Quantum Mechanics
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Lecture – 11
Tutorial 2

Okay, so in last few lectures you have already seen problems based on double delta potential, some problems are on harmonic oscillator, hermite polynomial, so we will be doing such problem based on hermite polynomials, few problems we did like a particle in one-dimension potential well and some in the lecture, some problems or some discussion was also taking place regarding particle in three dimensional box.

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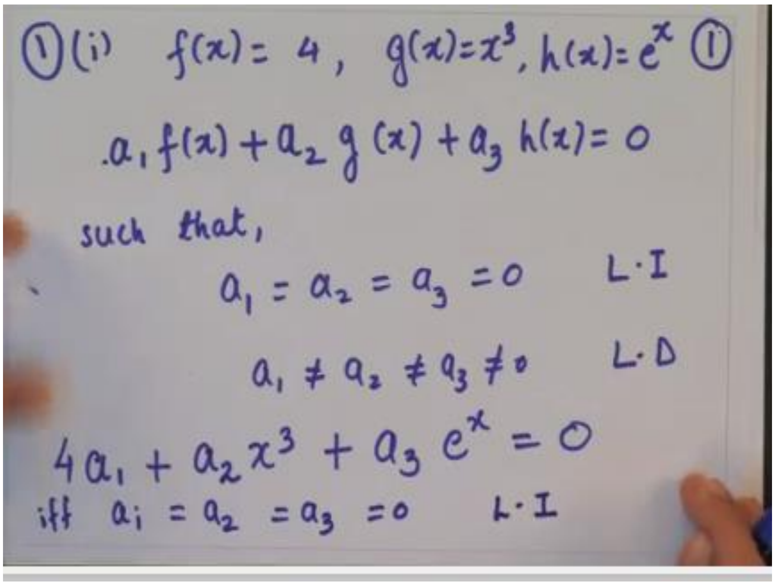

So, we will start by a very simple exercise, so the first problem is very simple and you can just to recap, we will start solving the first set which has 3 subsets.

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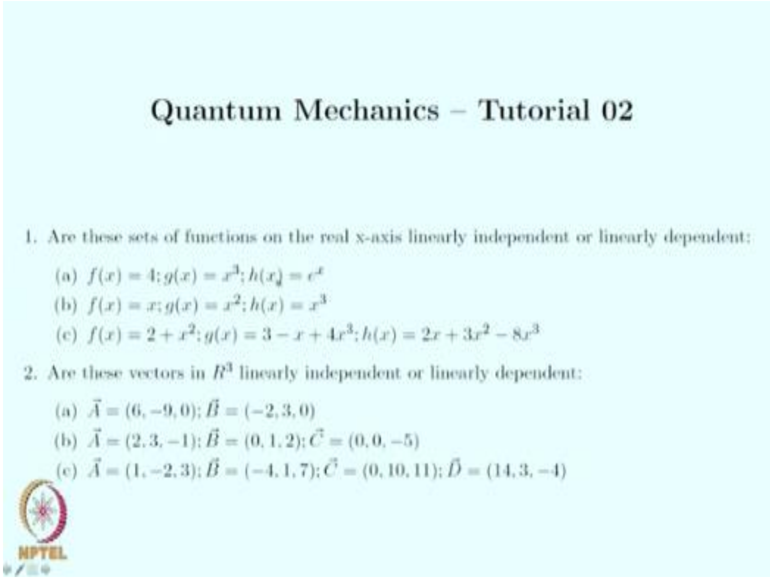

And you have to find out in this problem that these sets are or these functions are linearly independent or linearly dependent, so from the first exercise first problem, 3 functions are given to you, f(x) is 4, g(x) is x^3 and h(x) is e^x, so in order to find whether the functions or the set of functions are linearly independent or linearly dependent, the condition that this these functions must satisfy is a_1 f(x); f(x), a_2 g(x) + a_3 h(x) should be = 0 such that we have a_1 = a_2 = a_3 = 0.

Then, we call this set as linearly independent, if a_1, a_2 and a_3 are ≠ 0, then we have a set of functions which are linearly dependent, so if a_1 is ≠ a_2 is ≠ a_3 is ≠ 0, I mean to say a_1 is ≠ 0, a_2 is ≠ 0 and a_3 is ≠ 0 then these functions or these set of functions are linearly dependent. So, the first example; from the first example you can easily see that if I substitute these functions, I have a1 times 4.

So, $4a_1$ + a_2 x^3 + $a_3 e^x$ = 0, so this is 0 only if a_1 = a_2 = a_3 = 0, so this set of function are linearly independent okay. So, in the second problem you will see that it is very simple, you can repeat the same exercise okay.

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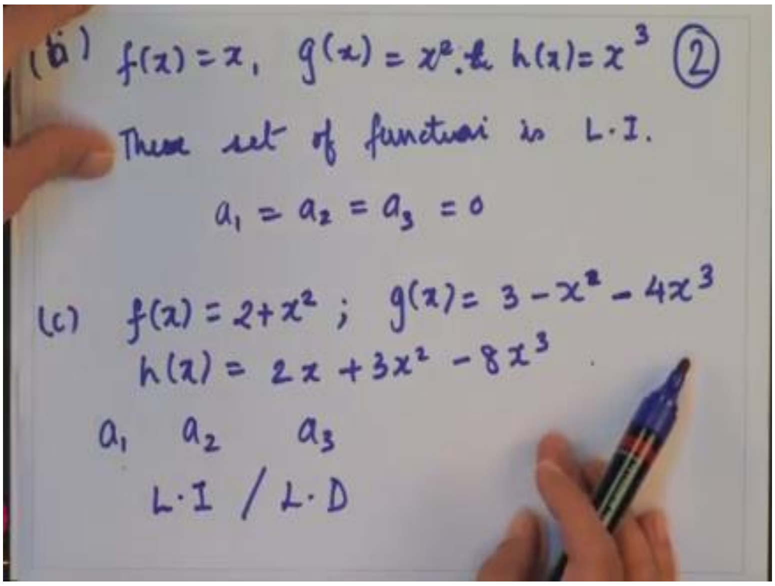

So, in part 2 or it is b; in 1b we have f(x) as x, g(x) as x square and sorry and h(x) is x cube okay, so this set is linearly dependent or independent that we have to evaluate similarly, you will substitute a_1x_1 + a_2 x_2 + a_3x_3, it can be seen from here that these set; the set of function is linearly independent, so when a_1 = a_2 = a_3 = 0, this is possible only when all the terms are 0 that means that it is linearly independent.

In c part again, you can easily check that this is f(x), g(x) is 3 - x square sorry, - x – 4x cube and h(x) is 2x + 3x^2 – 8x^3, so this you have to check whether it is linearly independent or not. So, now here what I do is; again I will use the trick where I will write; rewrite this as a1 f(x), a_2 g(x) and a_3 h(x) and equated to 0 and find the coefficient a_1, a_2, and a_3.

So, you have to find out the this these coefficients and check whether this set is linearly independent or linearly dependent, so this I leave it to you all, to do it, it is very simple, you just have to substitute for f_1, f_2, f_3 and you write it in the form, we wrote it before; a1 f(x) + a_2 g(x) + a_3 h(x), okay, if you find some relation between f, g and h, then it is linearly dependent okay.

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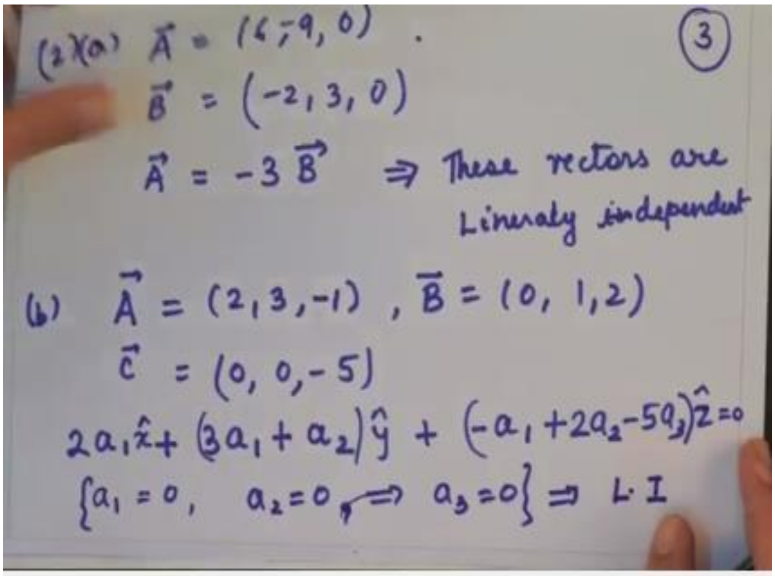

So, this is a part of exercise you can try so now, let us go to the second part of this question again, the problems what we did in first part, they will be similar to what we have done in the first part, here the coordinates are given, so first problem you have a vector $\vec{A}$ as; and $\vec{B}$ is - 2, 3 and 0. So you are; from this itself you can see that $\vec{A}$ is - 3 times $\vec{B}$, so this implies that this; these vectors are linearly independent, sorry, linearly dependent.

Because the coefficients are not individually = 0, here a_1 is 1 and your a_2 is 3, okay, I have to take it on the other side and I have $\vec{A} + 3\vec{B} = 0$, now in the b part again, we can check b part is again simple, very simple to check okay, this can be done, 0, 1, 2 and $\vec{C}$ is 0 , 0, -5 okay, now I skip the step of writing a1 $\vec{A}$ + a_2 $\vec{B}$ + a_3 $\vec{C}$, I can quickly write this as $2a_1$ okay, second term would be $3a_1 + a_2$, okay and the third term okay, do not forget, so these are in Cartesian coordinates, so x y and z.

So, I must write here, $\hat{x}$, this is $\hat{y}$ + the last term would be – $a_1 + 2a_2 - 5a_3$, $\hat{x}$, $\hat{y}$ and $\hat{z}$; $\hat{z}$ = 0, now you have to equate these each coefficient, so such that you can calculate the value of a, b and c, so from here, if this is equal to; if $a_1 = 0$, we can obtain a_2 which is also = 0 and when you substitute for a_1 and a_2, this would imply that a_3 is also 0, this would imply that these vectors are linearly independent, okay.

And the same exercise goes for the last part; last problem, it will be a little lengthy but not difficult, so you can just try this out.

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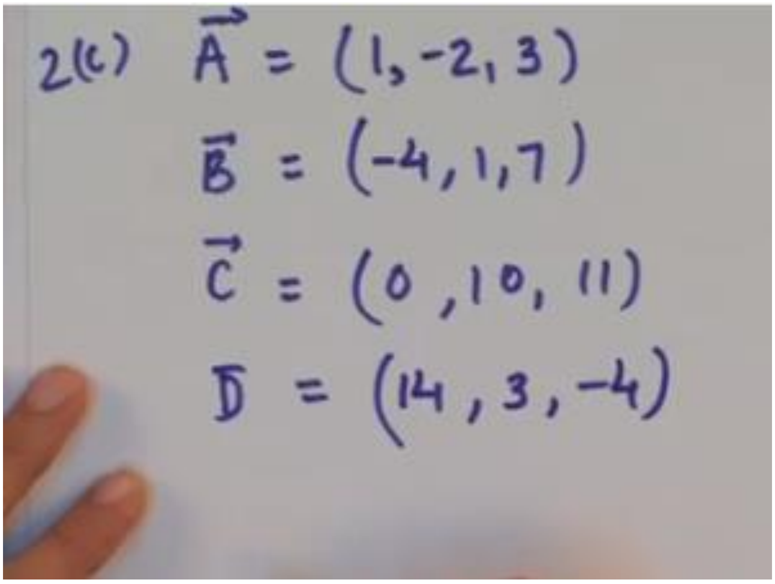

So $\vec{A}$ in the same manner as I did part 1 and 2; 1, 2 and 3, $\vec{B}$ is -4, 1 and 7, $\vec{C}$ is 0, 10, 11 and vector $\vec{D}$ is 14, 3, and -4, this you can work out and find out whether these vectors are linearly dependent or linearly independent, it is an exercise, it will take few steps, few minutes to do not that difficult okay.

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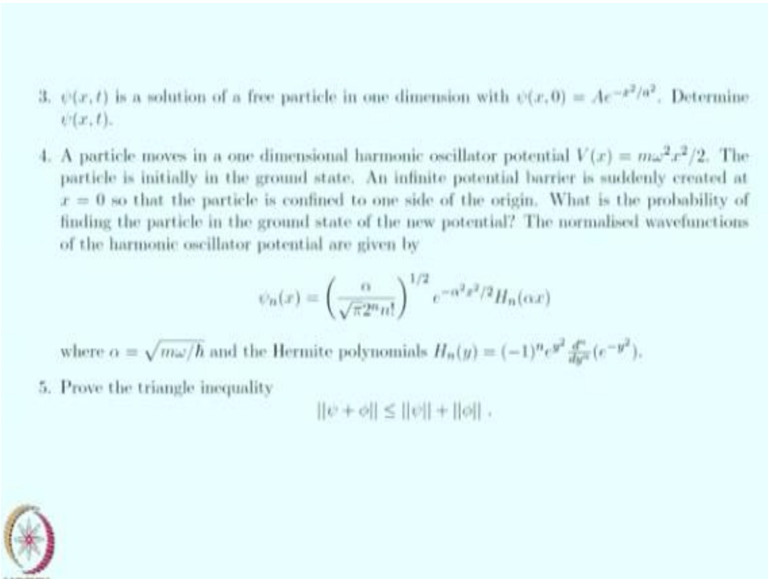

So, now we go to the next set of problems that is the third problem, in problem 3 okay, let me just recollect first tutorial in that we had calculated the time dependent wave function right, so this time dependent wave function we had simply calculated for a particle in one dimension.

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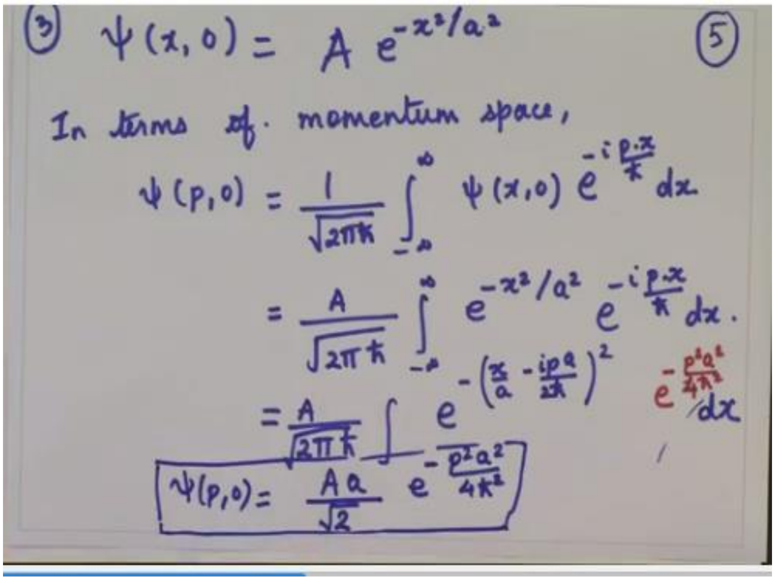

Now, here again, you are given a wave function, which is time independent wave function because I have ψ(x, 0) which is A e^{-x^2/a^2}, okay so now, in this what we do is; one way to do this is you write this in terms of, you can write this in terms of the momentum; in momentum representation, so in terms of momentum space, the wave function in terms of momentum space that is ψ(p, 0) this how will you obtain; we obtain this by taking the Fourier transform ψ (x, 0) $e^{\left(\frac{-ip.x}{\hbar^2}\right)}\, dx$, okay.

Then there will be some normalization factor also that is if I write this for one dimension, so I have; so for one dimension I have square root of $2\pi\hbar$ and if I write this for 3 dimension let me make it –i $\hbar$, so now if we write it in 3 dimension we will have $(2\pi\hbar)^{3/2}$ and this will be p.r, so now we substitute for the wave function, $\hbar$ and you have this as – ip.x $\hbar$ dx.

So, now here you have just substituted for ψ(x, 0), okay and you obtain the time, you obtain the wave function in momentum space, now you have to evaluate this term which is; you will have to rewrite this in the form of a complete square, so this; to this we add and subtract, so I will write the term; the term that we will add and subtract will be $e^{\left(\frac{ip^2a^2}{\hbar^2}\right)}$, you will have a factor of 2 coming from this is this, this is the square term, this is a plug term and then again a square term.

So, you will have this factor and you will write this as x/a - p ip/$\hbar$, so I will have a factor of 4, when I am squaring, 2 becomes 4, this is what I have, okay and then I have a dx okay, so now you have to integrate this and you obtain a final result at as A, this factor of a comes from the integrand and a factor of 2 and $\sqrt{2\pi\hbar}$ and the other term would get cancel, okay so this is what we get for the wave function of a particle in momentum space okay, remember this.

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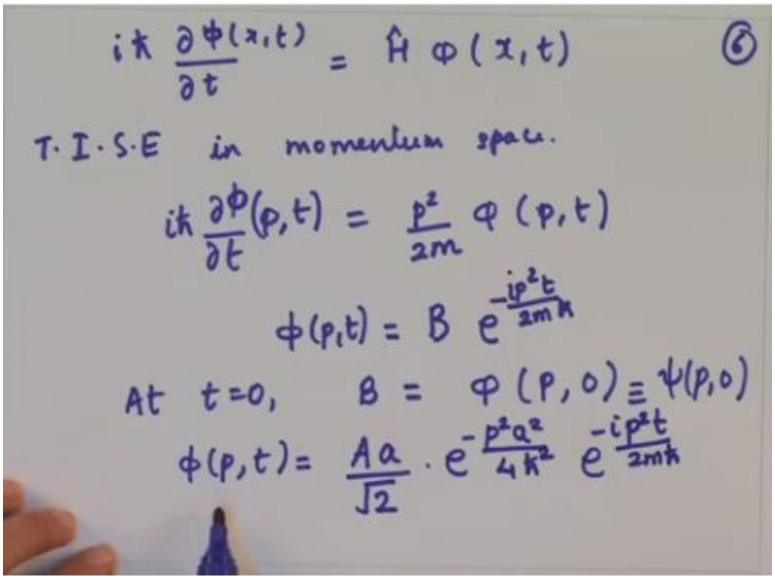

And then what we will do next is we will write the time dependent Schrodinger equation okay, we will come back to that expression, so we will write the time-dependent Schrodinger equation as i $\hbar \frac{\partial^2 \Phi(x,t)}{\partial t^2}$, this is the energy operator = H Φ(x,t) okay, now I will write the time independent Schrodinger equation in momentum space, this would look like $\frac{\partial \psi}{\partial t}$, have $i\hbar$, I have $i\hbar \frac{\partial \Phi(p,t)}{\partial t} = H\Phi(p,t)$, I can write as an $\frac{p^2}{2m}\Phi(p,t)$, if I use Phi let us use Φ throughout or ψ anything; any one notation.

So, $i\hbar \frac{\partial \Phi(p,t)}{\partial t} = \frac{p^2}{2m}\Phi(p,t)$ okay, so now let us write this as solution of this equation would be Φ p, t, you can easily take this $i\hbar$, so I will have a $\hbar$ upstairs i and you can obtain the expression p, you will have a p and you will have a $\hbar$ and 2m, so it will be $\frac{P^2 t}{2m\hbar}$ okay, so now with some constant term.

So, when I do integration by parts, this I need to do integration only no integration by parts only the integration would give us Φ(p, t) as this expression and remember, when I take this I, I have a minus sign okay and I have a factor of i also which I was missing okay, so I have Φ(p, t) as be, $e^{\frac{iP^2 t}{2m\hbar}}$ okay. Now, at t = 0, what do I have; I get value of p, p is p, 0 and we have evaluated p or Φ, 0 in our previous expression, so this is what was our ψ(p, 0) which is nothing but which I have defined here.

So, this is nothing but ψ(p, 0) which I have already evaluated so now, the wave function in the momentum space which is time dependent wave function in momentum space will be A upon $\sqrt{2}$, I am substituting for Φ (p, 0) times, you have $e^{-\frac{p^2a^2}{4\hbar^2}}e^{-\frac{ip^2t}{2m\hbar}}$ now, I have obtained the wave function in momentum space.

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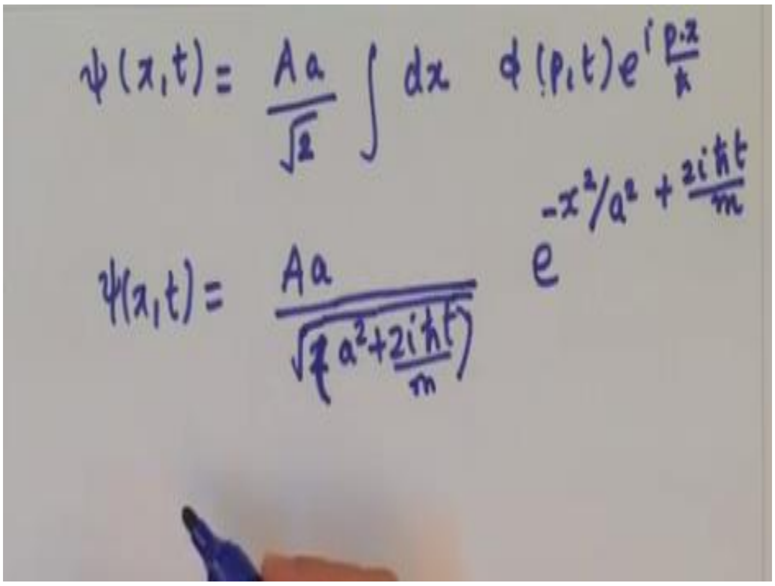

Similarly, we will again take a Fourier transform to obtain the wave function in position space, so ψ(x, t) will be nothing but again, I am skipping one step where you have to integrate this is what I obtain, integral, so I will just write down and then you have to integrate Φ(p, t); $e^{-\frac{ip.x}{\hbar}}$, okay and that will give me a final expression as to A upon $\sqrt{2}$.

And when I integrate this from the whole range, the final expression which I get is; let me write it here, I will have, so here from this I will get p square a square that is this term, I will have this term, $-\frac{p^2a^2}{4\hbar^2}$ and this term, so on integrating out this term , one can finally obtain $a^2 + \frac{2i\hbar t}{m}$ this is the expression times you will have an additional factor of $e^{-\hbar^2/a^2}$ +, so this is in the $e^{2i\hbar\, t/m}$, okay.

So, this is the expression for the time dependent wave function, so first what we did was that we first write the wave function in momentum space, once we obtain the wave function time dependent wave function; time independent wave function in the momentum space then what we did was we wrote the time independent Schrodinger equation in momentum space, you have to do all lot of integration wherein you complete the square and find out the definite integral.

And then again, we take the Fourier transform, so we go from the momentum; position space to momentum space, so this is a another way of calculating the wave function which is time independent; from the time dependent; independent wave function we obtain the time dependent wave function.

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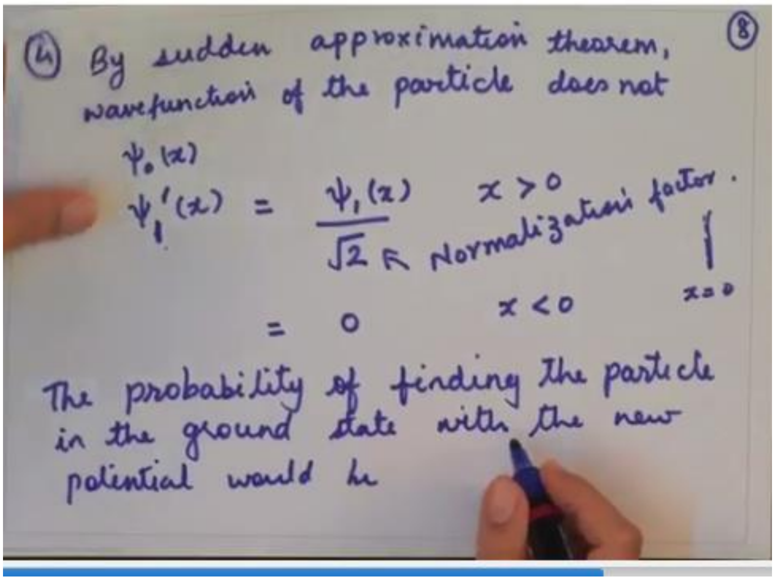

So, the next problem will be 4th problem involves a little bit of thinking, so in this problem you are given a particle, which is moving in one dimension in the harmonic oscillator potential that is the harmonic oscillator potential is given to you and the particle is initially in the ground state and an infinite potential barrier is suddenly created x = 0, so at x = 0, there is an infinite potential.

So, we can expect that at x < 0, the potential will be; the wave function will be 0 and so the particle is confined to one side of the origin and that is the probability of finding the particle in the ground state of the new potential, so what is that probability that so, what is the probability of finding the particle in the ground state of the new potential which was suddenly created then, the normalized wave function of the harmonic oscillator potential are given to you.

So, this is the normalized wave function, then the alpha is nothing but square root of m omega upon $\hbar$ and the hermite polynomial is given to you which we will require while we perform the integration, so now let us get started with now, what is given to you is basically, let us write by sudden approximation theorem, what we can say that the wave function for; wave function of the particle do not change or does not change with this sudden creation of the infinite barrier potential.

However, the wave function will remain the same and the new wave function you can see it has, so let me call it as ψ_1' will be the new wave function that is ψ_1 will now become ψ_1(x) divided by $\frac{1}{\sqrt{2}}$, the factor $\frac{1}{\sqrt{2}}$ is the normalization factor so, now the wave function of the new or the new wave function is nothing but the old wave function which is normalized.

So, the wave function remains the same and let us see what happens to the probability of finding the particle in the ground state for this new wave function, so the probability and one more thing that we must note that this wave function is true for x > 0 and our wave function is 0 for x < 0 because the potential or the infinite potential is suddenly created at x = 0, so at x = 0, an infinite potential is being created.

So, for x < 0, the wave function would go to 0 and so, the probability of finding, so the probability of the finding the particle in the ground state with the new potential would be; so in general how do we write the probability.

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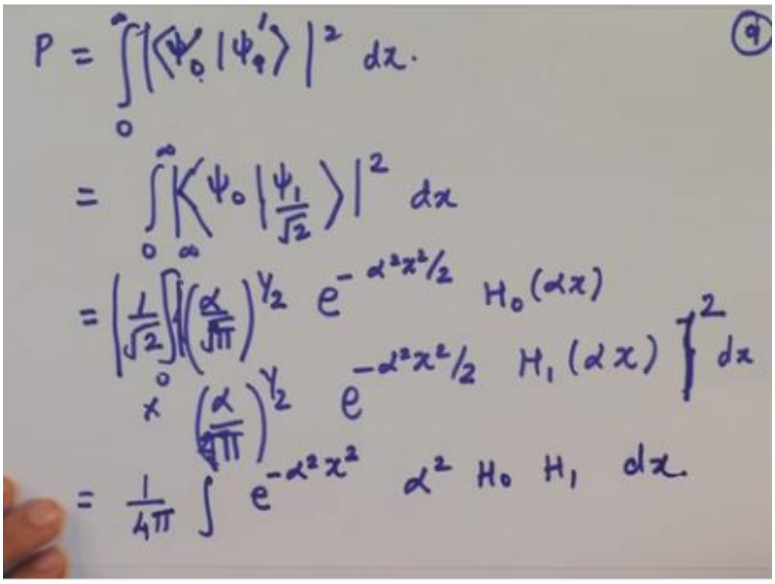

It would be; the probability would be given by ψ_1' or ψ_0, the whole square okay, so rather the other way round we can write ψ as ψ_1' okay and the integration is from 0 to infinity so now, you we know what is ψ_0 and we know what is ψ_1', so we have given the wave function which is normalized, so now let me write first this ψ_1' in terms of ψ, so this is ψ upon $\sqrt{2}$, okay and we have 0 to infinity sub square.

So, now when I rewrite the entire expression of ψ_1 and ψ_1' from the given expression, what I obtain is; 1/$\sqrt{2}$ times, I have $\frac{\alpha}{\sqrt{\pi}}$, this is for ψ_0, so when ψ_0, so I have here this expression where I have $\psi_0$will be α^2raise to 0 that is 1, so I have $\alpha/\sqrt{\pi}$ raised to the $\sqrt{\alpha\pi}e^{-\alpha x^2}H_0$okay.

And H_1; ψ_1 will be alpha upon 2 π/ n, so now let us continue with this, we will have $e^{-\frac{\alpha^2\hbar^2}{2}}$, I will have a $H_0(\alpha x)$, okay and another expression will again have alpha, so this power will go, so I have multiplied by, so I have integral, 0 to infinity, so I will just write here, alpha upon square root of pi the whole raise to $\frac{1}{2}e^{-\frac{\alpha^2x^2}{2}}$and I have a factor of 2 here.

And this will be $H_1(\alpha x)$, so now in order to simplify this, this entire thing multiplied by dx okay, I have dropped that in all the places now, we need to simplify this further, so here we can see we will have $e^{x^2\alpha^2}$and this part of the integral will also get simplified and you will be left with square root of; there is a whole square in this, right, this is whole square.

So, when you take this out, you will have alpha square, so I will just take the 4π term out okay, this entire square, so this is 2 and this is 2π, so I will have a 4π and I will have $e^{-\alpha^2x^2}$, I have an alpha square term H_0 and H_1, okay dx.

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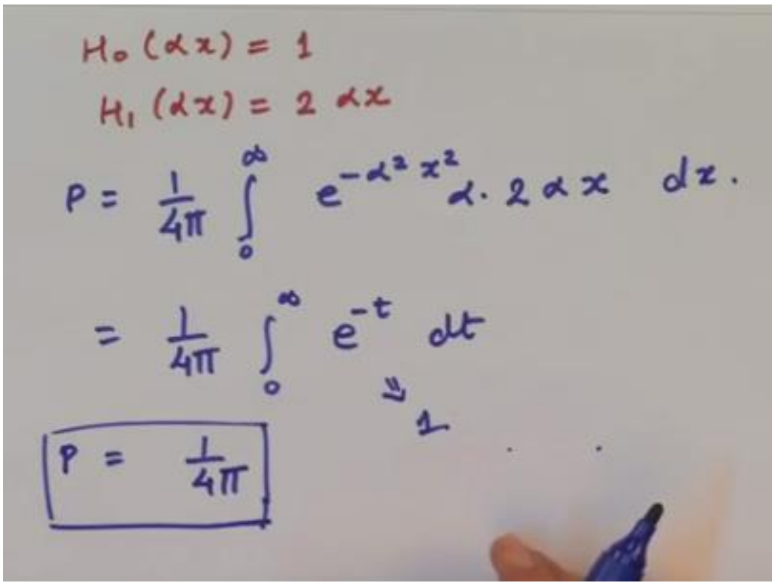

Now, remember $H_0(\alpha x)$, in this case will be 1 and $H_1(\alpha x)$ will be $2(\alpha\, x)$, so when I substitute this, I get 4π from 0 to ∞ and then I will have $e^{-\alpha^2 x^2}$, then I have from the previous oh sorry, here it was an alpha only, 1 α will be picked from H_1, so here H_0 is 1, H_1 is 2 α x, so I will have 2 α x dx and I had 1 alpha before okay.

So, this will be 4 π0 to infinity and then this I can write as standard form, you can just substitute alpha square x square = some t and this will give me $e^{-t\,dt}$ or just this is nothing but just 1, so the probability is $1/4\pi$, so now this was the second, this was the fourth problem on hermite polynomials and now let us discuss a very simple problem to end this tutorial session which I think it is very simple but let us do it.

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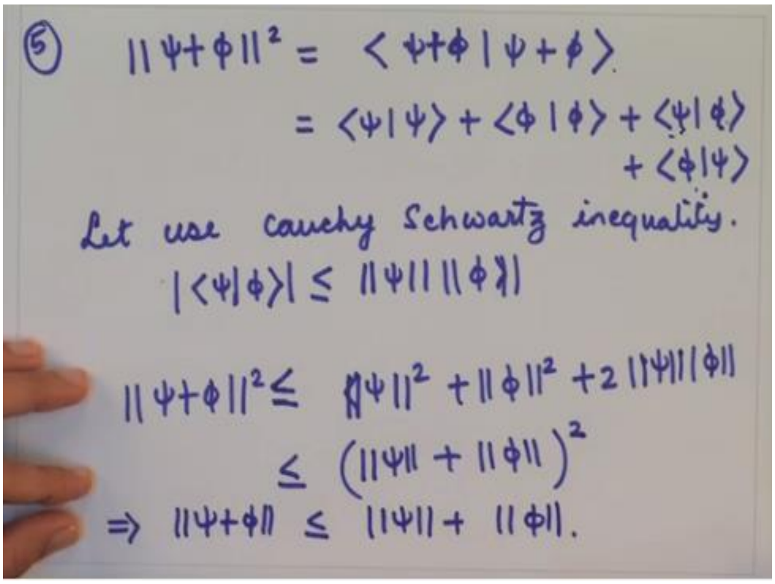

It can be done in various method and I will be applying this way of doing this problem, so what will be $\|\Psi + \Phi\|^2$, how will I write this; I can write this as $\langle\psi + \Phi|\psi + \phi\rangle$, okay I can write this norm as ψ + Φ, okay I am writing the $\|\Psi + \Phi\|^2$ as this now, this associative property we will use, this will be nothing but ψ on ψ, Φ on Φ + the cross terms, this plus okay.

Now, from here you can see that this will always be > 0 we know from the norm; the norm of a vector is always 0; is > 0 sorry, so now we have to worry about these 2, are they equal, so let us apply; let us use Cauchy Schwarz inequality, so from Cauchy Schwarz inequality, what do we know; we know that ψ on Φ that is any vector on the other is given by, okay, so this norm is always less than the norm of ψ and norm of Φ.

So, using that we can rewrite this as ψ, okay I have a $||\psi||^2$, this is nothing but I can write it as Φ square that is I can write this as $\|\psi\|^2 + \|\Phi\|^2 + 2\|\psi\|\|\Phi\|$, when I am doing this, this will be < or = this because I am substituting for the ψ on Φ and Φ on ψ, so this will be nothing but $(\|\psi\| + \|\Phi\|)^2$which would imply the expected result is always < or = the individual vectors okay.

So, now with this I think we can stop here and we will have more tutorial problems based on so what is next would be; coming up would be linear vector space and more interesting problems based on linear vector space, we will be discussing in the next class, they are more like math problems and you will enjoy doing them and the lectures would give you an insight of the math part, the relevance of the math part with the quantum mechanics, so see you later.