Quantum Mechanics
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Lecture – 01
Introduction to Quantum Mechanics - I

My name is Ramadevi. I am a professor at department of physics IIT, Bombay. So I have been teaching this quantum mechanics course. The tools of quantum mechanics are essential to perceive the microscopic world of atoms and nucleus. The main theme of this quantum mechanics course is to give exposure to direct bra-ket notations. Main emphasis will be on operator formalism and their applications to harmonic oscillator, hydrogen atom spectrum.

Every 4 lectures will be followed by a tutorial session by my colleague, Dr. Jai More who will discuss in detail some problems. This will benefit audience to appreciate the applications of the theory, lectures which we will be uploading. We will upload PDF and PPT files of video of every lecture prior, so that these files can be opened for clarity. The references are the first 2 books.

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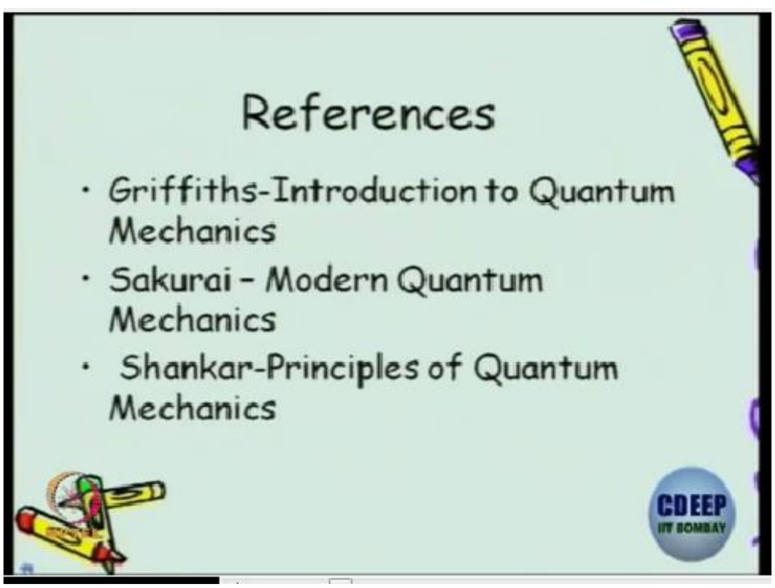

The third can be kept as kind of a reference book. It is really a pad book. So I would consider it to be like a reference book. It has everything. But I will prefer the first book to be followed by you which is Griffiths and the other books, Sakurai then slowly get on to Sakurai also. Shankar, you can use it as kind of a reference.

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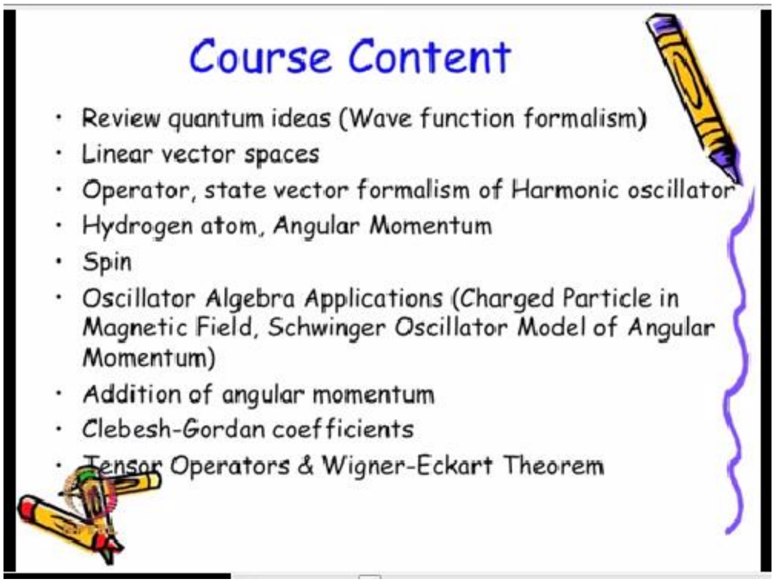

Okay, so this is the plan. I want to take you slowly from what you have learnt in your first year, into the second year, first course, quantum mechanics. So that will be review of quantum ideas. Then linear vector spaces, operators, state vector formalism of Harmonic oscillators. I am going to concentrate on hydrogen atom, angular momentum and then the concept of spin, addition of angular momentum. How does one add to angular momentums?

Then these things will force us to introduce a coefficient which is called as a Clebesh-Gordan coefficients. So we will spend some time in looking at these aspects at length, okay. So this is the main things which will be done. There will be some miscellaneous, okay.

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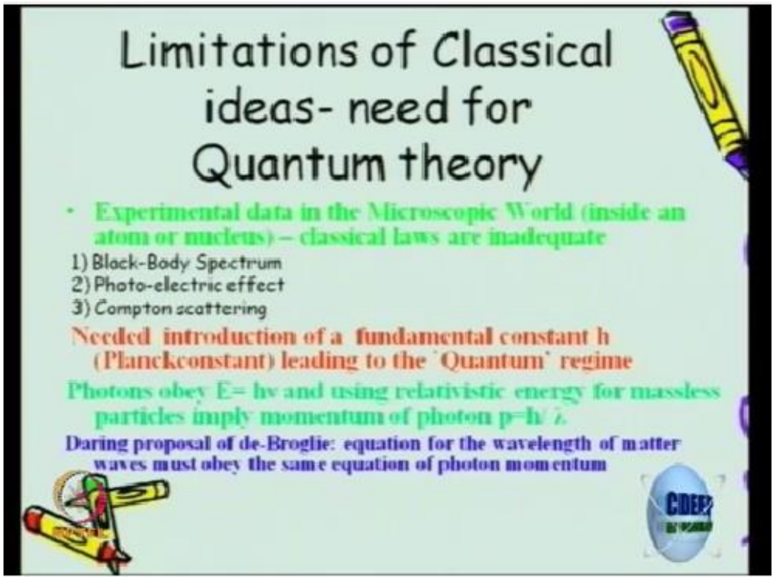

So to start the introduction, why do you want to do quantum. This you have already seen in your first year course that there are limitations of classical ideas, right. So what is the first one? You have seen microscopic world. Many of the experiments, you cannot explain using your familiar Newton's law of, or any classical law. So the first thing which you would have seen in the Black-Body spectrum which is the Planck's law which helped you to fix exactly all the points in the Black-Body spectrum.

Is that right? Everybody is with me? And then photoelectric effect cannot be explained just using classical laws. The more intense the beam, more photoelectric current will flow is not correct, right. You all know. So other thing is the scattering of, which is called as Compton scattering of photons with electrons, which also you have seen, how the wavelength changes. And these things can be explained only using quantum ideas, not classical ideas.

So one of the core important thing which went into all the 3 aspects of quantum aspects of photon, what is that? Is the universal Plank constant? You did not put a constant which was different for Black-Body spectrum, did not put a different constant for photoelectric effect for a different constant for Compton scattering. Is that right? You all knew it is the same h, it gave you an interesting, that is why it is universal.

The same h which you started with the Planck constant which was introduced by Planck to say that the energy of photons are quantized in quantas of integral multiples of $h\nu$. That h is the same h which happens even in photoelectric effect or Compton scattering. It may be mechanically using it but you know it is the universal nature, feature which shows up in that Planck's constant which is called as a Planck's constant h , okay.

You do not use different values, use the same value. So needed introduction of a fundamental constant leading to Quantum regime. So $h\rightarrow 0$ is a limit where you start getting to the classical, okay. So h not equal to 0 is the one which we call it is the Quantum regime, okay. So this is also familiar to you, Planck's equation, $E=h\nu$. And you can use Einstein's relativistic energy for massless particle.

What is that? $E=pc$ and you can try and write the momentum of a photon to be h/λ. Everybody is familiar with this, right.

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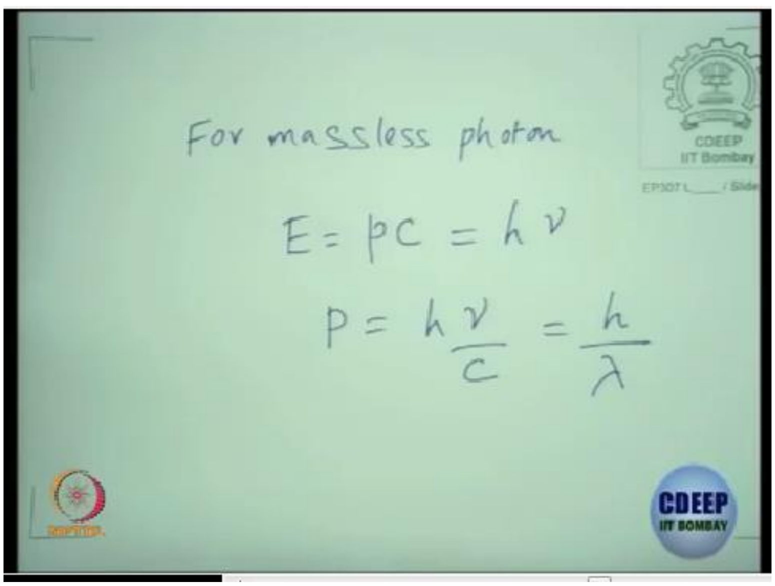

So for massless photon, $E=pc$ which is $h\nu$. Is it visible? And from here, you can write $p=h\nu/c=h/\lambda$, right. So this is what the first equation $E=pc$ is for the massless Einstein's energy momentum, relativistic equation and Planck's hypothesis is $E=h\nu$ from which you can show that the momentum is h/λ, okay. So that is what I have written here. So what was the proposal of de-Broglie?

He said why should it be that momentum, E=h/λ, it should be applicable for other particles as well. That was the daring proposal. He said p=h/λ should be true even for other particles, like electron, protons and so on. But E=pc is not correct. If you can write momentum to be h/λ, is the de-Broglie hypothesis. Which means that there is a wavelength associated with other particles, treat them as waves, okay.

This p=h/λ is the crucial point which tells you that you can look at any object both as particle, look at particle, you would talk about momentum. If you look at wave, you will talk about wavelength. And this is that equation which relates both, okay. So this is the daring proposal of de-Broglie. Equation of wavelength of matter waves must obey the same equations of photon momentum, okay.

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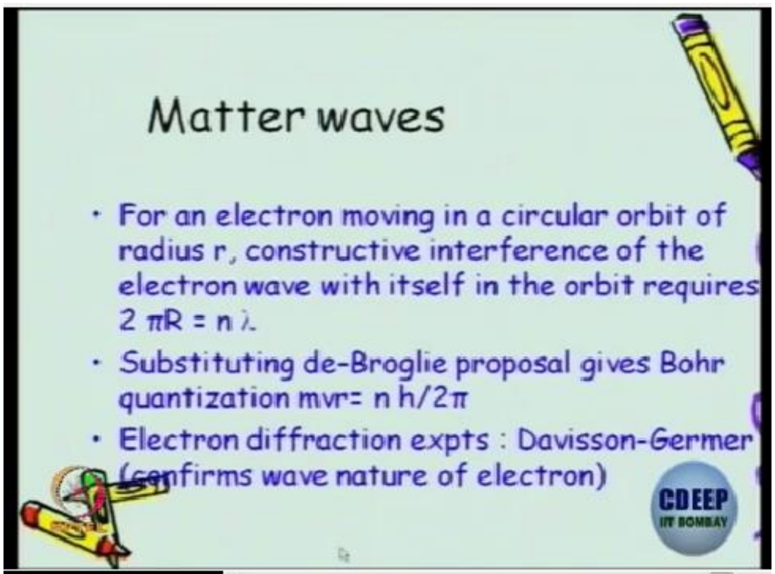

So matter waves, once you start looking at matter waves, you can look at them moving on an orbit and look at the circular orbit with constructive interference and the constructive interference will give you a condition for a wave as the circumference to be for the integral multiple of λ. And once you substitute, the de-Broglie proposal for lambda, you can also rewrite angular momentum quantization.

This is your famous Bohr quantization which you would have read as a proposal. I am just trying to say that once you start taking it as a wave and look at constructive interference on a circular orbit, you automatically get quantization of orbital angular momentum. These are the famous experiments, Davisson-Germer and these show that you can also look at electron beam and dissections of the electron beam.

And in that sense, it confirms what de-Broglie said that there is a wave nature for particles like electron. Usually, when we are given an electron or when are given a photon, we see photon as an electromagnetic light. We see electron as a point particle. But in this course, you should remember that a particle and the wave are part of every object and what we try to tap is what you will get, okay.

So depends on the situation. If you are doing photoelectric effect, you will get only the particle nature of photon. If you are doing interference experiments, you will get wave nature of photon. Similarly, electrons, if you doing collision experiments in your classical physics, it is like a particle nature. But if you are doing dissection experiment, it shows the wavelength, okay. Depending on the experimental setup and its energy, things are going to be different and that is what is happening in the microscopic world.

What we see in our day to day life, is the microscopic effect. But what is happening inside an atom or inside a proton, are governed by quantum ideas, okay. You cannot use your classical ideas and say that what I see here should happen in the microscopic world. We do see violations, that is why we started introducing the quantum ideas.

Like for example tunnelling, I cannot just go over, across this wall outside but you can still get, if it is a wave, a sound wave could cross across, right. That is once it is a wave, things can cross. That information can be there in a region where a particle cannot go out but a wave can go out, okay.

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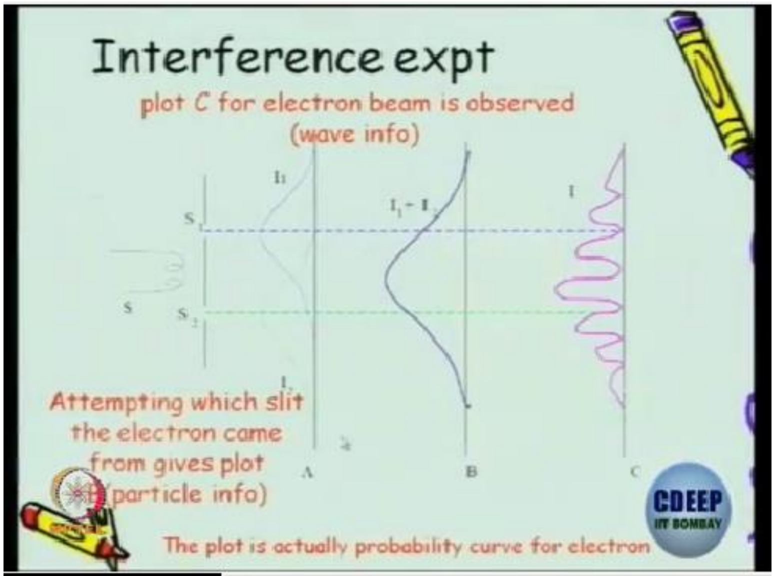

This is your famous experiments which you would have seen Young double slit experiment, right. And here you have 2 sources and if suppose you shut one of the source, you will have this blue line for light coming through S_1, similar intensity. Similarly, if I shut S_1, you will have a green line due to the light coming through the source S_2, the slit S_2. But if both are open, okay, what is your expectation? It has to be like the wave.

This is what you will expect, constructive destructive interference . But if you try to see which slit through which it went? Then what happens? The interference pattern disappears. You will get, if you try to see how, which way it goes? Then this is the pattern. So which way it goes if you are trying to see, then you are looking for the particle nature. Trying to look for the particle nature, it is just the sum of the intensities.

If you do not worry about which slit it went through and just look at the screen, it will have a superposition. Not intensity adding. It is the amplitude added and then what spread of it and you will get an interference, okay. So this is the experimental data, they have tried to find out by closely putting a, kind of, detector here to see whether the beam is coming through this or coming through that in that instant of time which they try to do that, then they will get the screen B.

They do not try to do that by putting some detectors here, disturbing the system. They will get this screen C. So you can get wave nature if you want to tap the particle nature, the interference pattern disappears, okay. So it was done even for the electron beam, not only for the photon electromagnetic field.

Of course, you could do this but this is done for the electron beam and it was proven that it has wave nature besides being a particle. So attempting which slit the electron came from gives plot B which is a particle info and neither particle of wave nature can be observed, is what I have tried to give you from this plot. Plot C gives the electron beam which is the wave info, okay.

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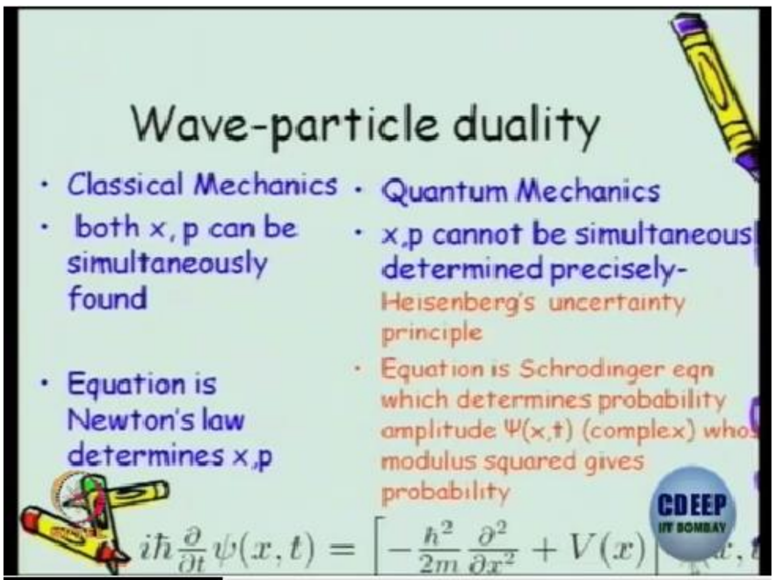

So I have slowly got you on to the wave particle duality which is how many of you have read Jekyll and Hyde. So the character is just, so same person seems to have 2 characters, right. Both the Jekyll characters are very good character and the same, similar thing but here, you know, every object seems to have wave nature and particle nature but if you try to probe what you want to find, it shows that character, okay.

So in some sense, all of us have both wave character and particle character. Why are our wave character not seen? Do you know why? Wavelength is very small which cannot be measured in the present day. So that is why you do not see the wave character. But if you take in electron whose mass is nearly small, then the wavelength is a measurable regime. So that is why you can have both wave character and; for everyone, it is there.

But which is dominant in our case is only the particle character. But wave character needs a measurement of wavelength which is within the experimental regime. And that happens only in a microscopic world, okay. So all the planets when you study, you will not do quantum. Why? So point, it is really a massive object and you will do Kepler laws and it is correct. There is no need to worry about the wave character.

So that is what you should appreciate that. Because I am teaching quantum regime, does not mean that day to day life requires but there are situations where you have to apply. So wave particle duality. So I am slowly taking you from classical mechanics when you do Newton's law, you write all the Newton's law where you do not worry about, you can have simultaneous values for position as well as momentum, right.

You do write the first equation or the second equation, $S=ut+\frac{1}{2}\,g\,t^2$ when you write or $\frac{1}{2}\,a\,t^2$, you know both, the position as well as the momentum, okay. It is noted that if you know the position, you do not know the momentum. You can measure the velocity or momentum. So that is why that is a very important point that x and p can be simultaneously found in your classical laws. Equation is Newton's law.

What is your aim in the Newton's law, you determine x as a function of p once you have x as a function of p, d/dt will give you momentum? So situation completely changes once you go into the quantum microscopic world and you want quantum mechanics. You cannot determine x as a one which gives you the position. p by de Broglie hypothesis gives you the wave nature, x gives you the particle nature.

As I have said, you cannot get both the particle and wave natures simultaneously, right, x, and I say the position of the object and looking at it as a particle. When I say momentum of the object by de-Broglie hypothesis, I am seeing the wavelength which means I am trying to look at the wave nature. But I cannot simultaneously, we have seen in the Young's double slit experiment, you want the particle info, you try to probe by putting a kind of a detector at the slits to see which slit it went through.

But then you lose the interference pattern. But if you do not see which one it went through, you get the interference pattern. You cannot go and which one it went through and the interference pattern. That is not possible. Is that right? So x and p cannot be simultaneously determined, is one of the core important things which I have tried motivating from the earlier slides.

And this is something which you all mechanical, you learn in your courses, earlier courses which is the famous Heisenberg's uncertainty principle that position and momentum cannot be precisely measured. Is that right? So here the equation is the Newton's law in classical mechanics. In quantum mechanics, the equation which you have to solve is the Schrodinger equation and the main theme of the Schrodinger equation is to find a solution for a wave function or it is also called probability amplitude.

In one dimension, it is written as ψ as a function of x. It can also be dependent on time. It is a complex; general it can be complex. And whose modulus squared will give you the probability. It is something which you have learnt in your first year. So this is the distinction between classical mechanics and quantum mechanics. Classical mechanics course when you are doing, you will be in this regime.

Now quantum mechanics course, you will be in this setting. So the familiar Schrodinger equation, the left hand side I have written is the partial derivative with respect to time and here, the right hand side I have written the energy operator or Hamiltonian we call which can be rewritten as double derivative of the position. I have written it in one dimension just for simplicity.

V is the potential energy which the particle is facing and we would like to solve this equation to find $\psi(x,t)$ which is the wave function, okay. So that is the theme. This is the equation and you know, it is the proposal, this equation. There are some indicative ways in which you can derive it by looking at their electromagnetic wave and then we propose for a nontrivial potential energy and that is the Schrodinger equation.

And many experimental curves, many experimental data are all explained using this which means this equation should be accurate. It is correct, okay. Just like Newton's law, there was no violation when you try to verify many things. Same way the Schrodinger equation is also useful, okay. So just to summarize, I will put it as a flow chart.

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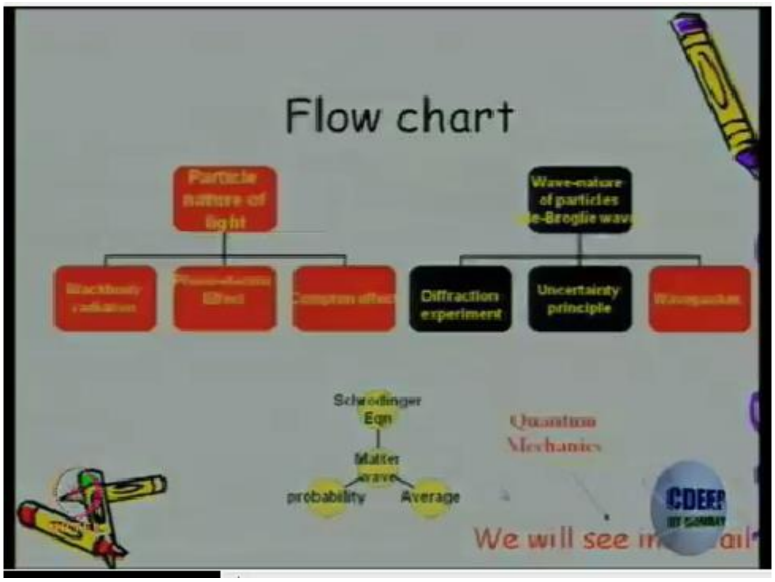

So you have seen the dual nature of light. Light is usually a wave but we can see the particle nature from black body radiation, photoelectric effect and Compton effect. Usually, particles are particles. You can look at wave nature of particles. They are called matter waves or de-Broglie waves. To explain them, you can look at diffraction experiment, uncertainty principle. You can also look at the superposition of various waves.

You look at the group velocity to see the wave packet. So these are ways of looking at the wave nature of the particles. And Schrodinger equation is the equation which describes the matter wave and some of the concepts which you will, which you would have already studied as the probability finding expectation values or they are also called average values of position, momentum, energy and so on, okay. So this is quantum mechanics and we will see this in detail.