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Quantum Information and Computing

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Modul No.08

Lecture No.46

Experimental Aspect of Quantum Computing - II

Hello and welcome to this lecture on quantum computing and quantum information this is a continuation of our theme of discussing experiments in quantum computing let us look at what we have installed today.

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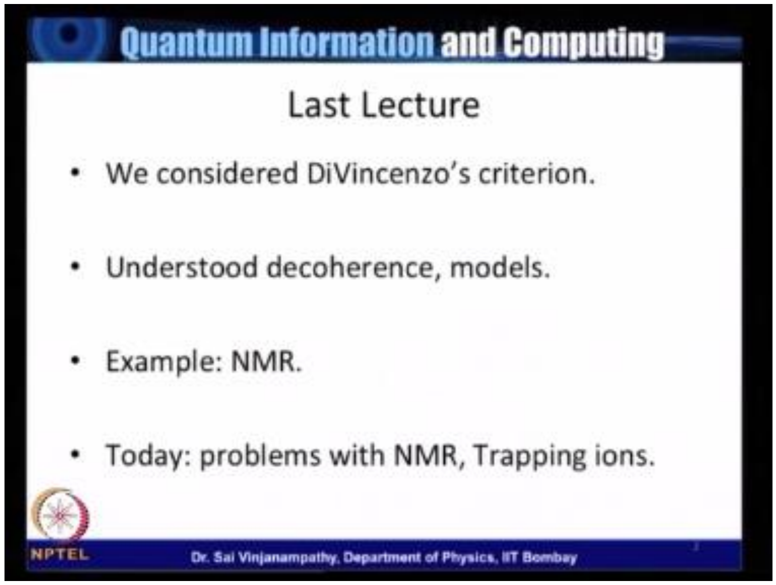

Last lecture we discussed the DiVincenzo's criterion which where the set of constraints that an experiment in system was fulfill to be able to do quantum computing we understood decoherence and we also discuss some models of decoherence we actually considered NMR nuclear magnetic resonance is an example of such a system today let us see why there are problems with the NMR qubit why it is unsuitable for quantum computing and let us then move on for an alternative which is trapped ions.

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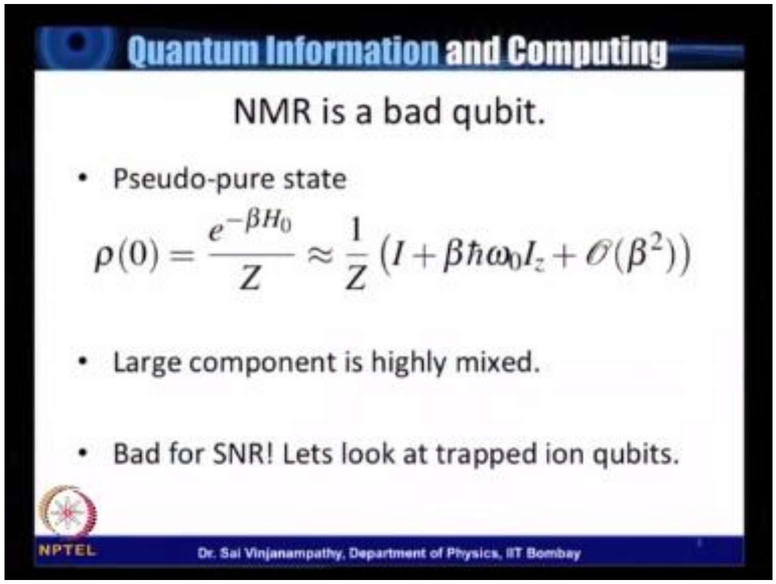

So let us see why NMR is a bad qubit NMR is a bad qubit because the nuclear magnetic resonance qubit is in what is called a pseudo- pure state it is at an inverse temperature β which is given by 1/ KT so the initial state of an NMR qubit can be written approximately as $1 + \beta\hbar\omega_0 I_Z$ let me remind you that the I_Z is just half σ_Z and this tells us what the problem is the problem is that even though we have access to a single qubit in the form of the σ_Z or the I_Z part of the density matrix almost all of the population is in this maximum mixed state which is given by the identity matrix.

So what this means is that this qubit actually exhibits really bad signal to noise ratios and this unsuitable for large-scale quantum computing it is unsuitable for this scaling relationship in quantum computing so let us look at an alternative and as an alternative let us look at trapped ions to consider trapped ions as qubits we one must first discuss how to trap ions.

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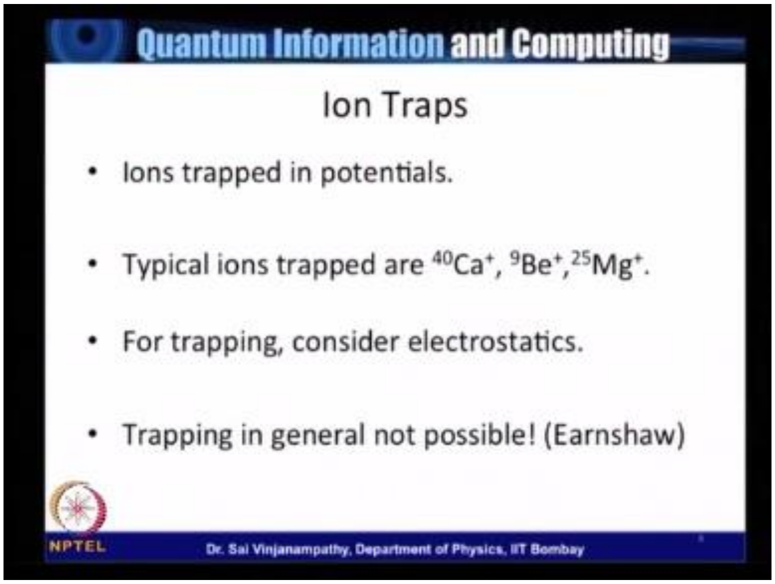

So let me mention that typical ions that are trapped are calcium, beryllium, magnesium and for trapping one would like to take the simplest approach possible so the simplest approach is to see from electrostatic potential is able to trap an ion after all it is a charged particle this is in general not possible and these are consequence of a theorem in electrostatics called Earnshaw theorem let us take a look at it.

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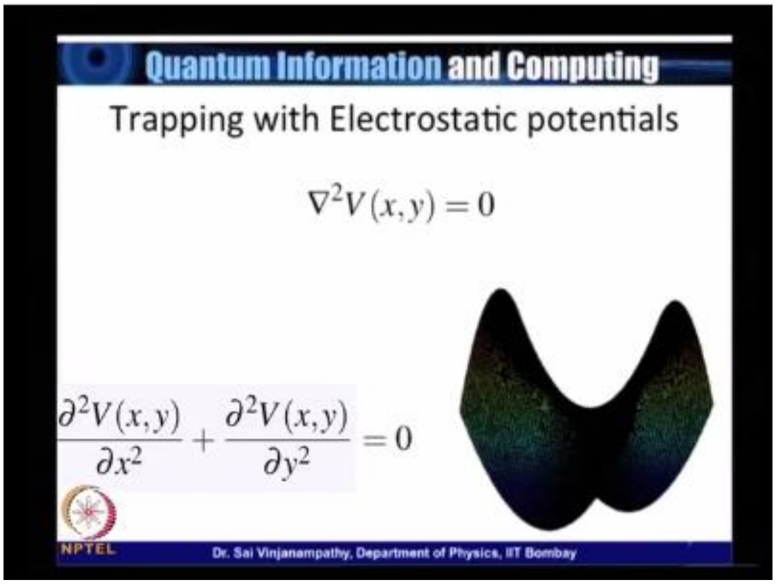

Now Earnshaw theorem is just a simple statement that a source free potential must obey the Laplace equation $\nabla^2 V = 0$ now if you expand the ∇^2 we can write this for a two dimensional potential as $\nabla^2 V/ dx^2 + b^2 V/dy^2 = 0$ and now we see what the problem is the problem is that if the curvature of the potential along the x axis is positive let us say the first term here is positive then the second term has to necessarily be negative for these two to add the 0 what this means in a picture is that if we was the potential that I wanted to consider and if I imagine that instead of a charged particle I just have a marble and that V is a gravitational potential.

Then if the marble were moved a little bit along this axis it will roll back gently to this place just like a marble would sitting in the bottom of a hill but with respect to this direction it is on top of a hill and if I perturb the marble a little bit towards this direction on the opposite direction going down if you just roll away and so this requirement that $\nabla^2 V = 0$ basically translates to the fact that there are no stable trapping potentials electrostatic offers to us, so what is the solution around it? The solution actually is available even in this analogy that I pointed out. So let us stick with the analogy of a saddle and the marble in a saddle and let us see what can be done.

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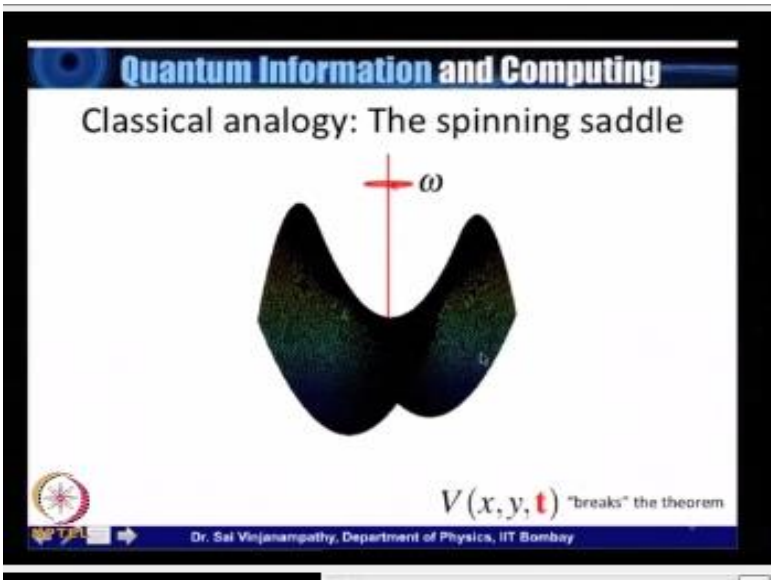

So now imagine that instead of a stationary saddle we have a saddle that is spinning about one of the axes that is indicated to it. So consider this saddle to be spinning with a frequency ω, if this marble stations at the center of this potential is not displaced a little bit, if ω is slow enough then the marble simply rolls away, the saddle is turning the entire time but the marbles motion is much faster than the saddles.

So the spinning of the saddle has no effect on the marble being in an unstable potential this problem has not been resolved, now imagine the opposite limit where the frequency with which the saddle is fun is rather fast, if the frequency is fast in comparison with the typical dynamics of the marble let us the amount of time that the marble takes to go from here to up to a little bit far away from here.

If this frequency is much faster than that typical dynamics then what happens is that before the marble has a chance to get away too far, the saddle has spun to a place where this potential is in the place of this potential. So what this means is that while the marble was trying to roll away the potential spun around so that now it offers resistance to the marbles motion in by increasing this height, right.

So if you do this spinning quickly enough what happens is that the marble finds itself to be localized around the center and this basically is the solution to the problem that we are facing, this spinning saddle basically the way it breaks Earnshaw's theorem to use a colloquial world, is that it takes this potential V (x, y) and introduces a primes component to it, the frequency implies that there is a time component.

And this time is, is crucial in this entire picture because what it does is it allows us this additional degree of freedom to control this, this part. So let us see how to translate that with trapping ions.

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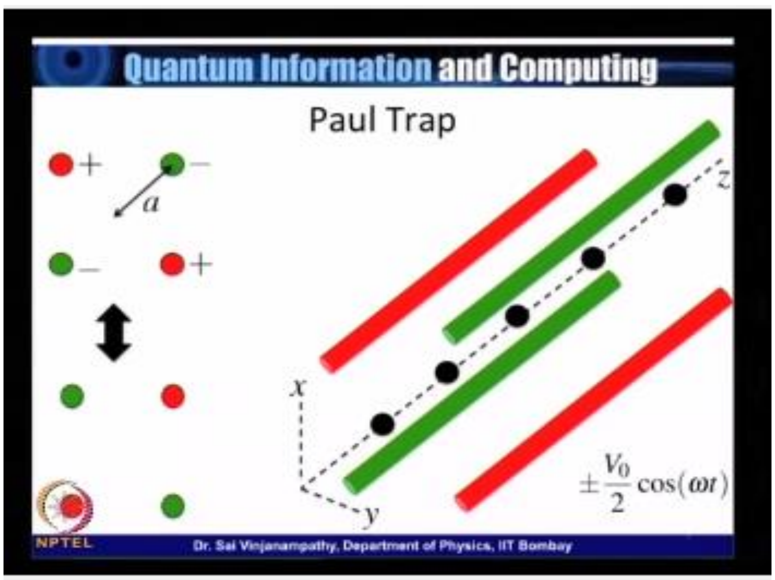

So the spinning saddle idea inspired what is now called the Paul trap, the Paul trap consists of four electrodes which I have indicated in red and green here and the ions are along the z-axis on the center. Now I described the fact that one must change the potential in the time depend fashion and if you look at what happens to these electrodes we see that while is at time T = 0 along the diagonal the two electrodes are positively charged which is to say that if the potential is + $V_0/2$ cos (ωt) along the anti-diagonal the potential is negative which is to say it is - $V_0/2$ cos (ωt).

In a short time what happens, is that the polarity flips and along the diagonal the potential is negative and along the anti-diagonal the potential is positive, this spinning saddle is now emulated by the Paul trap by simply transforming the time-dependent potential in this way.

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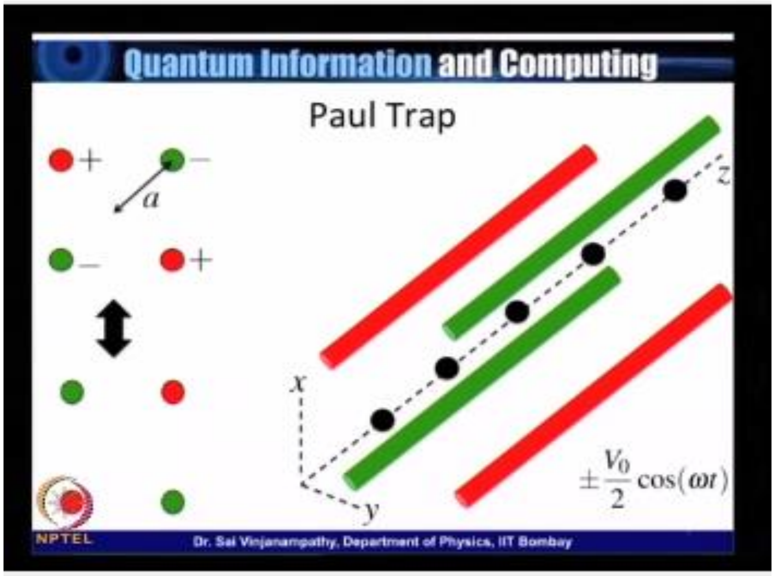

What happens is that this prescription traps the ions along the axial direction.

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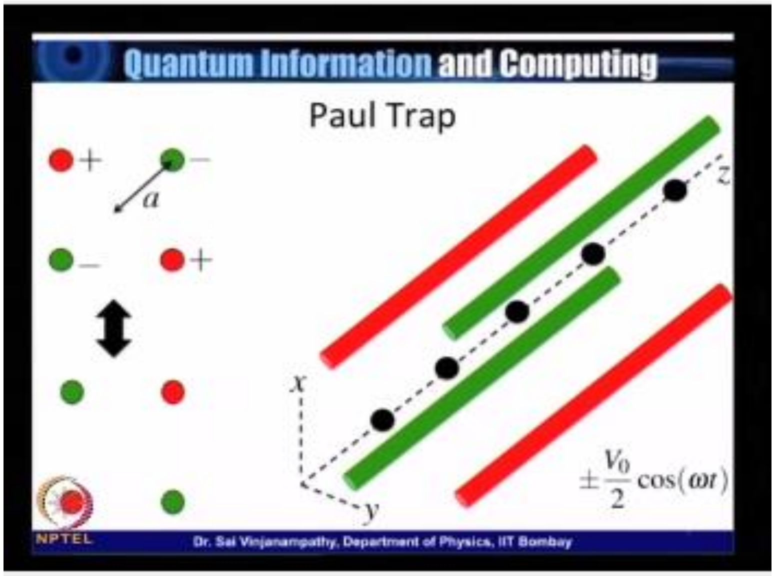

Let me point out that the axial Direction is along this Z, to analyze this a little bit further let us look at the equations of motion involved. Le t us start with the simplest equation of motion which is the z equation of motion one can derive that the Z equation of motion is that of a simple harmonic oscillator Md^2z/dt^2 is proportional to the –z this proportionality constant which is the frequency has terms such as Q which is the charge of the ion, M which is the mass of the ion, L which is the distance between the end gaps and V_{cap} which is the cap potential. Here I am referring to two end caps which are on both sides of this, of this diagram which are also charged by an amount recap.

So this equation of course means that the motion of the ions is that of a simple harmonic oscillator. A detailed analysis also shows that there are additional modes along the z axis like this breathing mode that I have derived here. Let us now consider the motion along the other two axis which is the x and the y axis.

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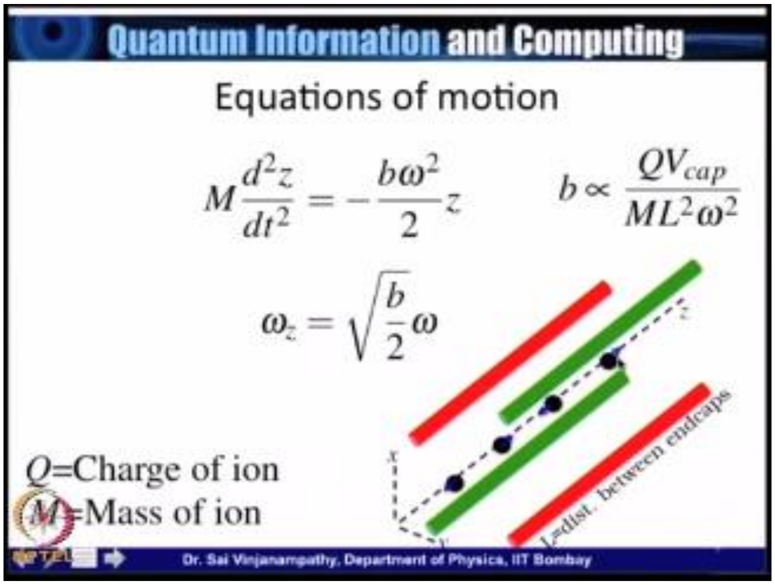

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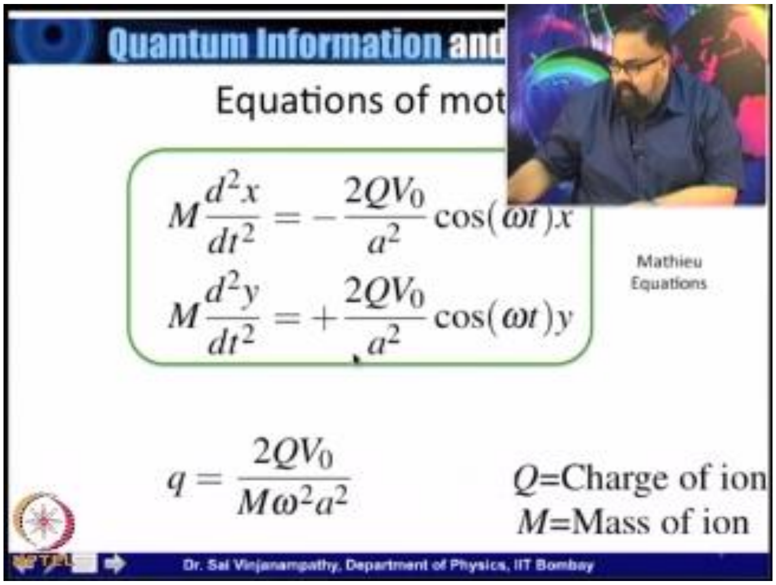

One can again derive the equations of motion along the x and the y axis and these equations of motion you are given by what are known as the Mathieu equations. I will describe these equations to you the differential equations have the familiar form of d^2x/dt^2 is proportional to x and d^2y/dt^2 is proportional to y. But there is an additional time dependent term which is cosin(ωt) that enters both equations.

We know where this cos(ωt) comes from it comes from the fact that plus or minus V_0 cos(ωt) divided by 2 was the choice of the modulation that we have chosen. Again various constants enter this, enter this equation. Let me point out this constant a.

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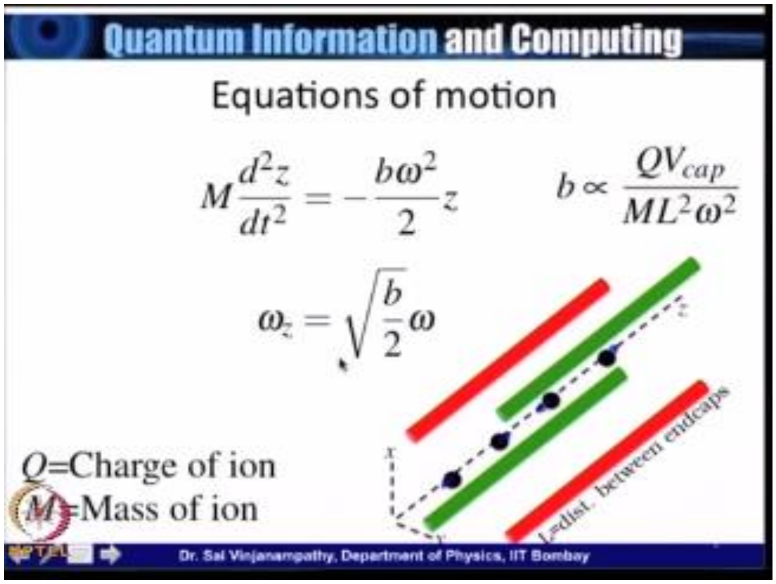

Which is related to the distance between.

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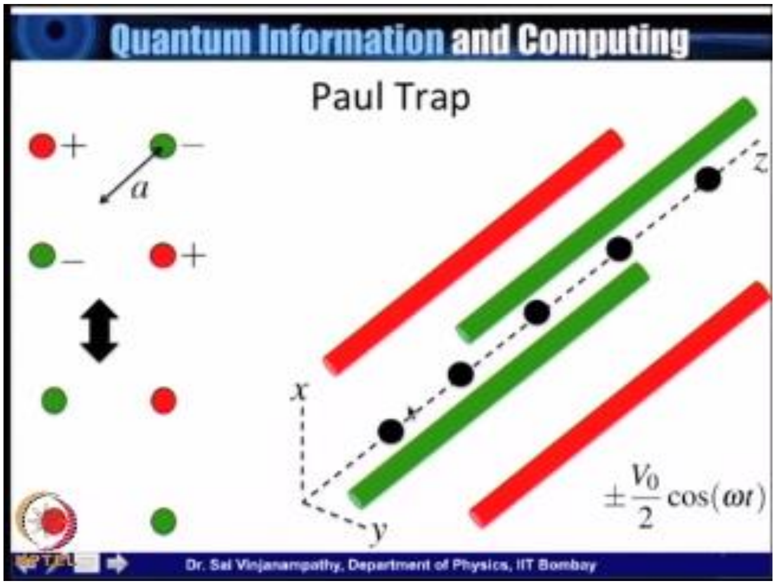

The distance between the ions in the center and one of these rods which is in the corner, right? So this ion and this rod so along this direction which is in the xy plane that is the, that is the distance a.

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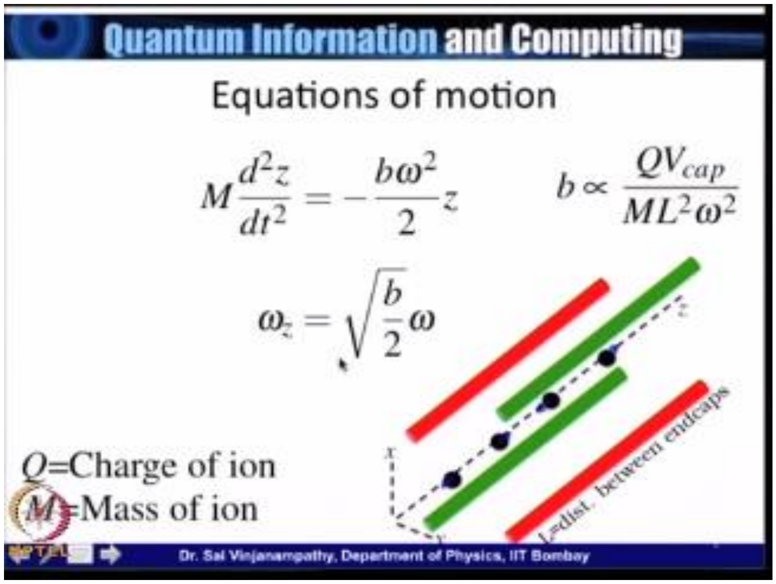

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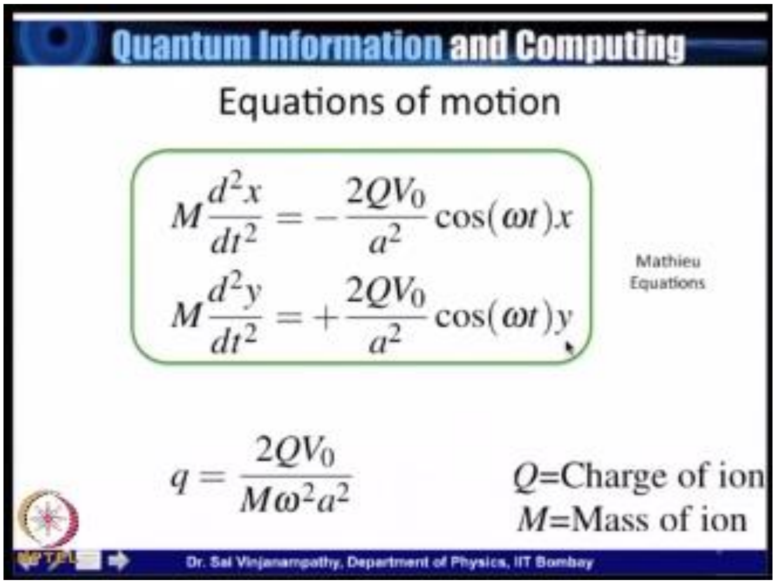

Now these equations can also be solved, but let me actually show you a picture of what is happening.

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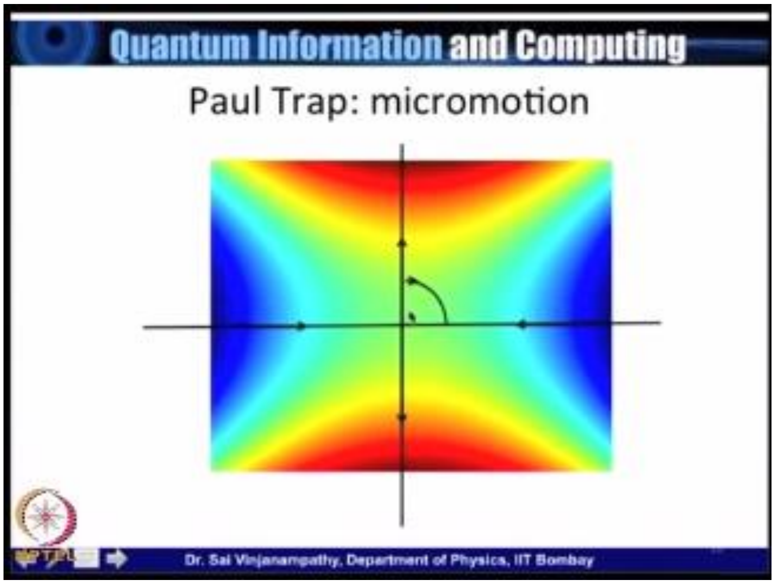

To do that let us actually draw the saddle in a color block, so here what I am showing you is a contour plot where red indicates a lower potential which means that an ion placed here and then gently push with this run away in that direction that is the arrows. And blue indicates a higher potential which means an ion that is placed here and push would, would gently roll back into the into the center. As we spin this potential very, very quickly the ion basically executes motion about the center, this motion looks like this.

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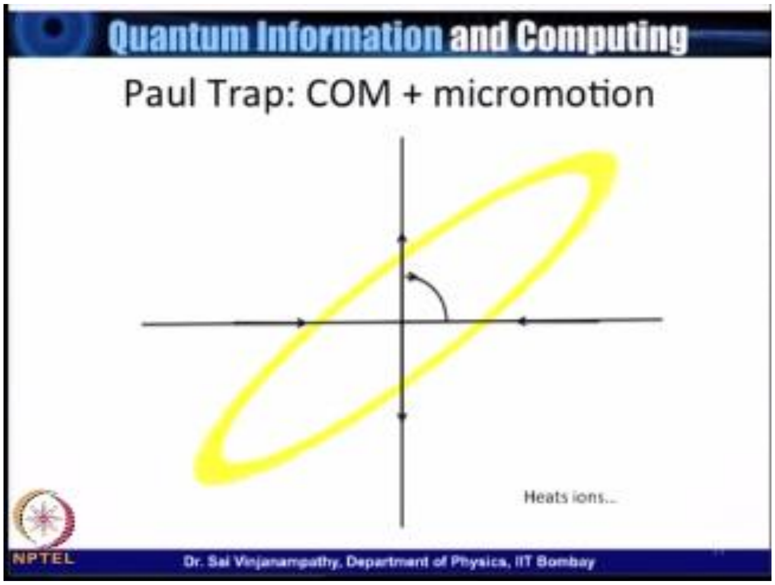

What I have done here is of greatly exaggerated both axis and I am describing to you two kinds of motion, the first of these is what is called the center of mass motion which is just the fact that this ion just rolls around the center. The second is an extremely rapid motion that I have depicted here and this is called micro motion and as a consequence of the fact that the potential is spinning rapidly under it.

Micro motion contributes to the heating of ions, because it increases the number of photons in the vibration degrees of freedom. So what we have are trapped ions that are rather hot and this is no good for quantum computing again. So what do we do about this we have to cool these ions? So let me describe to you two mechanisms that one uses to cool ions. The first of these is what is known as Doppler coding.

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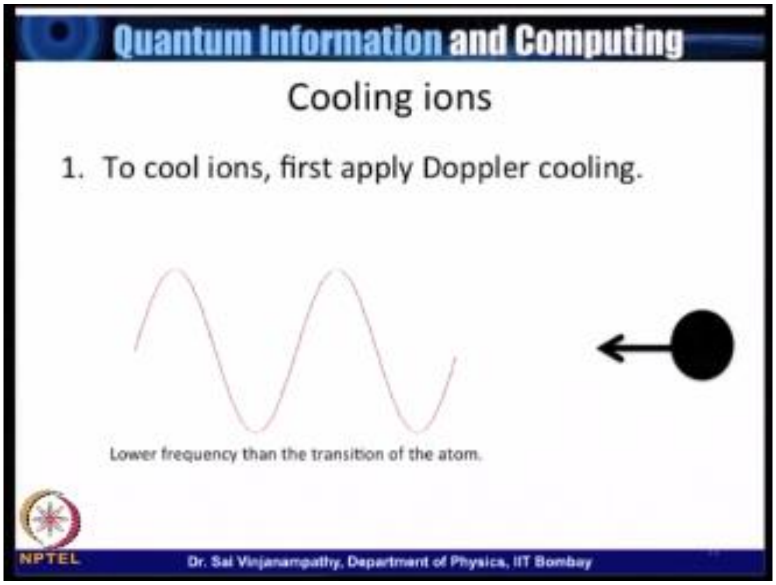

If you want to cool ions down to around 15 photons we can appeal to something called Doppler cooling. Doppler cooling is based on the Doppler Effect.

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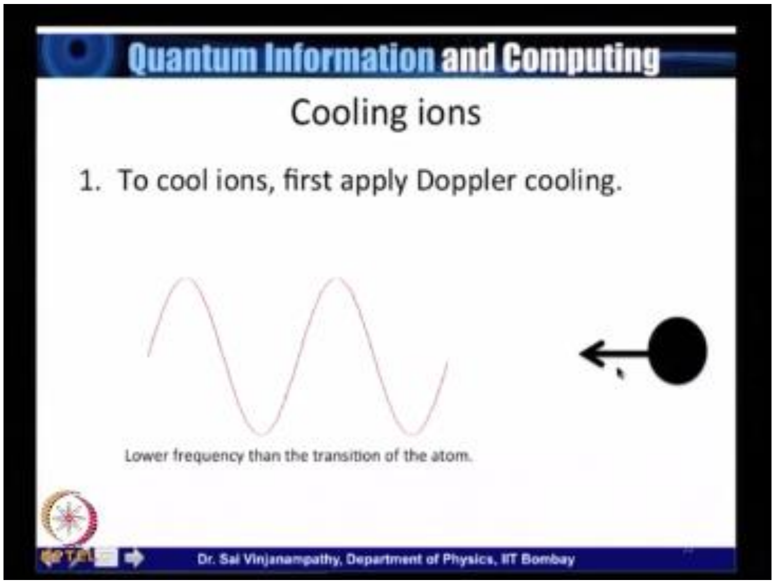

And is described by the cartoon that is given here so imagine that an atom is moving in a particular direction and if, if electromagnetic radiation is incident on this item such that it has a lower frequency than the transition of the atom then we see the atom is moving towards the electromagnetic field it can absorb a photon when it absorbs a photon because the photon is moving in the direction that is opposite to that of the of the eye on the ion loses momentum and hence queen by repeating this over and over again one can cool these ions to around 15 for 15 phonons it turns out is not good enough for quantum computing still.

One must get these ions as cold as possible and to do that one has to do what is known as sideband cooling.

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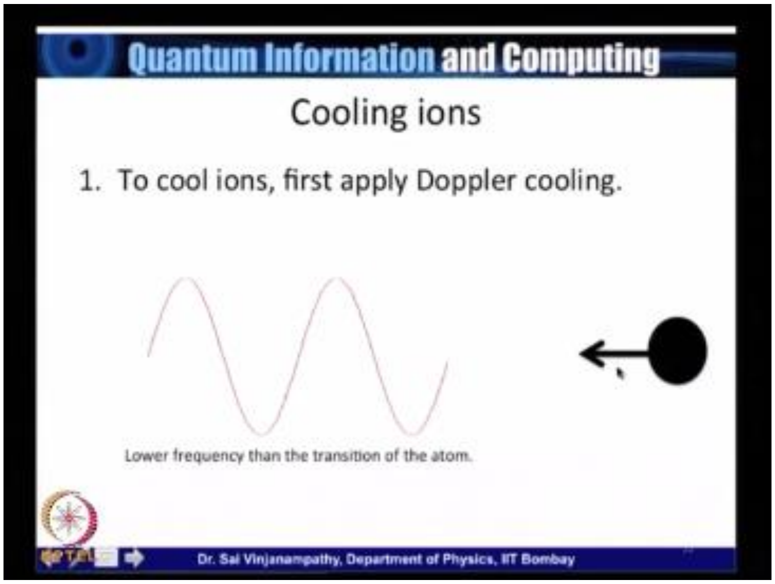

So let me describe to you sideband cooling briefly now.

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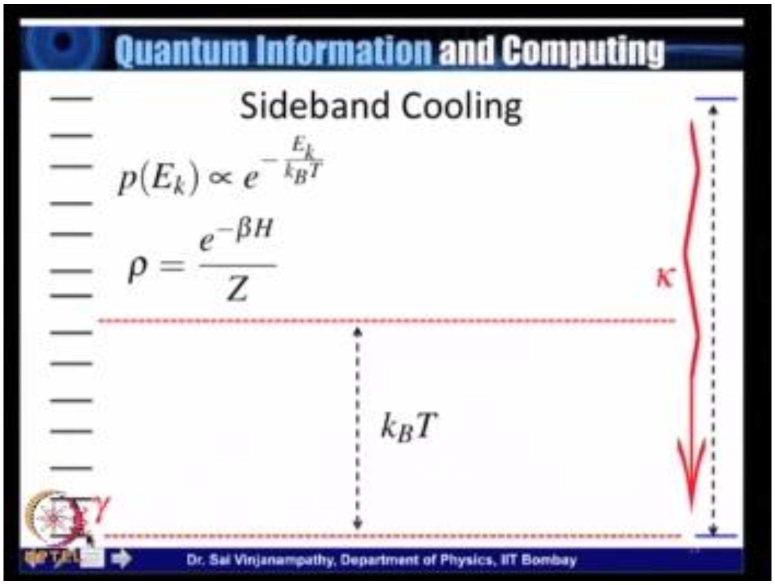

Consider the following abstract problem so imagine that you have an atom which has many energy levels and which has a particularly narrow energy gap furthermore imagine that the damping rates of this atom are rather slow as well now consider the following scenario where the temperature sets the scale K_BT which is much larger than the atomic frequency gap what this means is that the state of the atom is a thermal state given by e to the - β H divided by the partition here H reference to just the bare Hamiltonian of the atom consequently an energy level E_K somewhere in the middle is occupied with probability e – β E_K this is the Boltzmann distribution factor.

What this means is that this qubit is no longer the ground state and it breaks one of the assumptions that is made in the DiVincenzo criterion listen remember that criterion 2 was that we must be able to initialize this quantum system in the ground state and because of this thermal occupancy this, this atom is no longer than the ground state so we wish to cool this atom what do we do what we should do.

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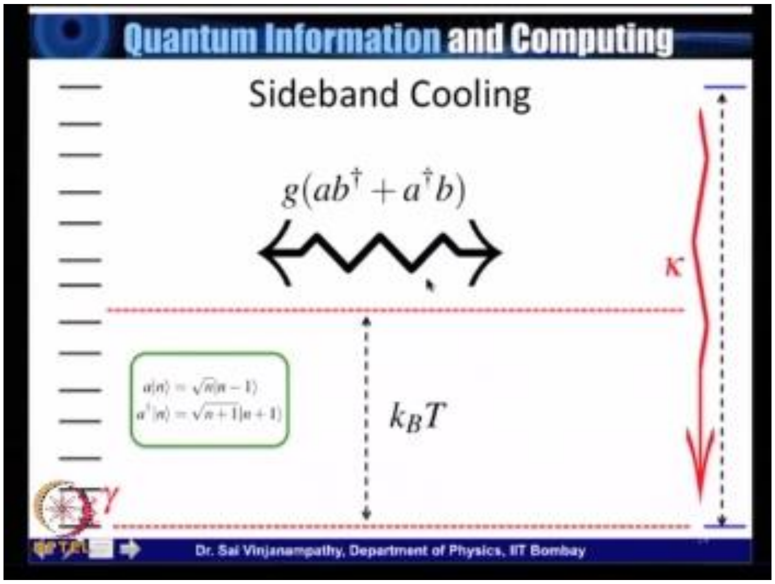

Is in fact to bring another atom which has a much larger spacing and which is damped really, really fast by somebody to cover now because this atom has an energy level spacing much larger than the temperature scale and because it is damped really fast it is almost always going to be in the ground state which is to say that the density matrix of this item is quite well approximated by the ground state now what this means is that our target atom has a lot of entropy in it where is the auxiliary atom that we have just brought in has almost no end to the image side band cooling is a technique that allows us to shunt entropy from the target atom.

To the auxiliary air to do this what we need is some sort of coupling between these weapons a typical coupling is of the form ab dagger + data will be what is the physical meaning of this term ab dagger to say that will be note that a is the annihilation operator for this atom and v is the annihilation operator from this happen further more remember that the annihilation operator reduces the occupancy of the foxed state by 1 right and the creation operator which is a^+ increases the occupancy of the foxed state by 1. So what is the meaning of this Hamiltonian.

The meaning of this Hamiltonian is that we would actually like to reduce the number of excitations in this atom and transfer the excitations to this atom which will then subsequently be transferred. For Hamiltonian set we need to be and this is conjugate of this which is a^+ this explains that Sideband cooling interaction Hamiltonian. So how is this implemented for our physical system which is the high on top, so let us look at that.

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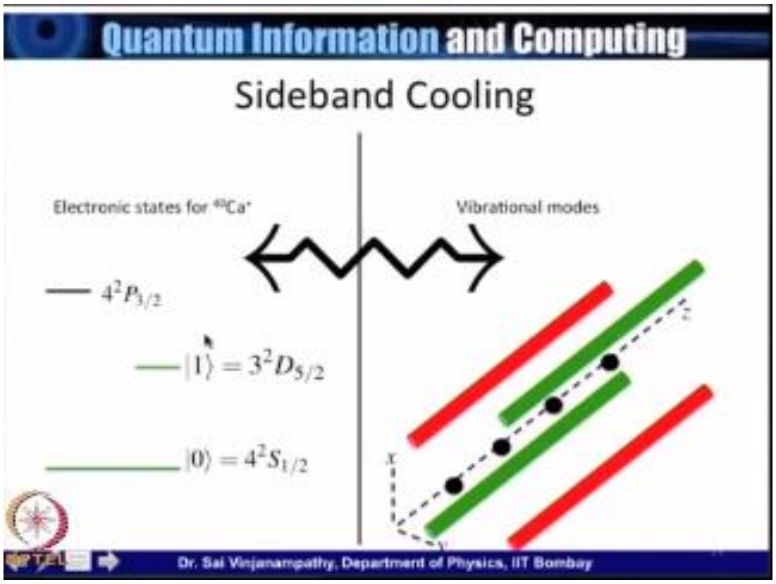

In our physical implementation there are two modes as well, the first of these is the electronic states of the trap up for instance let us consider calcium a good qubit is represented below the vibration modes of this atom represent the other degrees of it. So let us see how both of these modes were used to cool the atom.

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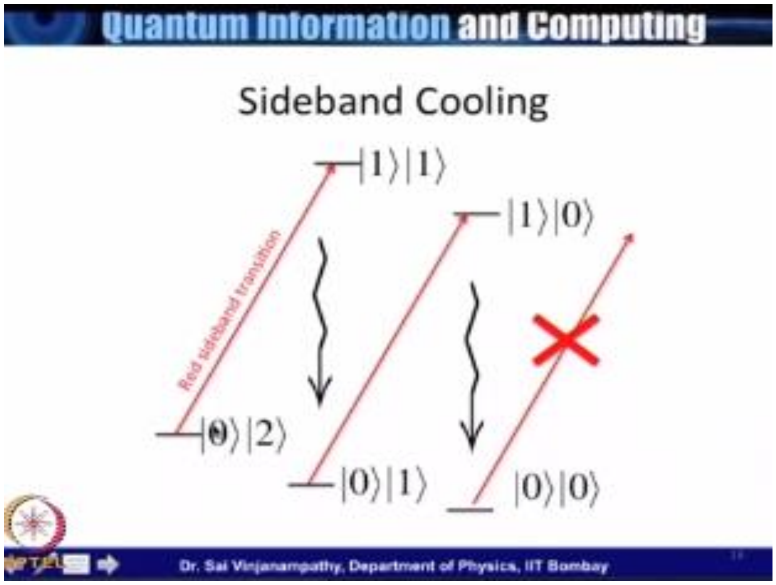

Consider the electronic state 0 and the phononic state 2 what this means is that the electronic state is in the ground state and the phonons which are quantization of the vibration modes, so let us see how sideband cooling is implemented with these two modes.

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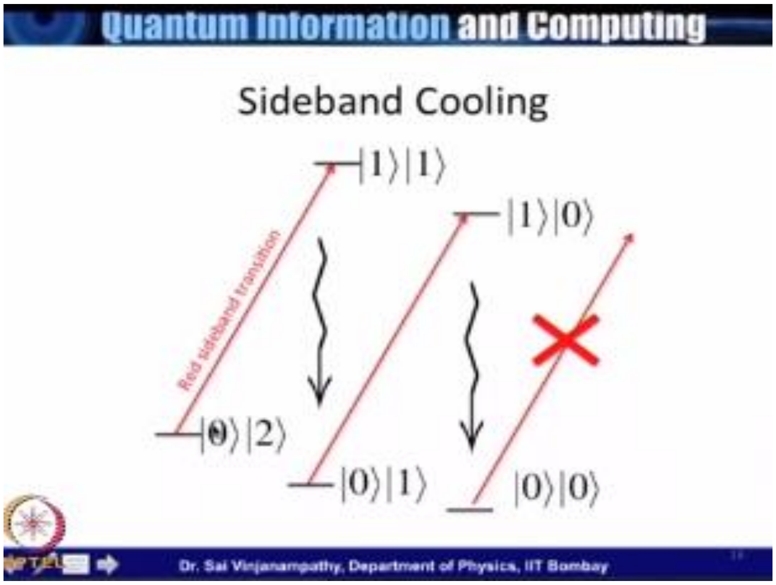

Consider the following transitions so consider the state 02 where zero reference to the electronic degree of freedom and 2 reference to the vibration degree of freedom what we want to do is cool this vibration degree of freedom. So what we would like to do is transfer this excitation to the electronic degree of freedom so there is something called the sideband transition, the red sideband transition in this case which I will describe in the notes for you which does exactly this it is the $a^+ b + b^+ a$ transition and what it does is it takes 02 and it outputs 11.

Here this 1 refers to the electronic degree of freedom having one, having an occupancy of one and the other one reference to the fact that the phononic degree of freedom still has an occupancy of 1 which is still 1 phonon. This is dumped quickly and we find ourselves in the states 01 again repeated application of the red sideband transition takes us to the state 10, so this excitation is transferred from here to here and it takes us to the state 1 and it leaves the phononic degree of freedom in the state 0 and this again decays quickly to 00.

And because this is now the ground state there is no occupancy for this transition to go anywhere and the cooling has been completed so this procedure of combining doctor cooler with sideband cooling allows us to get extremely cold atoms which are then useful to do quantum computing. So let me briefly discuss how gates are implemented in an ion trap quantum computer.

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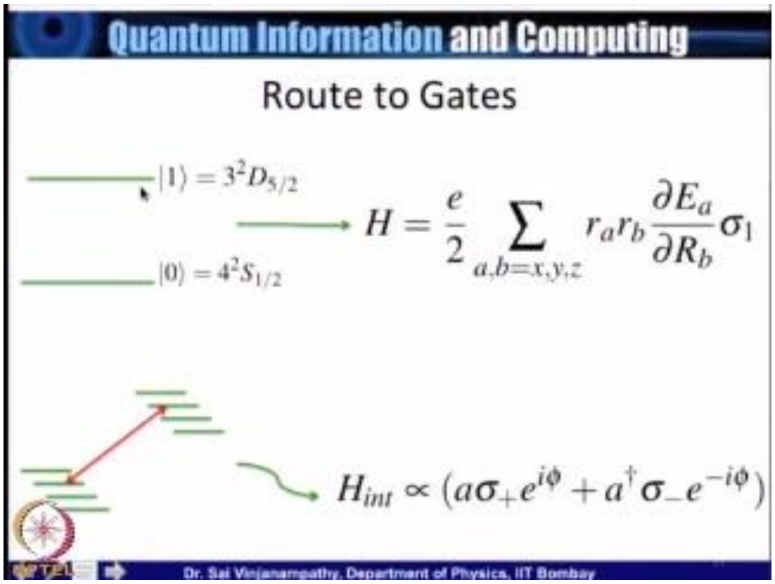

The route to implementing quantum gates is represented in this slide we have a quantum Hamiltonian which is proportional to the σ_X and this allows us to make transitions between 0 and 1 further more to perform multi qubit operations the following interaction Hamiltonian is available which is also form $a\sigma+ e^{i\pi +} a^+ \sigma\text{-}e^{-i\pi}$ note that this is exactly of the same form as before it has the form a b^+ + a^+ b but instead of the second harmonic oscillator mode we have a single spin this I will show in the notes.

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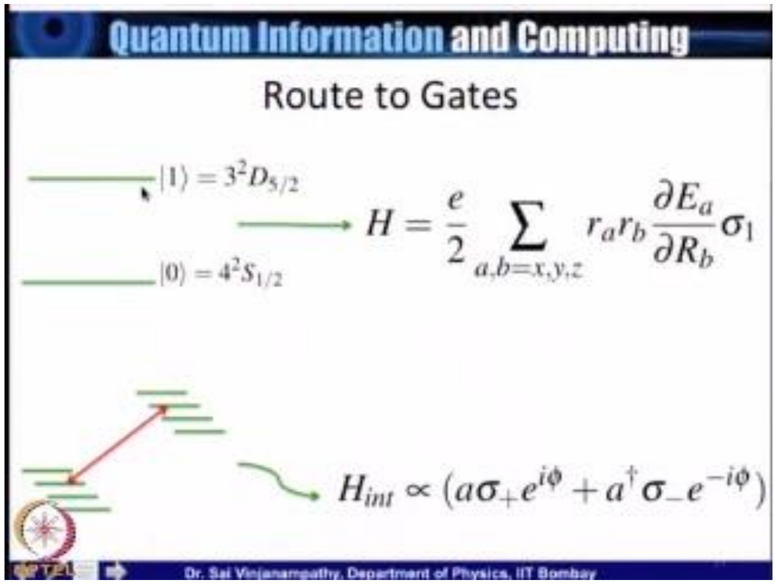

Allows us to perform both single qubit gates and two qubit gates let me summarize briefly.

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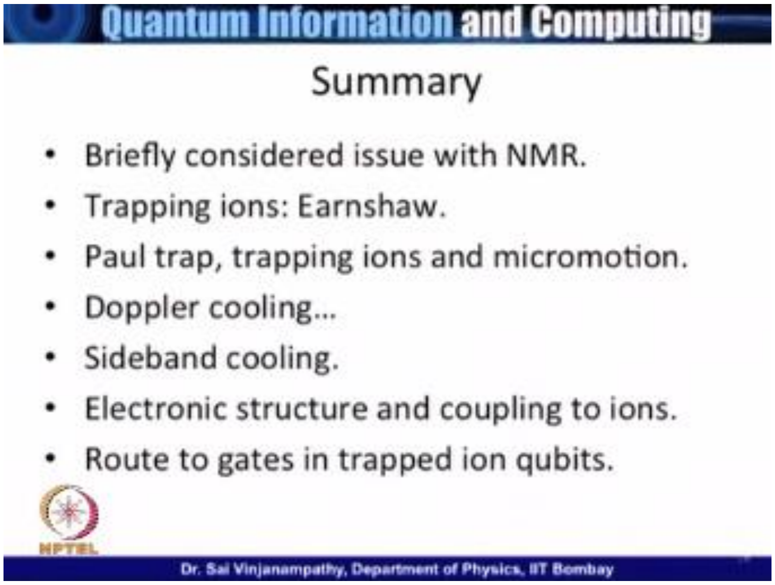

In this lecture we saw that nuclear magnetic resonance qubits are not good qubits. We considered an alternative which was trapped ions we consider the issue of trapping ions and we saw that Earnshaws theorem implies that electrostatic potentials cannot trap ions. We then consider the spinning saddle example and motivated the fall trap. We saw the vibration modes of the fall trap and considered motion along the axis and in the transverse direction.

We discussed heating and we discussed how to resolve heating in cooling we discussed two separate things one was doctor cooling very briefly and the other was sideband cooling. Finally we discussed how gates could be implemented in the ion of quantum computer, thank you very much.

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