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Quantum Infromation and Computing

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Modul No.06

Lecture No.32

Quantum Error Correction-III Shor's Qubit Code

In the last lecture and the previous one we talked about Shor's error correction codes talking about three qubit error correction code as we have seen this is not a complete code and takes care of bit flips, but let us talk about the more general code which will take care of both bit flips and phase flips but before doing that let us make a few observations.

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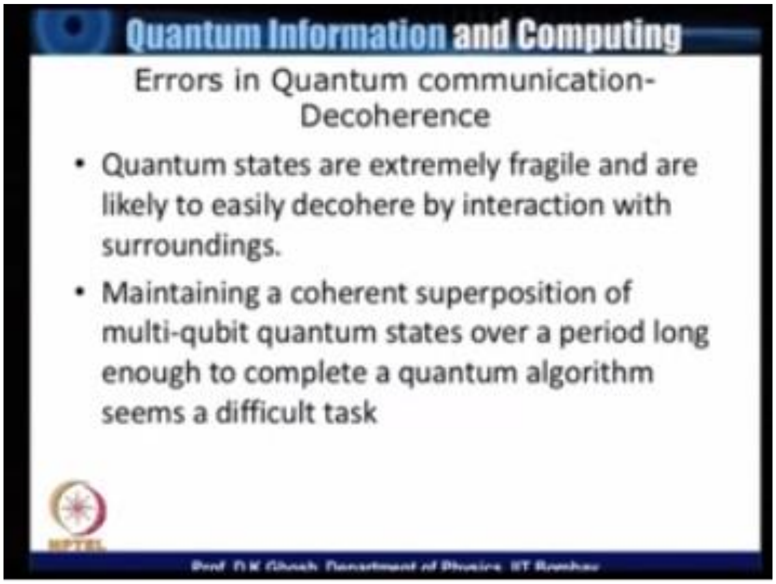

Which are very easily switch we are going for error correction we said that the quantum states are extremely fragile there likely to easily decode and that happens by interactions with surroundings. And it is necessary that we maintain a coherent superposition of multi qubit quantum states if you want to do any quantum algorithm satisfactory, and this becomes a difficult task because of the fragile nature of the qubits and the qubits do not maintain their coherence. So bit flips and not the only errors.

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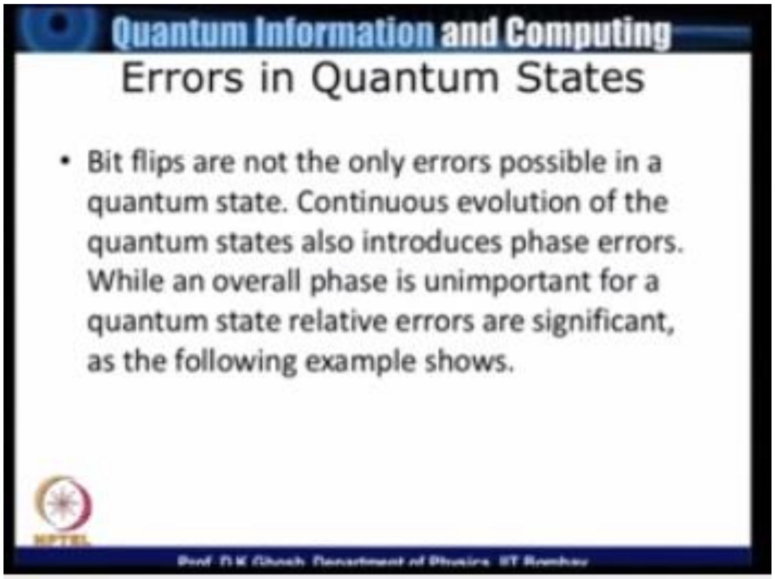

That are possible in a quantum state continuous evaluation of the quantum states also induce as they said phase errors and the while a general overall phase error is unimportant for a quantum computation for the simple reason that two states which differ by an overall constant represent the same state the if there is a relative face between the components of the same states then of course that creates a problem. So for instance, suppose I start with their initial state.

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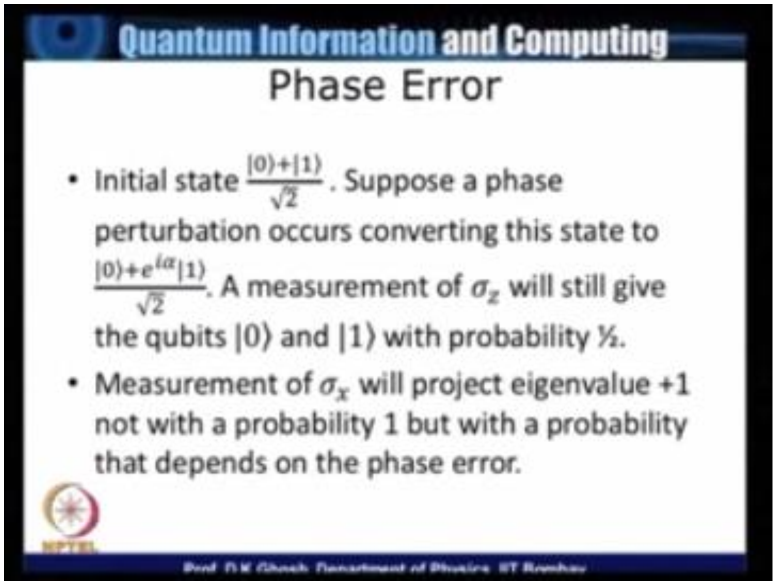

Which is $0 + 1/\sqrt{2}$ and let us suppose that a phase perturbation occurs to a state which has a relative phase between them. For example, $0 + 2^{ei\alpha}\ 1/\sqrt{2}$ now if you make a measurement of σ_z you will still get either 0 or 1 while in this particular case you have $0 + 1/\sqrt{2}$ so therefore the σ_z expectation values will be ½ for both 0 and 1. But however if you made a measurement of σ_x remember the original state was an Eigen state of σ_x with an Eigen value + 1 but now what happens is this because of this relative phase we have seen already that the Eigen value +1 and the – 1 will not be 1 and 0 respectively.

As would have been the case if there are non relative phases and it would depend upon how much their phase is affected.

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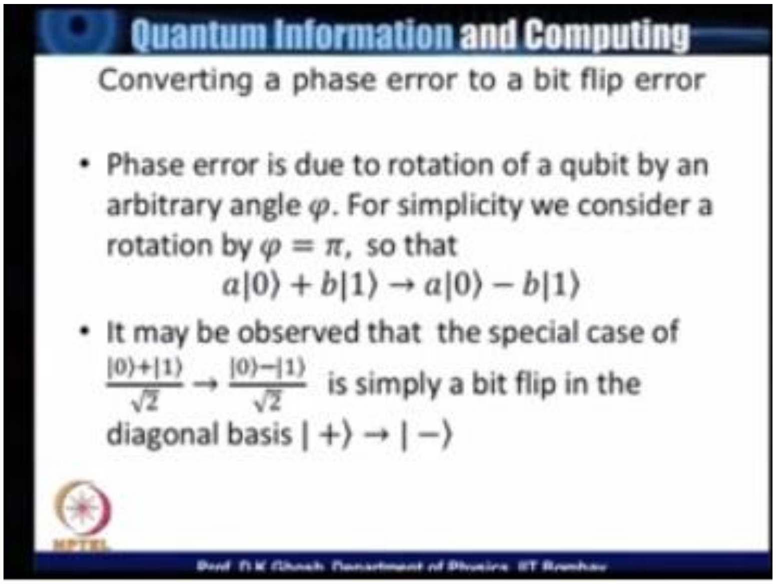

So let us look at what can one do for the phase error, one of the things that you would like to do is to talk about a phase error due to rotation of a qubit by an arbitrary angle ϕ, but in order to make discussions somewhat simple let we can just take the case of ϕ= л. Now if ϕ= л a0 +b1 goes to a state a0 – b1. Now you notice one thing that supposing I look at the case, special case of a= b = 1/√2 then this simply means a state 0 + 1/√2 has become a state 0 – 1/√2.

Now but it may be observed that while 0 + 1/√2 going to 0 – 1/√2 but suppose you are looking at it in a diagonal basis. Now if you are looking at a diagonal basis 0 + 1/√2 is nothing but the state plus but 0 – 1/ √2 is the state – so it means what was a phase flip or л in the computational basis is actually a big flip in the diagonal basis, and this is this is a useful information to have as is shown in the diagram.

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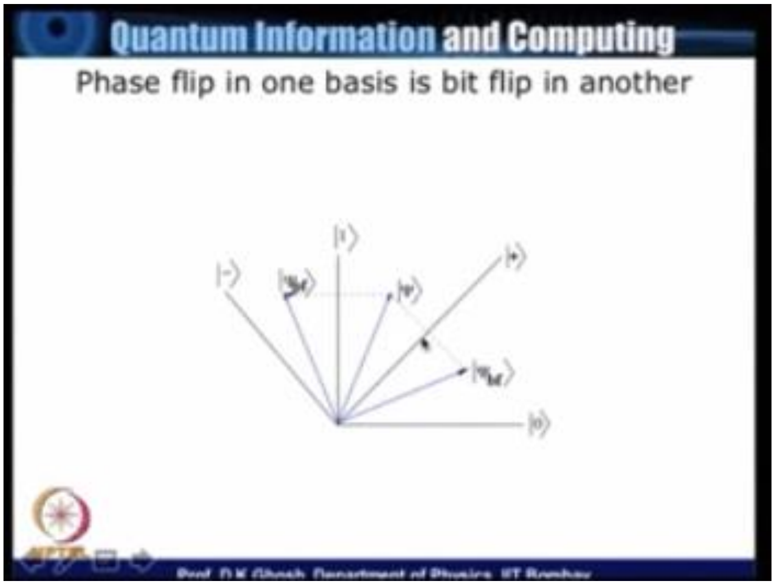

That showing here, so look at this, this diagram source this is your computation this is 0 and 1 and the diagonal basis they make an angle of 45 degrees with the computation basis so this basis is + and – now suppose I consider now an arbitrary state ψ. Now let us look at what it actually means, now if you are looking at a computational basis then this ψ which have this component the x with the 0 component is this much is shown by this bottom line here.

And of course, the component 1 would be come here, now so if there is a let say bit flip no if there is a bit flip what happens 0 goes to 1 and 1 goes to 0, now that would make this state go to this point because all that it means is the width here would be same as the width here and their length here would be the same as length there. So this would be a bit qubit. Now if you look at this geometry it simply means that this is the reflection of the state ψ about this diagonal basis.

On the other hand consider a phase flip, now what does a phase flip do so you have a a0 + b1 and let us suppose it become a0-b1. Now you can immediately see what will this become and since we have seen that a overall phase does not matter then I can bring this math and say that this state is here, if this state is here then exactly means this is the reflection about the, if you want to call in the vertical direction the vertical direction. So there is a nice geometrical interpretation which shows that a bit flip in one phase may become the phase flip in another basis.

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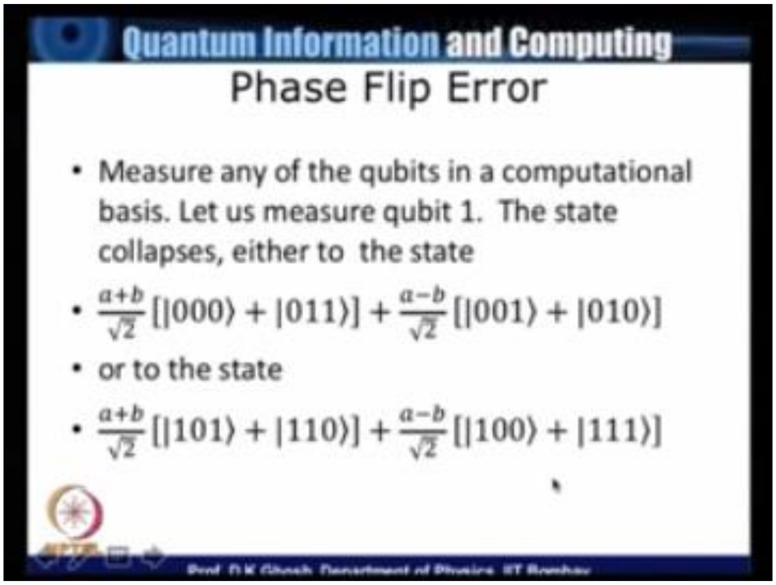

So let us look at what we could do to take care of the phase flip error and so what we need to do is to talk about a encoding that triplets in a diagonal basis. Now if you do this that this is your diagonal basis and you want to compute this in a computational basis now this is very simple as you know that $+10^2 + 1/\sqrt{2} - 10^2\ 0 - 1/\sqrt{2}$ and what you do is you simply multiply that. Now if you multiply them you will find that this will naturally give you eight states there because this is + + + and this is - - - and this is what you would then get 000, 011, 101, 110 and with the a-b/ $2\sqrt{2}$ this thing.

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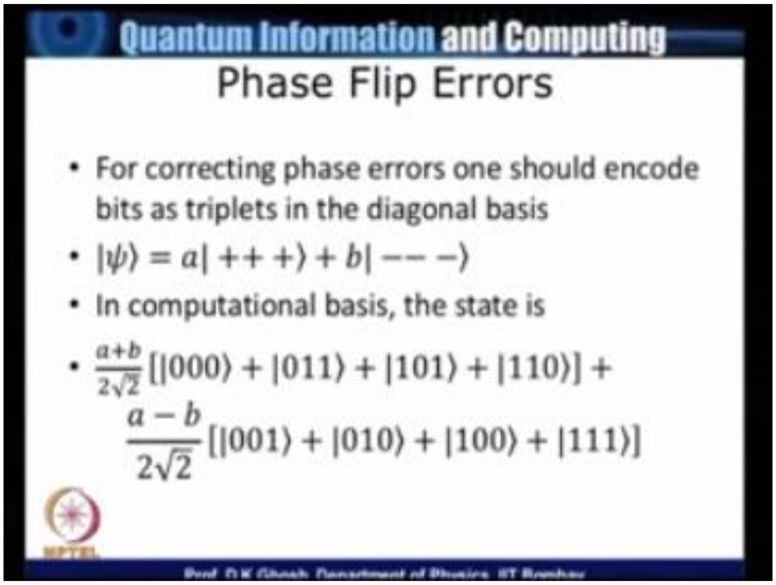

Now let us see what would happen if you did a measurement of any one of the qubits and we have illustrated in the computational basis so suppose I make this measurement.

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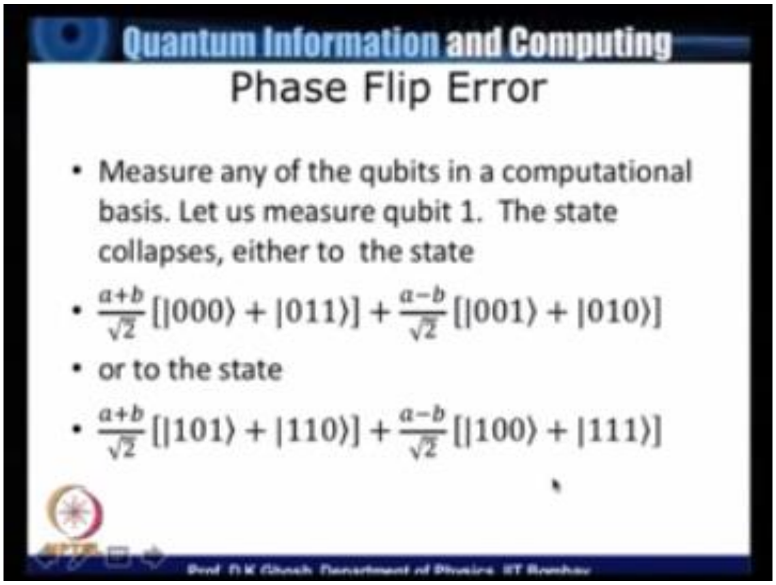

In the computational bits. Now let us measure basis number 1 now if you look at the basis number 1 flipping between the two slides. You notice here the qubit 1 which I measure will either have a value 0 in which case that the system would collapse to this term, this, this, this and that. Alternatively the qubit 1 could collapse to value 1 in which case I will have a+b/2√2 this state plus this state and a-b/2√2 this state plus that state.

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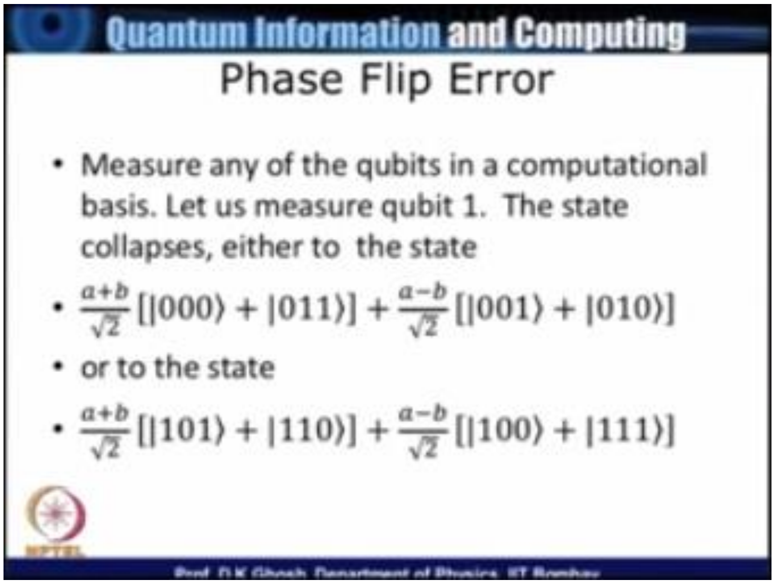

And this is what is the illustrated here, that the state collapses either to the state on the first line for which the first qubit have the value 0 or to the state which is on the second line for which the value of the first qubit 1.

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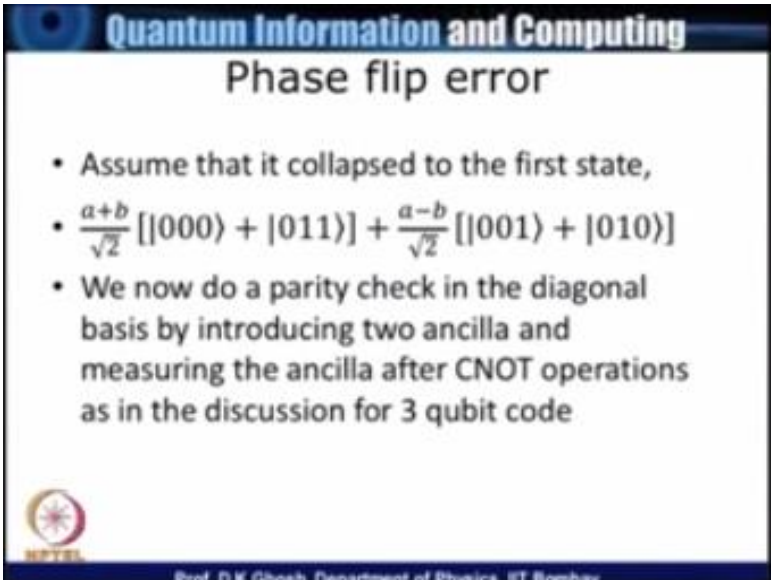

Now let us suppose that it has collapsed to the first state, so first state.

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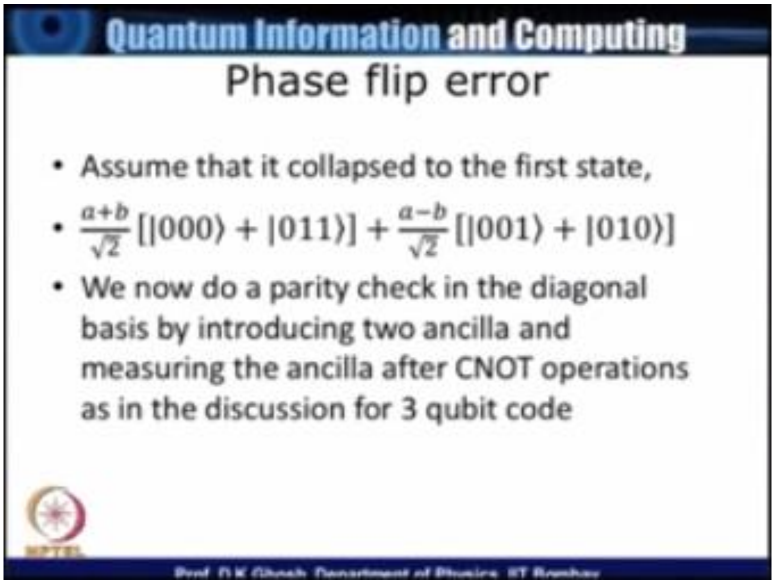

As I have shown you here is a+b/√2 [|000>+|011>] + a-b/√2[|001>+|010>] so the, now what we do is this that we need to do a parity check in the diagonal basis. Now how it is done, this is done by introducing two ancilla and measuring the ancilla after the CNOT operation as we have done and discussed it in detail while discussing 3 qubit code.

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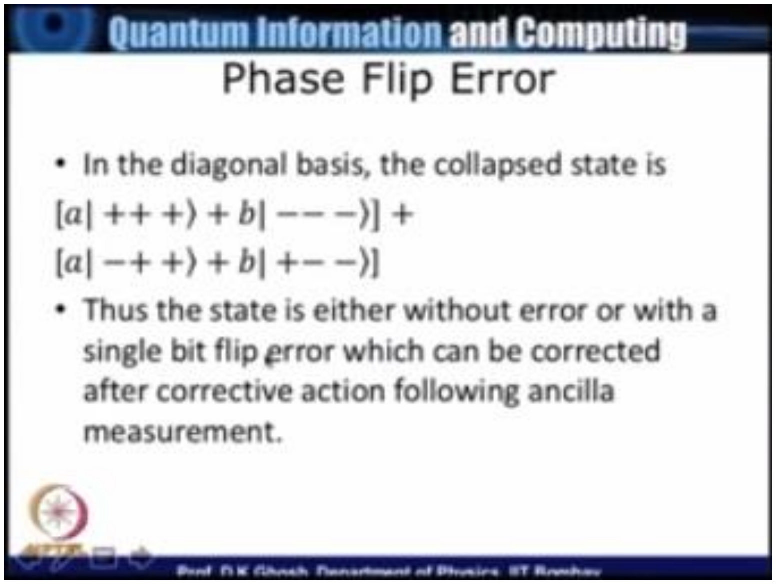

So in the diagonal basis my collapsed state is [a| times +++> + b| times ---> or a times -++>+ b times +--> you notice what is happened here, what happens is this that this is of course without error, because that is what we started with and this says that there is a single bit flip error but this bit flip error is in the diagonal basis and this we can take care by doing the corrective action as we talked about in the 3 qubit basis.

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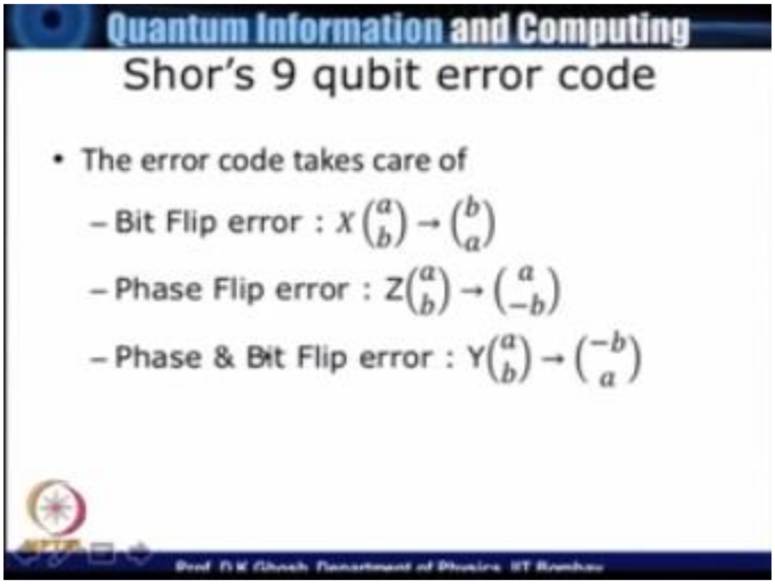

So we will now go to a situation where we take care of bit flip as well as phase flip in the same code and this is known as Shor's 9 qubit error code. Now this error code takes care of bit flip error we know that bit flip error arises because of application of an X gate on the state ab, and that gives me ba. On the other hand of phase flip error is taken care of by a operation of Z gate on ab which gives me a-b and if you have both phase on bit flip errors that essentially equivalent to Y gate and this is of course there is a overall factor of I which one has to take care. But the Y gate acting on ab gives you –ba. So these are the three types of error that we would like to take care.

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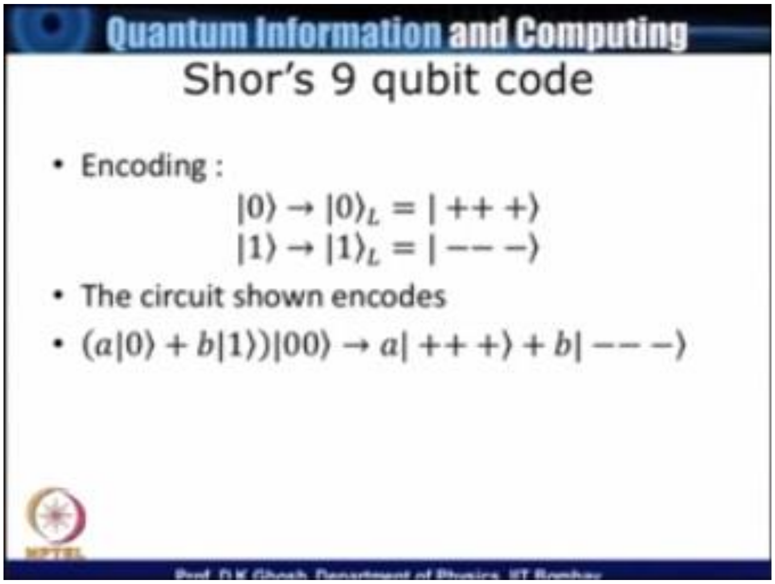

And that is that Shor's 9 qubit code takes here.

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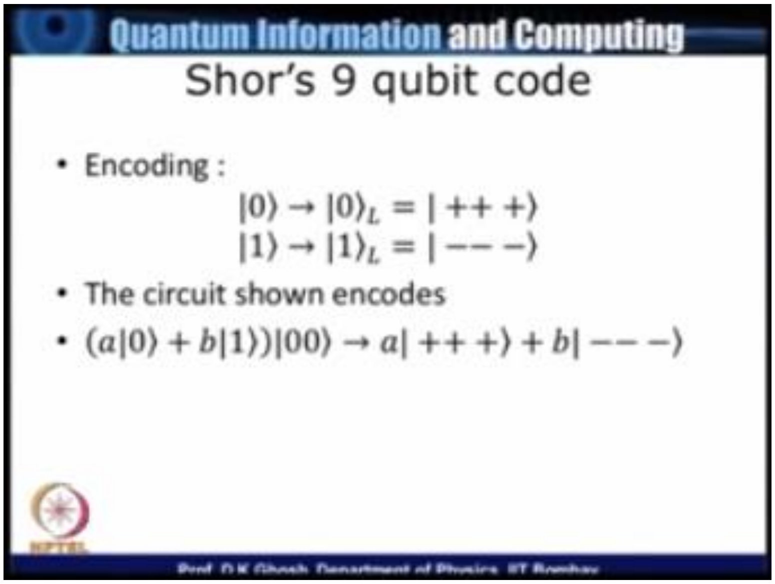

Now let us see how does Shor's 9 qubit work, so in Shor's 9 qubit.

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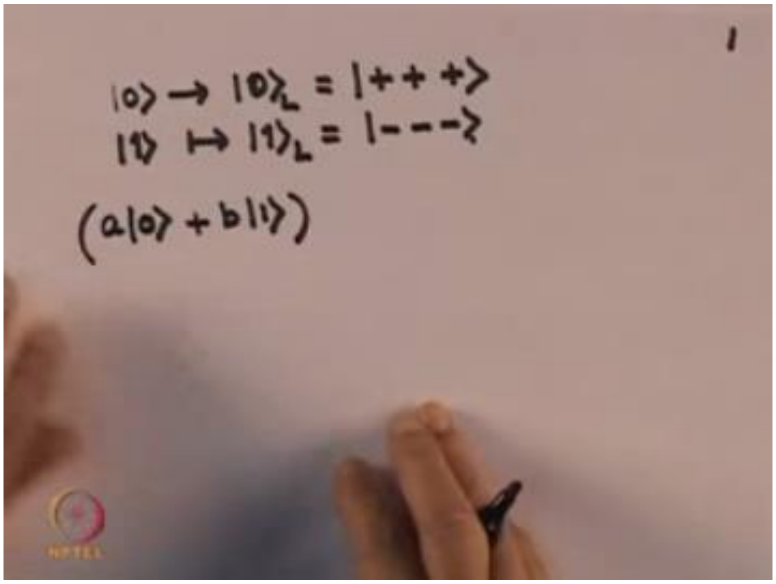

What we do is, we encode |0> as we have said several times as |0> logical and this is encoded as |+++> and |1> is encoded as |1> logical which is |--->. So what actually happens in the following that we are interested in sending through a quantum channel a state a+b|1>, so the thing is this I am showing the circuit here on the slide.

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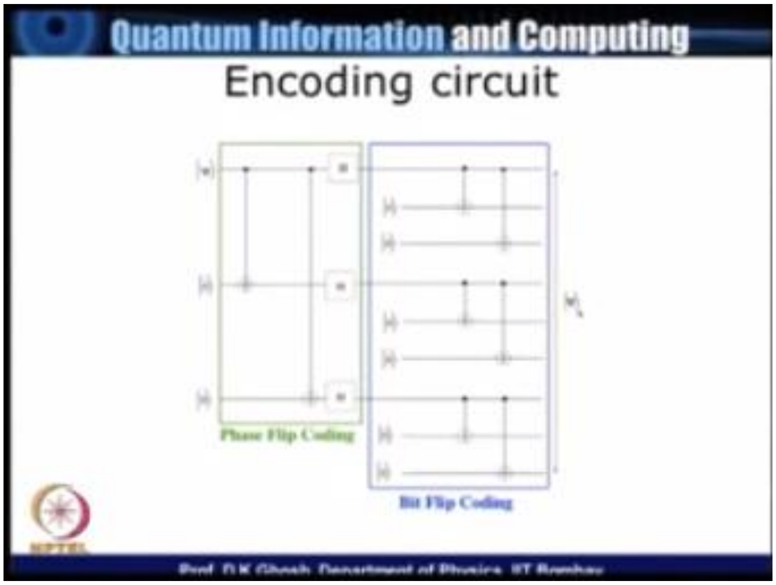

And I will go on explaining how, what does this encoding circuit actually do? So first thing is the following.

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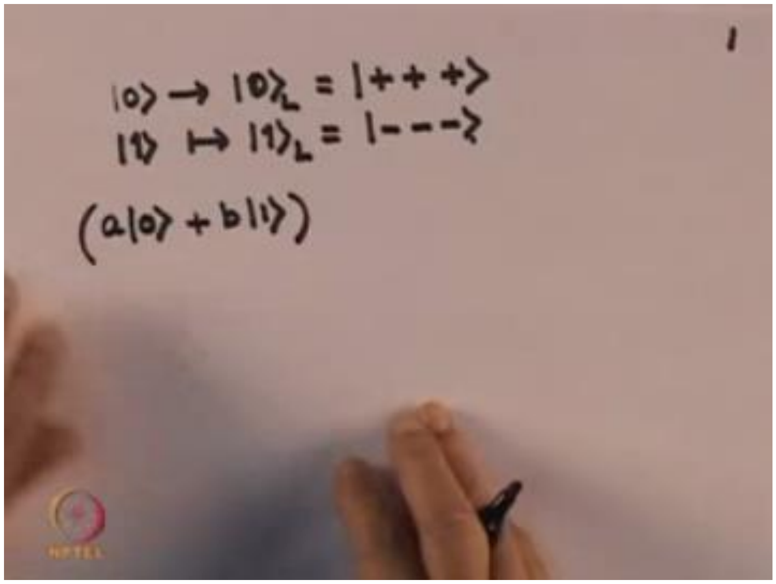

That we want to do what I am called as the phase flip encoding.

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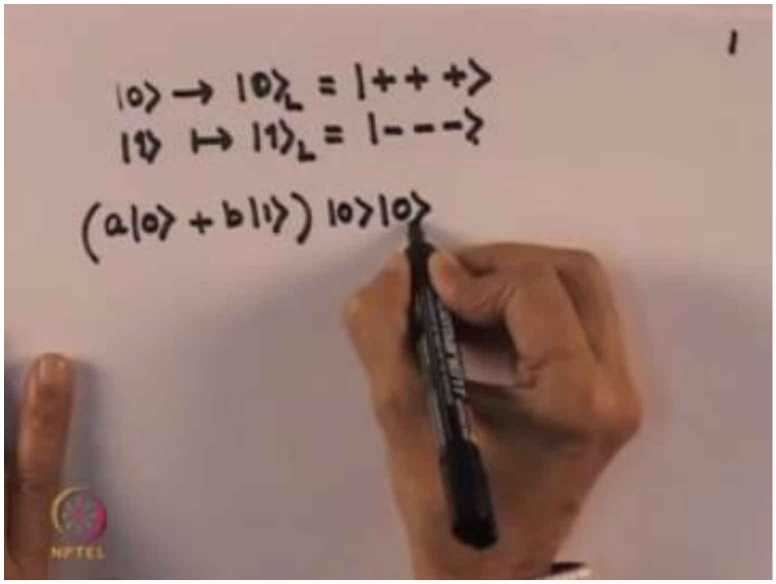

Now so I start with a|0>+b|1> and the first thing that we would do is the, introduce two ancillas here.

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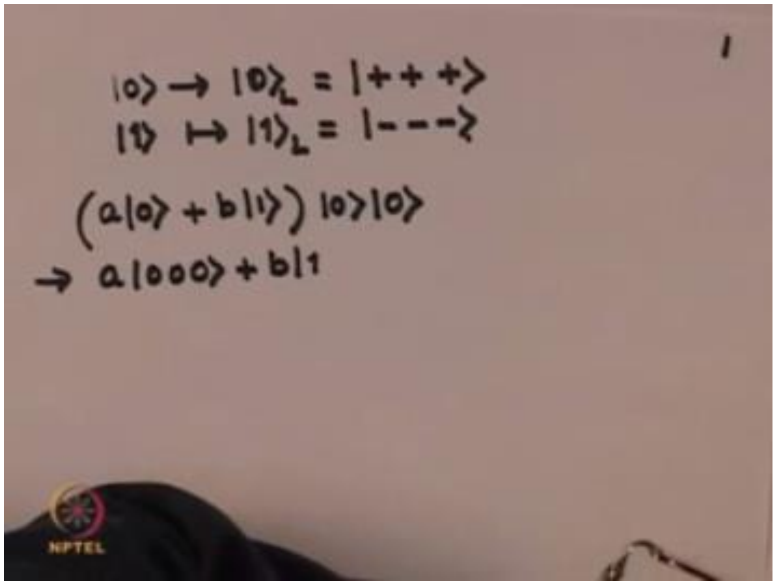

Going back to a slide you can see it here under the green thing I have introduced two ancillas 00 and I had started with ψ which is all a|0>+b|1>, so what do I have is this, I have a|000> writing it in all the 3 qubits together, I have b now let us look at what has been done here on the slide.

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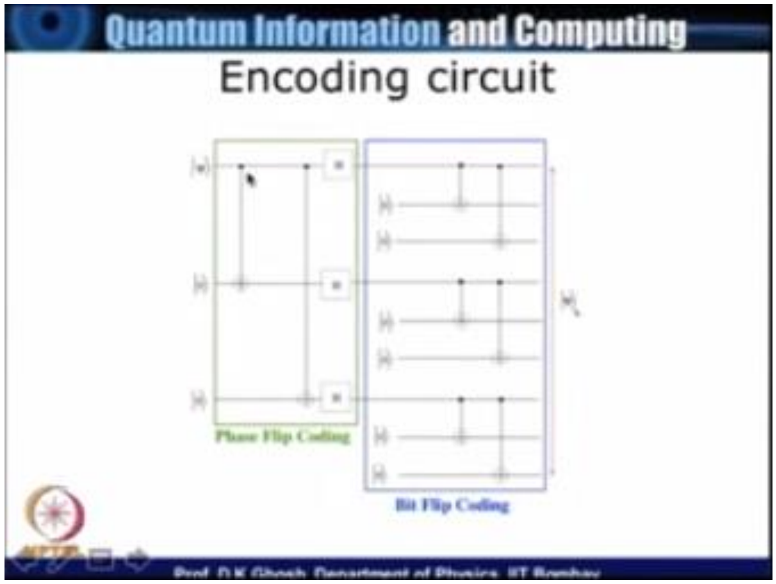

So the first ancilla bit and the second ancilla bit that we have introduced I use a CNOT gate with the ψ as the control and since ψ is a|0>+b|1>.

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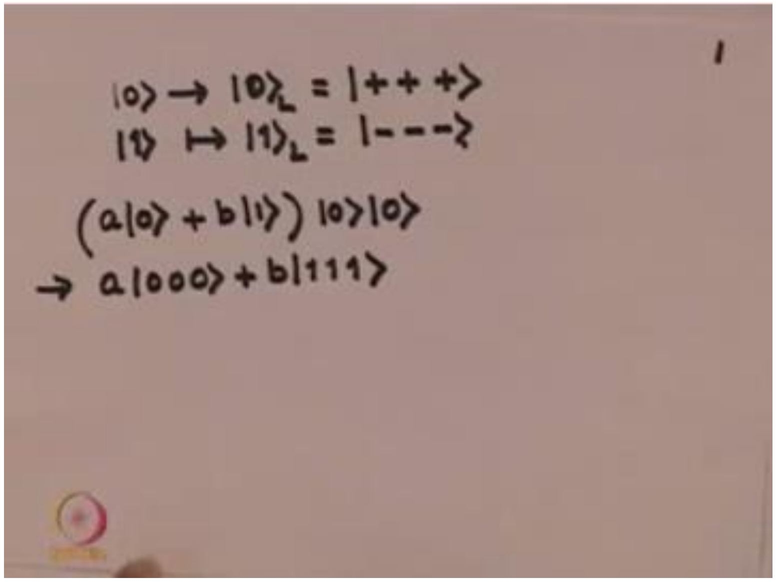

So a|000> nothing happens on the CNOT but on the other hand b|100> becomes b|111> so that is the effect of inside the slide.

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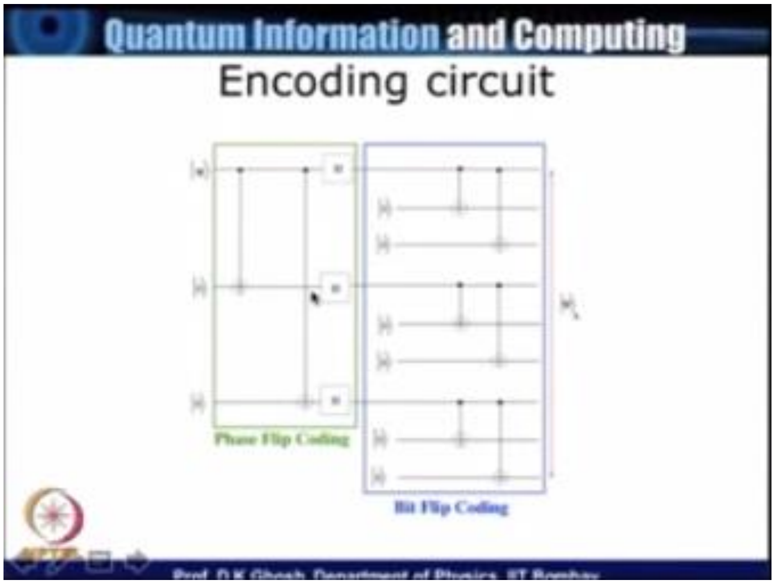

If you look at this step here, now having done that what we do is to apply.

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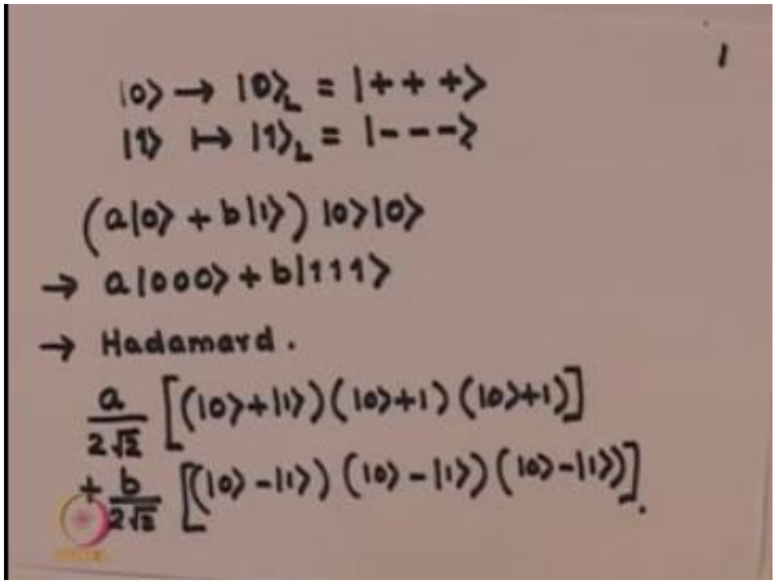

Hadamard gate on all these 3 qubits, okay. Let us look at what we get so we would have A, now there are three of them which are being used for Hadamard gate so I will get $a/2\sqrt{2}$ then I have [(|0>+|1>) (|0>+|1) (|0>+|1)] and likewise the second term which is $b/2\sqrt{2}$ [(|0>-|1>) (|0>-|1>) and (|0>-|1>)] so this is the result of this Hadamard gate which is shown in the slide.

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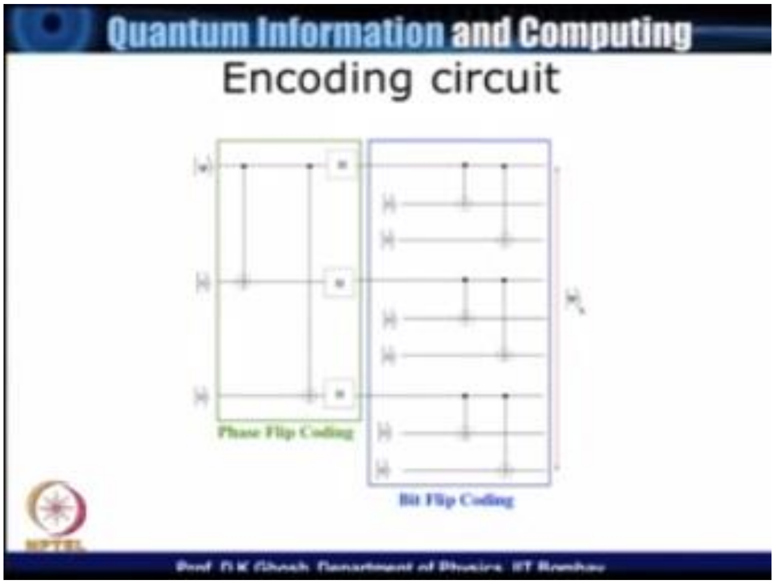

Now at that stage what we do is this that after the Hadamard operation has been done, after the Hadamard operation has been done we introduce two more ancilla with each of the 3 qubits there. Remember I started with 1 qubit introduce two ancilla subjected this ancilla to first CNOT gate and then a Hadamard on all three gates. Now what I do is each of these three, I introduce two more ancilla, so that is the reason for calling it Shor's 9 qubit codes. So let us see what we do in, with this, so what we are going to do in the following that remember now go back to the slide again.

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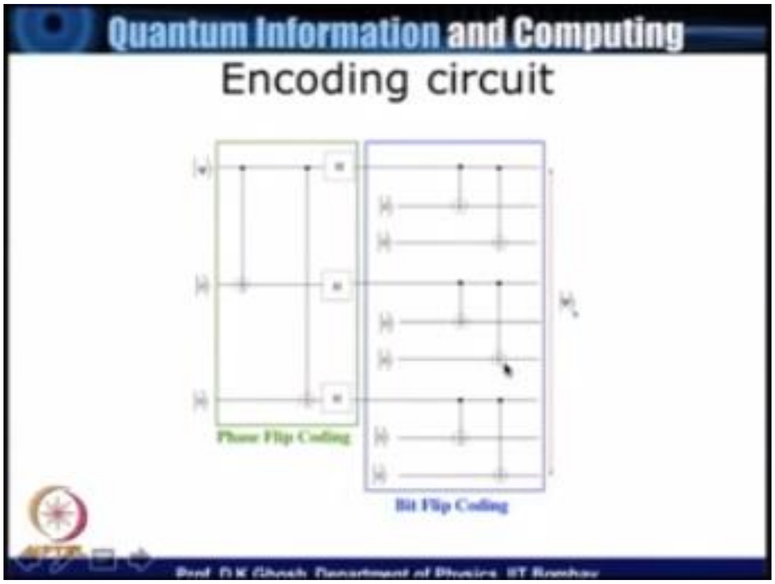

We will switch between slide on this, so this one was that 0+1/√2 or in one part and 0-1/√2 these are the three states which were entangled together. Now I have introduced two more ancilla which have been two times 00, and what I do is to each I apply a CNOT gate and the way it has been shown. So line 1 acts as the control does the CNOT gate on line 2 and 3 the qubit four acts as the control does on 5 and 6 qubit 7 acts on the control and applies the CNOT gate on 8 and 9.

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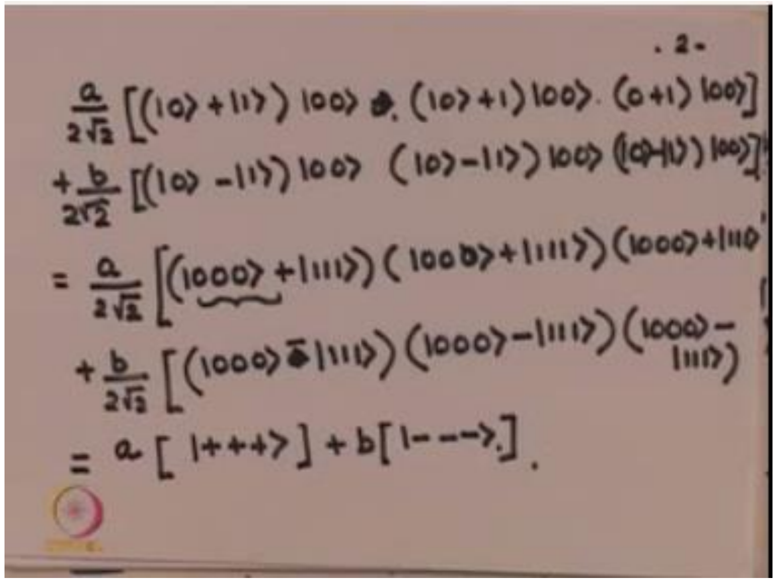

So what happens when we do that so we had of course, remember I had a/2√2 so I to start with I have this these are nine states that I have 0 + 1 and these are the two ancilla that we have put in 00 again multiplied by 0+1 00, 0+1 00 likewise I have b/2√2 0 – 1 00, 0 – 1 00 and again 0-1 00. What happens with this that we are applying using this as the control CNOT on each of the bits, so look at what will I get, I will get a/2√2 I will now need to write them down together.

So in the first block of three I will get 000 + 111 this is the effect of CNOT and since the identical situations I will get 000 +111 and 000 +111 and then + b/2√2 I get a very similar structure excepting that the + sign between the each block to that it become -1. Now instead of writing this nine things together what I will do is call this thing here as +++ so this whole combination we will say a/ or a times I will say this is +++. So this is the shorthand notation for this thing that each + stands for 000 +111 not to be confused with 0 +1/√2 that as we have been doing and + b times --- so what I have done basically is to start with one qubit introduce two ancilla.

And then do certain operation for these two ancilla, so that I have three qubits at the end of the first group and then with each of these modified groups I apply two more ancilla and then I have

9 qubits and this is the final coding that I have received. Now what we do is this let us assume that out of the nine qubit better going through the logic channel almost one qubit may be affected.

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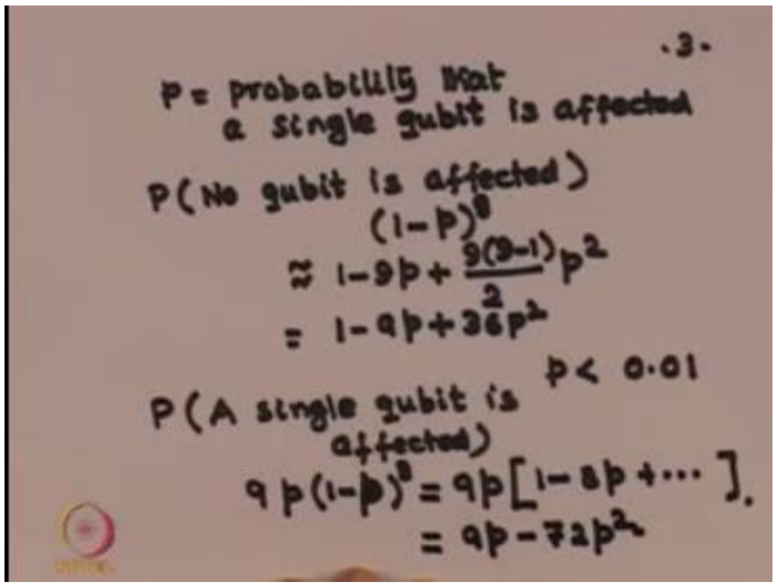

Let me say the p is the probability that is single qubit is affected. Now when I say affected it means it could be either a bi flip or a phase Flip or a mixture of the two bit flip along with the phase flip. So then the probability that no qubit is affected is clearly $1 - p^9$ let me also tell you typically you do not expect a very noisy channel and this p is usually a small number and I expect it be usually much less them even 0.01 or so.

So since p is small I can expand this $1-p^9$ in a binomial series and get this is $1 - 9p + 9x\ 9\text{-}1/2\ p^2 = 1\text{-}9p + 36p^2$. The probability that is a single qubit is affected is clearly the case where any of the nine and hence a factor of nine is subjected to an error which is p and the remaining 8 which 1 – 8, 1- p is the probability race to the power 8.

Now expand this also in a binomial expansion the term is simply 9p because this expansion is 1 – 8p okay so that is so let us write this down so 1-8p then of course we have +…… etc. So this quantity would be $9p - 72p^2$. So if you know try to find out what is the probability.

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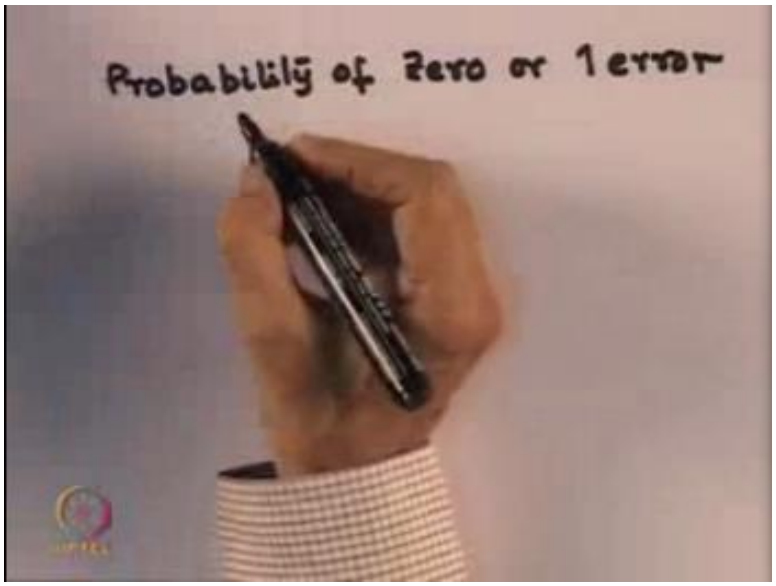

Probability that I have either 0 error or a maximum one error is simply obtained by adding up these two.

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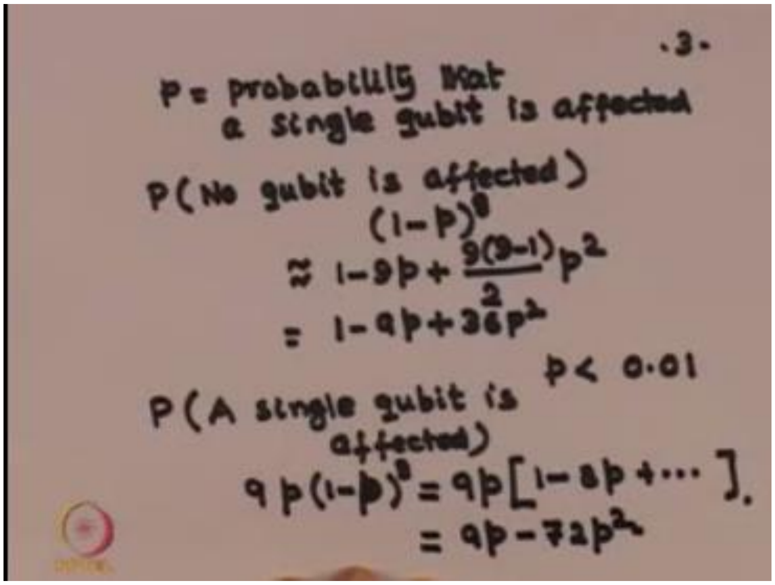

So I had here $1-9p+36p^2$ and I have $9p-72p^2$.

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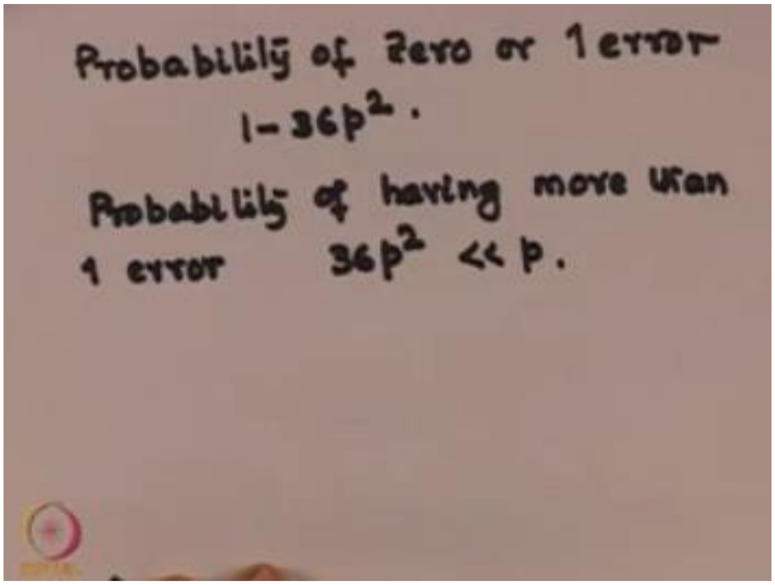

So therefore which is simply $1-36p^2$ could means the probability of having more than one error is just 36p2 and since we had said that probability p is small so this is of course a much smallest than p with which we started. So that essentially is the coding circuit of Shor's 90 bit code we will start from here and in the next lecture will try to see how decoding of such a circuit will be done.

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