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Quantum Information and Computing

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Modul No.07

Lecture No.40

EPR and Bell's Inequalities-III

In the last couple of lectures I have been talking about what we said is Einstein Podolsky Rosen paradox or in short the EPR paradox the paradox and a discussion of it is very central to quantum computing for the simple reason that if Einstein objections you are right and if there were hidden variables in the problem our entire philosophy of doing computing through quantum mechanics would have has been changed.

In other words we want to find out whether the Copenhagen interpretation of quantum mechanics is right or are there quantities which are hidden from our views hidden from our consciousness which actually code the information about the physical properties of the system as I would like to repeat that the Copenhagen interpretation of quantum mechanics says that a when you do a measurement any one of the Eigen values of the operator corresponding to the physical alternatives will be realized with a probability .

In other words it is the process of measurement which gave it away Einstein is idea of realism says that an object must have a physical property independent of the measurement process measurement process is simply a means of revealing the value of that physical property secondly there is a question of locality which says that if you make a measurement on one particle at one point in space and you make a second measurement on another particle at a far enough distance.

So that disturbance of having the measurement in the first instance cannot influence what measurement you are making in the second place and this is what is called the locality, it is this dual property of local realism that is cornerstone of EPR paradox or Einstein Podolsky Rosen objection to the standard or the Copenhagen interpretation of quantum mechanics, and what we did in the last lecture.

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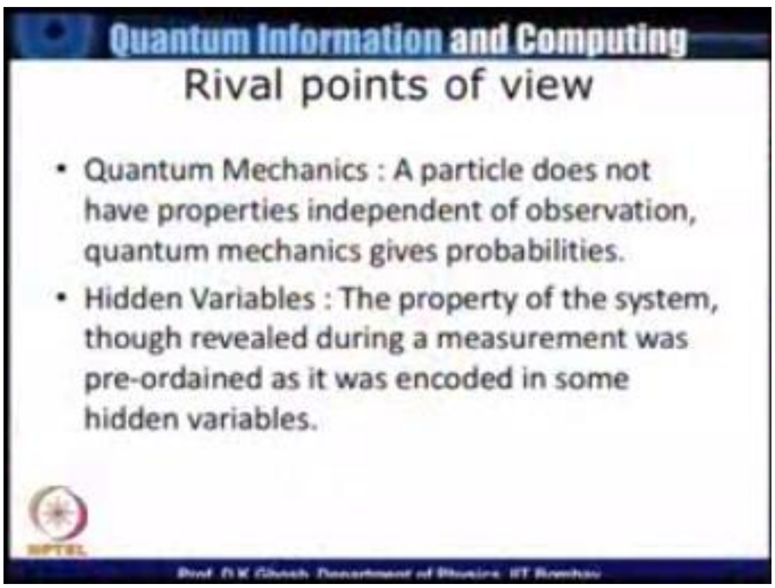

Is to give you a gedanken experiment where we put quantum mechanics and hidden variables to test and we were able to derive an inequality which is a class of inequalities which goes by the name of Bells inequality to check or provide a confirmatory test on which one is right there are many such in equalities in physics I would end this discussion on hidden variables and quantum mechanics by another inequality which can provide a test for the correctness of either theory. And this is what is known as CHSH inequality.

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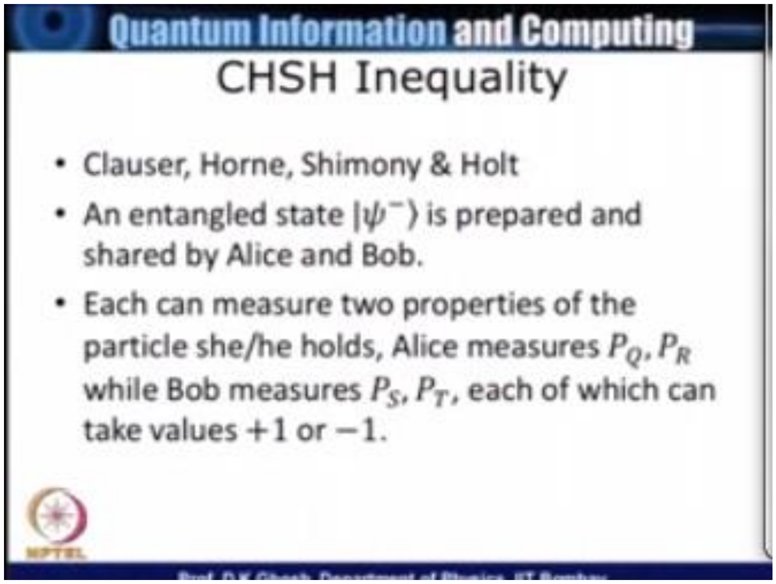

Due to Clauser, Horne, Shimony and Hold. So the problem is we again come back to our well-known entangled state ψ - which is 0 1 – 1 0 x√2 the first particle is with Alice and the second particle will Bob now I do a slightly different measurements let us say Alice can measure any two properties of the particles she wrote now remember in order to do this I assume there are identical copies of this entangled states that are available to me.

So that I can make experiments or I can observe any property that I like and take a statistical data. So Alice can measure two properties which we will call as q and r they PQ or PR P for property while Bob can measure to other property S and T and each one of these four quantities can take a value either + 1 or a - 1 now let me illustrate it by.

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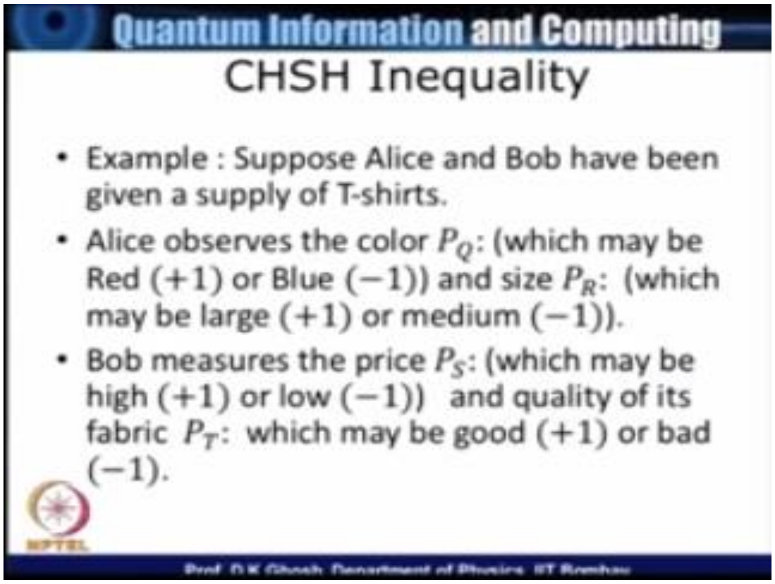

Long quantum mechanical example what do I mean does suppose Alice and Bob have been given a supply of T-shirts and the T-shirts are distinguished by four different properties out of it Alice can observe two properties and Bob can observe two other properties they have the measuring equipments for doing those. So for example, Alice can observe the color of the shirts which come in two colors it could be red in which case he assigns the value one to one of the measures with you or it could be blue in which case it is assigned a value -1.

She could also observe the size of the T-shirts which could be large in which case C assigns a value +1 or medium which case is assigned the value -1 Bob has two other properties with the same supply see he is interested in observing the price which is s if it is high we call it plus one if it is low equal to -1and the quality of the fabric which may be good of course in the first case we give it a value +1 and the second place we give it a value -1.So these are examples though in classical field of.

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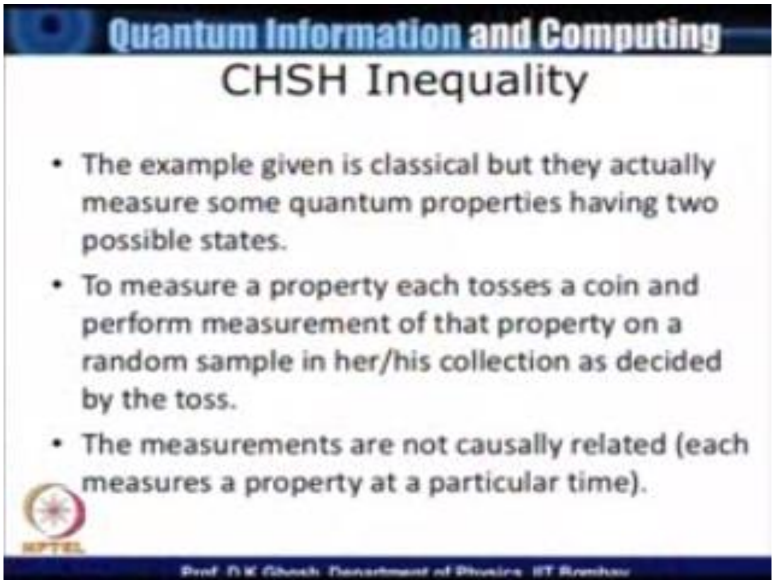

Observations which could be assigned values +1 or -1 now what do they do, so firstly they just decide to measure a property at random what does Alice do and Alice takes a coin process it and for example if it is head then she would look at its color and if it is tail then she would look at it size. Likewise Bob will also do an independent experiment tossing a coin if it is head he will look at the price if it is tail you will look at the quality of the fabric.

And get a result and this result they will keep that that is at the same instant Alice measures one property and Bob measures another property. And so therefore, these are not casually related because we assume Bob and Alice are separated by space line resource. Now let us look at.

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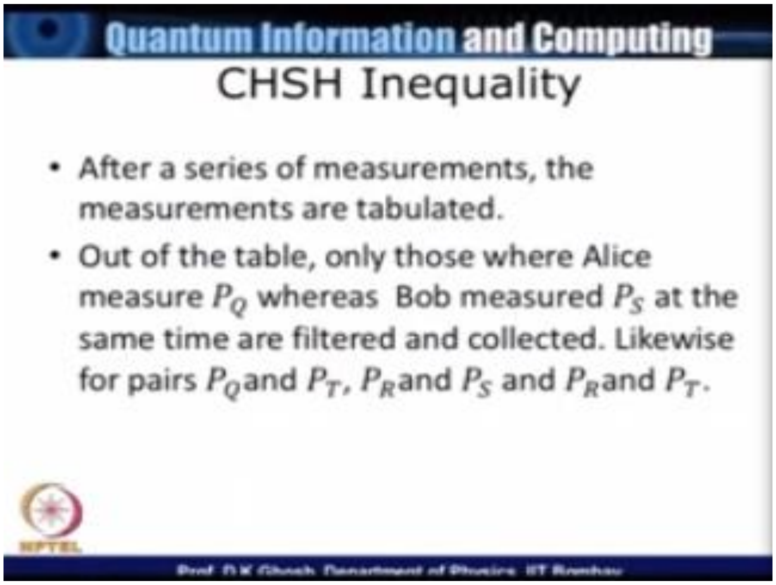

The what will they do they will make a table so that we naked table the way we do in a lab observation number one what it means is Alice is first observation Bob is first, so Alice has measured one of the properties and Bob has measured another we also write down which properties each has measured at a given time. So what it means is that there are various pairings, so since Alice is measuring the property Q and R and Bob is measuring the properties Sand Q, so i can have Q with S, Q with T, R with S and R with these are the various collections of pairs that were possible.

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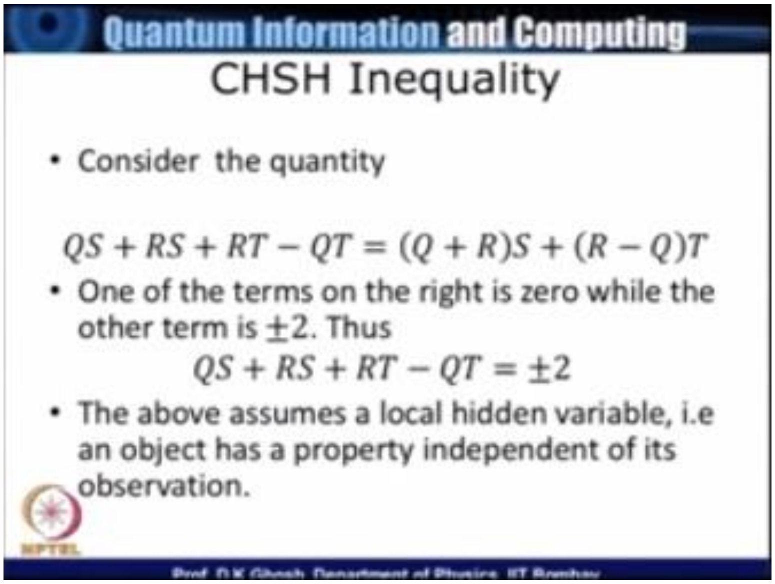

Now let me look at the value of the quantities which are QS+RS+RT-QT now what does it mean I pick up maybe first from set of observation where in the table I have Bob is measuring S Alice is measuring Q so that is a QS and from another part of the table I pick up an RS another part RQ how the profanity and I look at this quantity which is QS+RS+RT- note the – sign –QT RS and this quantity I keep on tabulating it in sets of four, so this is the so four observations will be mixed, another four observation out of the table will give me this again like this. Now if you look at this expression.

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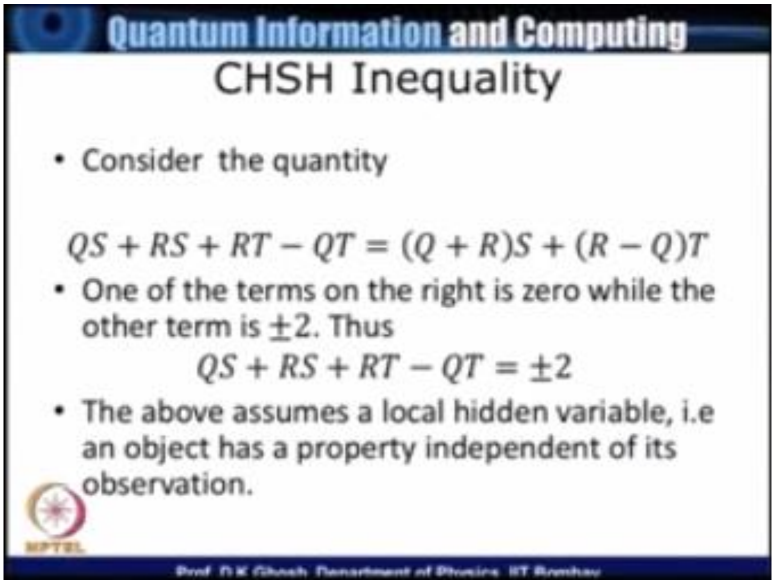

QS+RS+RT-QT I can combine them into (Q+R) S+(R-Q)T now look at this, look at the right hand side since Q and R can take values either +1 or -1 so one of these terms must be equal to 0 so for instance I could have Q = 1,R=1 in which case the first Q+R=2 where R-Q=0 I could have Q=-1, R=-1 in that case my first term is -2S because and second term is still there. But suppose Q and R are different so suppose Q is +1 R is -1 then the first term is 0, but the second term is -2T and likewise for Q =-1 and R=+1, so one of those terms on the right-hand side either Q+R or R-Q term is 0.

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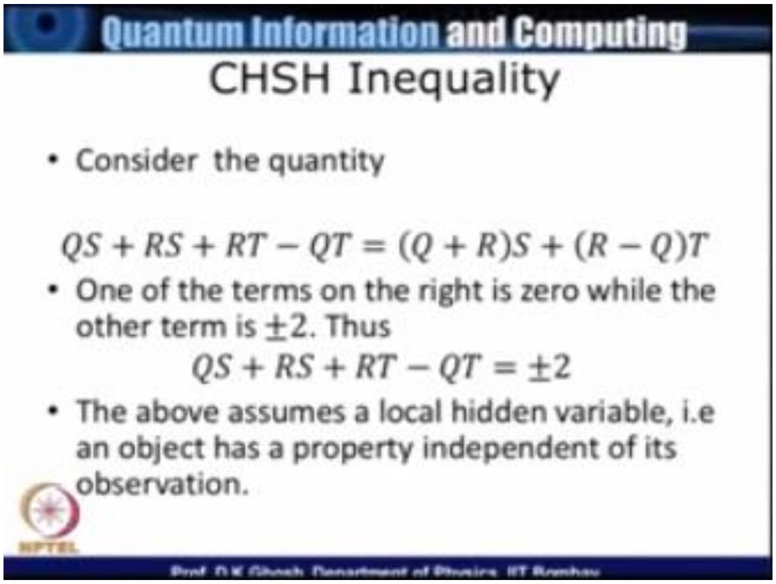

And when one term is 0 the other term can have either a value +2 or a -2 so therefore the quantity on the left hand side which is QS+RS+RQ-QT for a set of four observations takes the value either +2 or -2 okay, and so what we have said is that this is what would happen if there are local hidden variables. If an object has a property independent of its observation, because when we say I gave you that classical example color fabric prices quality of fabric price and all that and these are essentially classically fixed quantities that is the property which is independent of the observation. Only during observation I you know what is it, so if you now consider several sets like this since each term is either +2 or -2.

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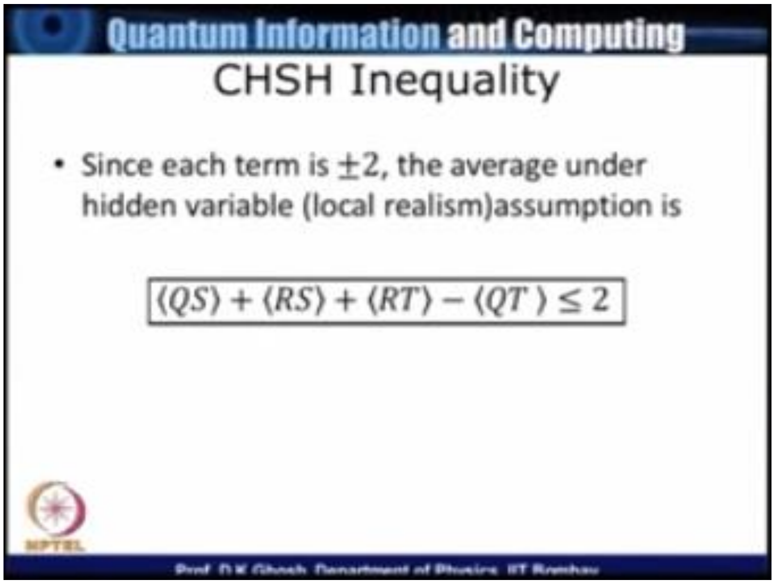

If I take an average of this, now average of the sum is some of the average there I can find out what is the expectation value or average value of QS+RS+RT-QT and since the each of the term gave me +2 or a -2 this sum has to be less than or equal to 2, because remember because of that - sign which is there. So let us look at.

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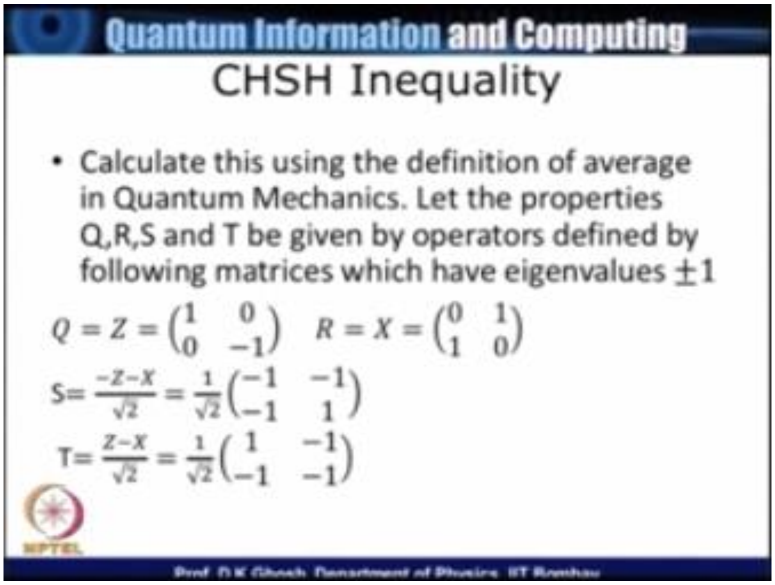

What would happen if I believed in quantum mechanics, now in quantum mechanics what I do is to consider similar similarly four properties two of them with Alice given down two of them with Bob ST and since in quantum mechanics observables are given by operators having certain eigen values and I am saying that these eigen values can be either +1 or -1. Let me take QS is equal to just the σ_z the Pauli Z matrix, R=σ_X Pauli X matrix and S and P are combinations of Z and X in the way they appear on this line.

You can check of course you already know that σ_Z and σ_X have eigen values +1 or -1 but it is trivial to check that S and T defined like $-Z - X/\sqrt{2}$ extra, it also gives eigen values +1 or -1. Having done that let us look at what do these things give, now I want to calculate expectation values of these four things that I wrote down QR QS QT RS and RT and do it in the entangled state which we have been talking about mainly 0-1 01 -10/$\sqrt{2}$, so let us do that calculation. So I have a look at.

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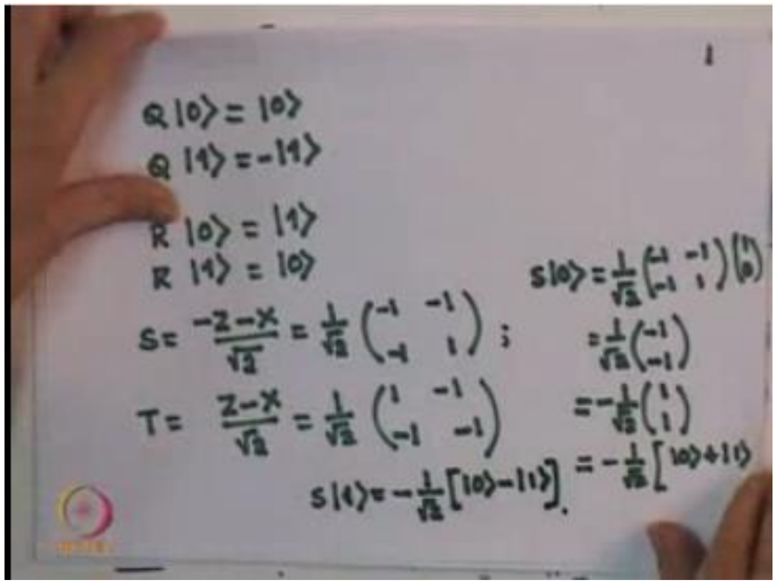

What is Q, Q is simply Z, σ_Z that so Q acting on 0 is nothing but 0, but Q acting on 1 is -1 because that is a σ_Z. R since it is a X gate this will give me flip the bit so R acting on 1 is 0 and R acting on 0 is 1. Now we define S as –Z-X/√2 so I have got this as 1/√2 Z do you remember is 1-1 since there is a -Z so I get -1 +1 and this is –X, X only had off diagonal component 11 and because of this -sign i get -1-1.

So this is operator S and the operator T is Z-X/√2 and once again I have 1/√2 I put the Z then in its place and X with a -sign in its place. So let us calculate what does S acting on 0 will be, you can check immediately this is one of them only I will work out -1-1-11 acting on 10 that is 1√2 minus 1 and here i have got another - 1 so if you take out a - 1√2 i get 11 so this is nothing but a Hadamard game active on the state 0 and then flipping a sign Hadamard gate followed by a face so this is equal to -1 √2 + 1now you can easily calculate what does S give you on 1 is giving on one gives you same way - 1 √2 instead of 0 +1 you get 0 -1 that leaves us with T.

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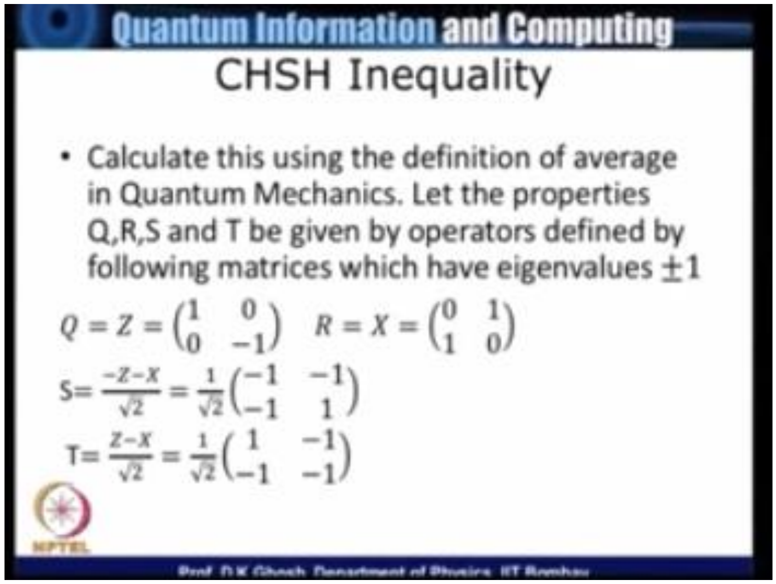

So T is.

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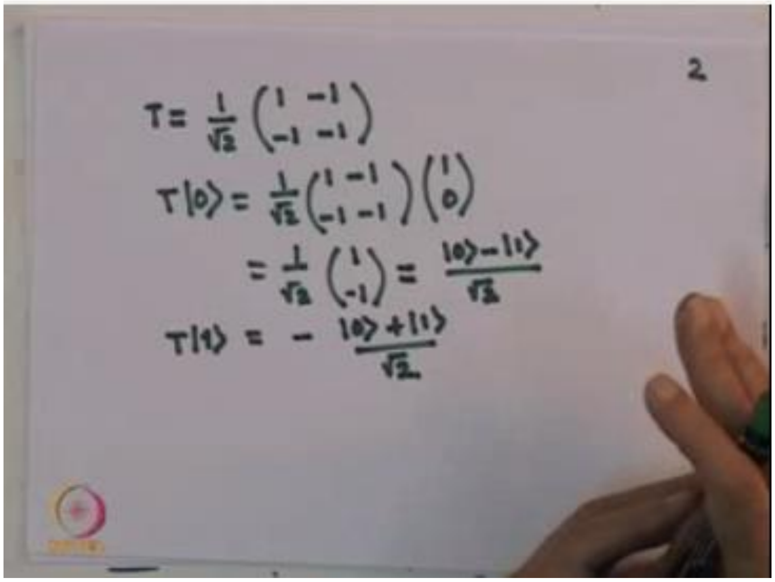

We have worked it out 1√2 1 -1 -1 -1 so T acting on 0 is 1 / √2 1 -1 -1 -1 acting on 10 and that is 1 √2 you can check this one and this is minus 1so this is nothing but 0 - is and T active on known similarly will give you 0 - of 0 + 1these are a trivial algebra which you can work it out so what am i interested in I am interested in the calculating the quantities expectation value in the state side so now that I have worked out what does q r s t acting on 0 & 1 give me so let us look at so for instance I will just do one calculation.

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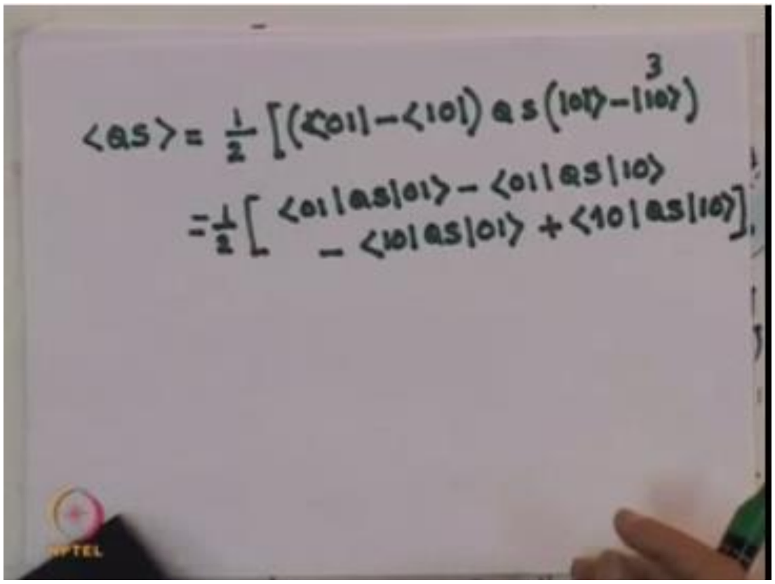

What does QS so what am i doing I have a 1√2 in my state twice so I got ½ bra 0 1 - bra 10 now I have a Q so let me write this as QS and 0 1 - 1 this obviously has four terms so I got 1/2 let me just expand this out i get 0 1 q QS 0 1 - 0 1 QS 1 0 - 10 QS 0 1 and +1 0 QS now since i already know the properties of Q and S so let us look at what this give me.

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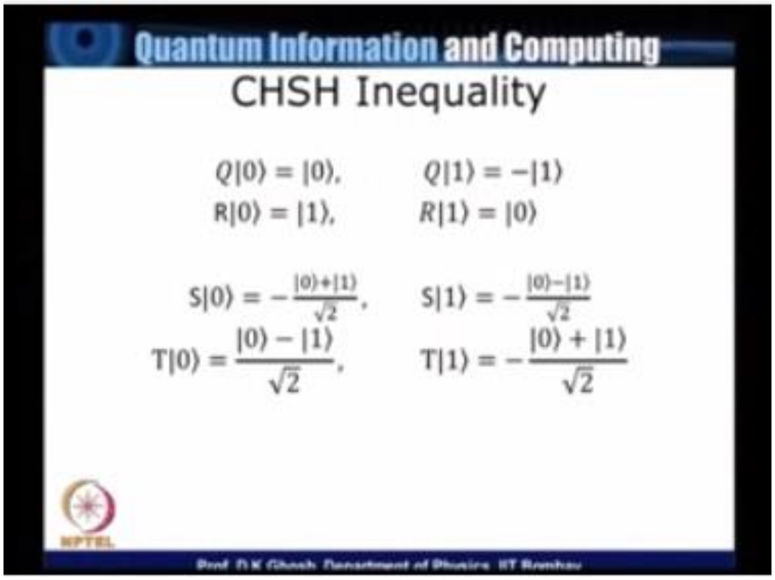

So I get.

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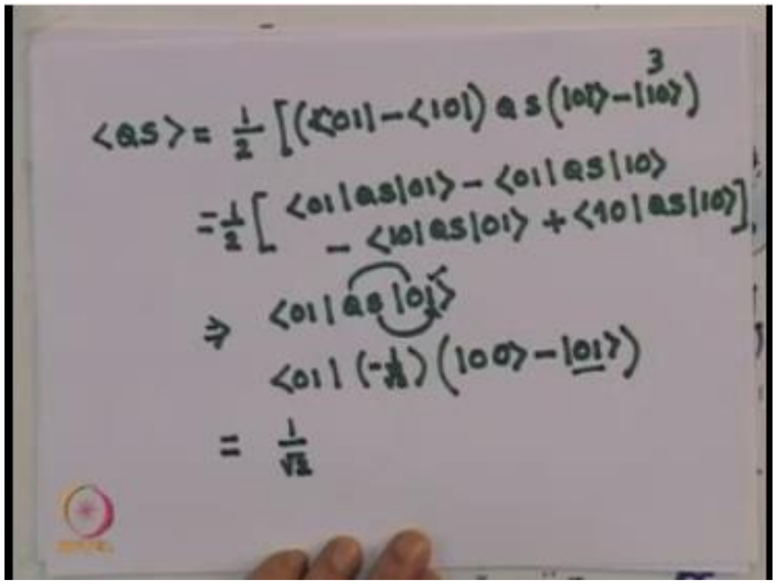

Let us look at the first off so i have 0 1 QS 01 so Q acting on 0 is 0 now s acting on one we are written down so this is 0 1s acting on 0 is -1/√2, I have got a 0 and s acting on one gives me two things there so I get either a 0 or I get a 1 so therefore I get a 0 0-0. So look at this so I get it down here there is a 1/√2there but in fact the only nonzero term comes from this term this 0-1 with this 0 and there is already a minus sin there, so I get it 1/√2 all others you notice this is 0 0 so therefore this would be orthogonal to this as well as to the other term.

And so I will be left with simply a 1 over school and likewise you can calculate the expectation value of each time.

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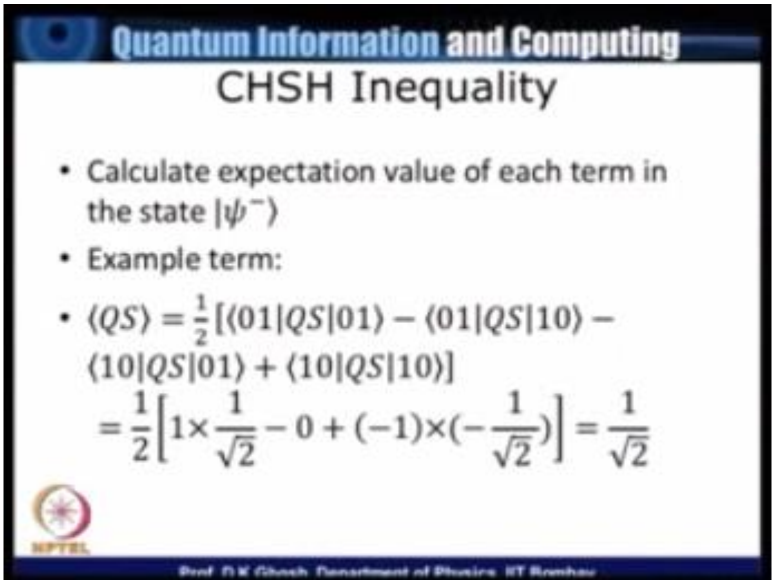

Now if you do this they it turns out that first three terms are 1/√2 and the last term is -1/√2, so that when you add them up you get total of four terms each having 1/√2 in other words if I add them up I get this one.

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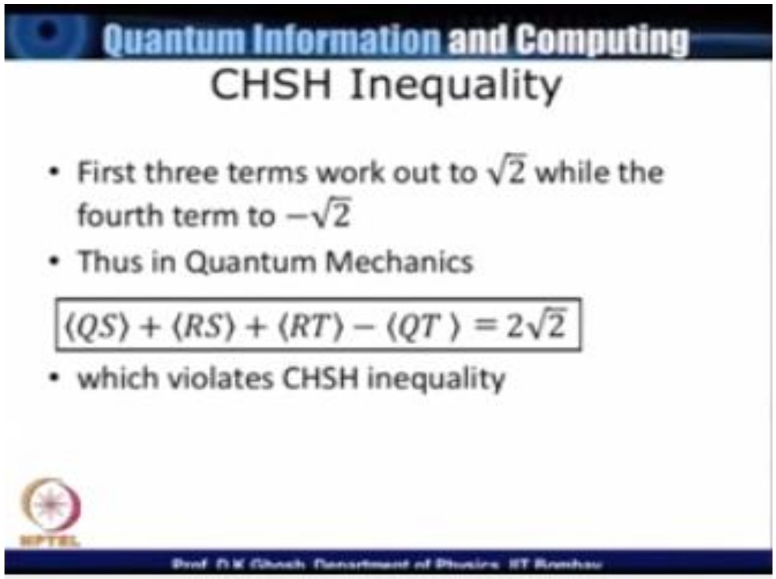

First three terms are $1/\sqrt{2}$ the fourth term is $-1/\sqrt{2}$. So therefore the inequality in this case will be QS + RS + RT - QT is two x squared up, and this violates CHSH inequality which we worked out for the classical, in other words it provides a test of each one.

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