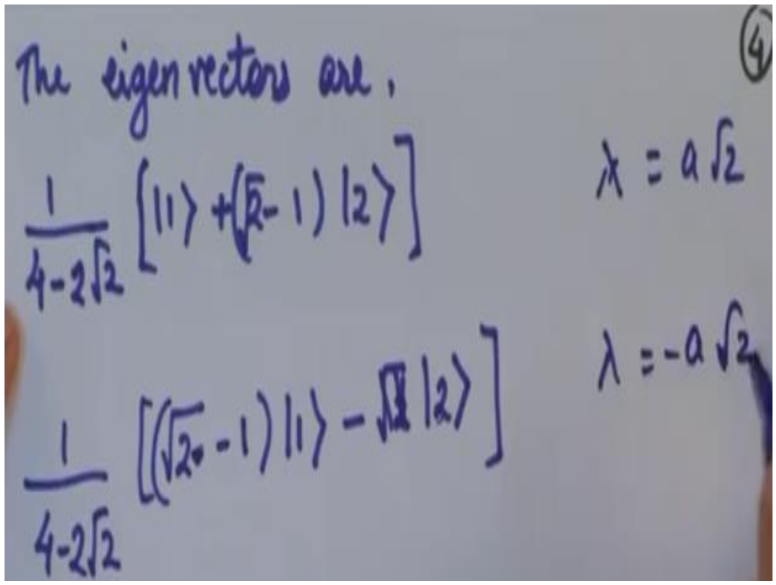

Eigenvectors are, so the first one which I just now demonstrated will give you the result okay. This is very simple to check. So this is the first eigenvector and the second one you obtain it is just a few steps you have to calculate and then you obtain this (√2 -1) |1> - √2 , sorry – |2>, not √2 . So the first one is this normalization |1>+(√2 -1) |2> and the second vector eigenvector will be square (√2 -1) on |1> – |2>.

So this is the eigenvectors corresponding to the eigenvalue. This so the eigenvalue for this was a √2 and for this it was –a √2 . This you can easily check. So once you obtain the result from the previous this part, this part when you simplify this, you will obtain an eigenvector you rewrite it in terms of |1> and |2> okay. Go through this problem 5. Consider the state |ψ> as 3i $|\phi_1>$ -7i ϕ_2 and |χ> is given by this expression.

That is –$|\phi_1>$ + 2i $|\phi_2>$ where these kets $|\phi_1>$ and $|\phi_2>$ are orthonormal. They are orthonormal and you have to calculate now 3 things, one is you have to calculate | ψ + χ> and bra <χ + ψ|, <ψ + χ|, then in the B part you have to calculate the scalar product of <ψ| χ> and <χ | ψ> and check whether they are equal or not. So from B part we will discuss, B and C part we will discuss later.

So C part says that you have to find show that the state| ψ> and |χ> satisfied Schwarz inequality and triangle inequality. So you have already seen these inequalities in your regular lecture this triangle inequality last time we discussed. So just recollect and just check this is a numerical check. So you can just try out this.

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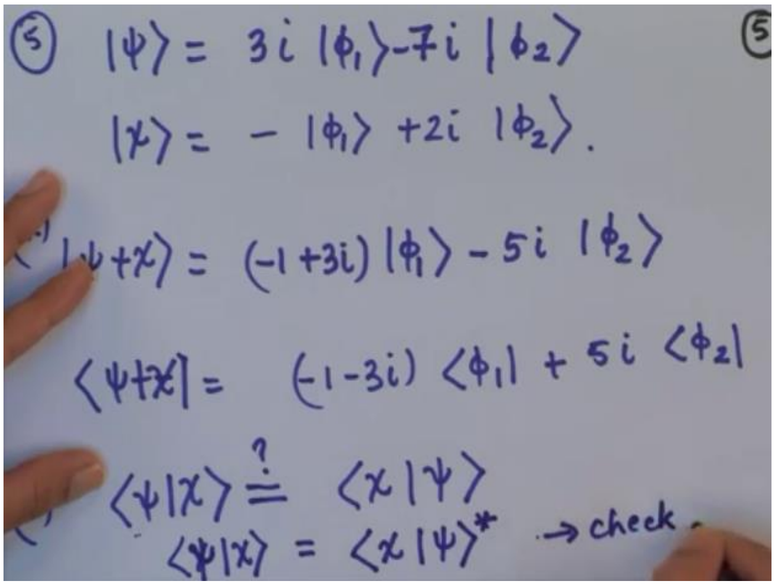

So let me start with the first part |ψ> is given to you as 3i $|\phi_1>$- 7i $|\phi_2>$, right and |χ> is given to you as -$|\phi_1>$ + 2i $|\phi_2>$ okay. It is very simple. It is just a simple addition, simple exercise. So this will give me, this is very simple. It will give me 1+3i $|\phi_1>$ okay. I am adding the kets and this is 7i -7i and this is +2i. So simply -5i $|\phi_2>$ okay. So this is very simple, when you are adding the 2 kets, you can add them okay.

You are using the property of, you can add this linearly and you obtain this result now you have to find out the <ψ+χ|. So simply what would you do is just take the complex conjugate of this, okay. You just take the complex conjugate, so you will have what you get is -1 -3i $|\phi_1>$, correct + $5i|\phi_2>$ okay. It is very simple to check this okay. This was the A part right. In the B part, you can attempt this problem.

You can simply attack it by just putting the < ψ|χ> and simply by and you know that $|\phi_1>$ and $|\phi_2>$ orthonormal. That means $<\phi_1|\phi_1>$ will give me 1 and $<\phi_1|\phi_2>$ will give me 0, simply that. That is a hint. So you have to check whether this and this are equal. So what is the guess? The guess would be that <χ| ψ> on χ that is the product ψ and χ will be nothing but so check this whether you get.

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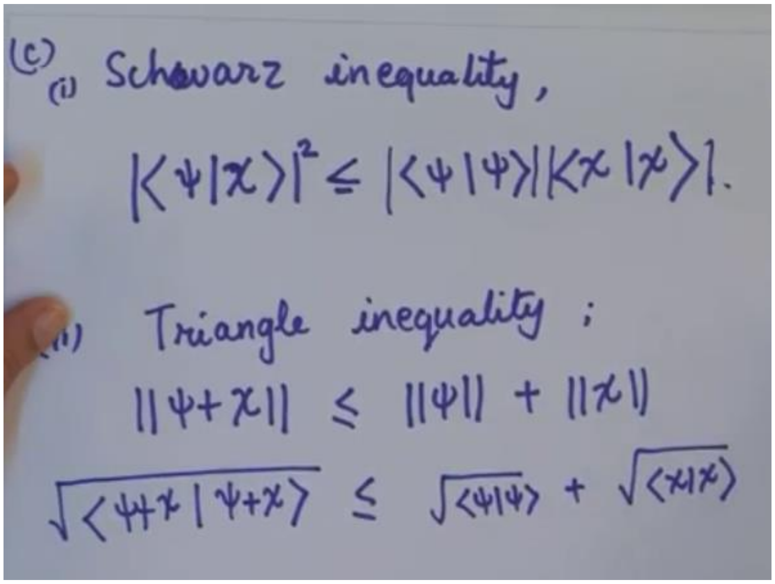

Recall Schwarz inequality would be you have to see whether this $|\langle\psi|\chi\rangle|^2 \le |\langle\psi|\psi\rangle||\langle\chi|\chi\rangle|$. This is very simple to check. You have to calculate the ||ψ|| okay. So the norm of ψ will be, rather you calculate the eigen, the ψ square okay. ||ψ|| will be square root of this. So basically we are not calculating the square root of this so or you can just make it okay.

So you have to check whether the $|\langle\psi|\chi\rangle|^2$.is equal to or less than or equal to this. This can be solved by just calculating the $|\langle\psi|\psi\rangle||\langle x|x\rangle|$ okay, which is few steps and then χ on ψ, which you have already calculated in the previous example. So whether they are equal. So this is 1 part and in the second part what we do is we have to check the triangle inequality.

So triangle inequality would be, we have seen it last time that you had checked that norm of ||ψ+χ ||should be less than or equal to || ψ|| + ||χ||. This is the triangle inequality we have seen last time. In the last tutorial we have proved this. So which is nothing but here it would be $\sqrt{\langle\psi+\chi|\psi+\chi\rangle} \le \sqrt{<\psi|\psi>} + \sqrt{\langle\chi|\chi\rangle}$. It is again you have already calculated in the previous problem <ψ|ψ> square and <χ| χ> square.

So |χ> square you have calculated, |ψ> square you have calculated. This part you have calculated in the first part of the problem. You have to just take the bra-ket and do the bra-ket multiplication and take the square root. So it is, it will involve few computation. I am deliberately skipping this, so that you do it yourself, try and calculate it yourself as an exercise. So such problems would be very useful and in further tutorials, we will see more problems.

More physics based problems, these were few math based problem except fourth, third and the fourth problem. So this was just to give you an exposure to the bra-ket notation. So you have to get used to with this bra-ket notation. It is very useful and you will see it has a, it is very useful for doing the calculation. So you must see to it that these notations, when you do these calculations, these notations are very useful and you need to have your hands on these expressions I mean these calculations.

So try these problems yourself with the hint that I have given you in this tutorial. We will continue next time.