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Quantum Information and Computing

Prof. D.K. Ghosh
Department of Physical IIT Bombay

Modul No.03

Lecture No.14

Measurement Postulates

In the last lecture we have talked about the formalism of density matrix which is useful for describing satiations where we do not have pure states but we deal with states which interact with environment or states or states where we consider an ensample so that when you pick up a particular member out of that ensample that may be different from if you pick up another member.

One of things that makes quantum computing different from classical computing is the question of measurement we have pointed out that when you have a situation where you have a super position of states as an input and a super potion of sates as an output the output though in principle it contains the results are the suppose you are calculating a functions so that the output could be function corresponding several inputs at a time.

Because of the quantum parallelism which is inherent in such systems. The problem is that in spite of the fact that the information regarding the value of the function this example that I gave you is there in the output register when you actually want to extract this information you cannot get all the information at a time, but you will get firstly only one of the results but that is the only issue the issue is that you will get one of the random results out of that.

Now this causes a lot of problem in quantum computing and one as to really do as find a smart way of how to extract the informations that you want. The question of measurement has always been a very central issue in quantum mechanics and quantum computing, and what I want to do today is to talk about the postulates of measurement. I call it the postulate because like other postulates for example, that state is represented either by A vector or a ray in the Hilbert space or in terms of a density matrix in case of a mixed states.

The measurement is also a fundamental postulate and what we will do today is to see what this measurement postulates mean.

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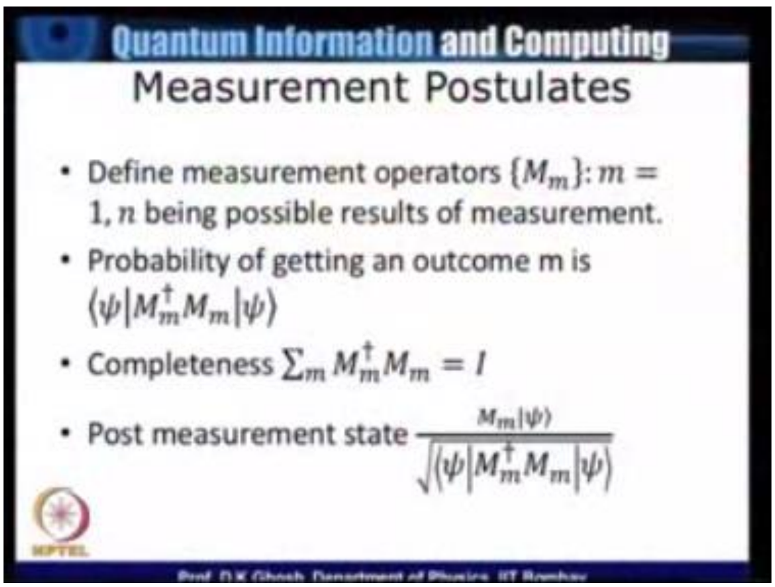

So let us define a set of what we call as a measurement operator suppose I have in principle I am making a measurement it could be for instance of the energy of the system any physical observable, the momentum, the angular moment or whatever you have. Now we have seen that when you make such a measurement the system would according to Copenhagen interpretation collapse to one of the Eigen states. So let us assume that out of the possible results.

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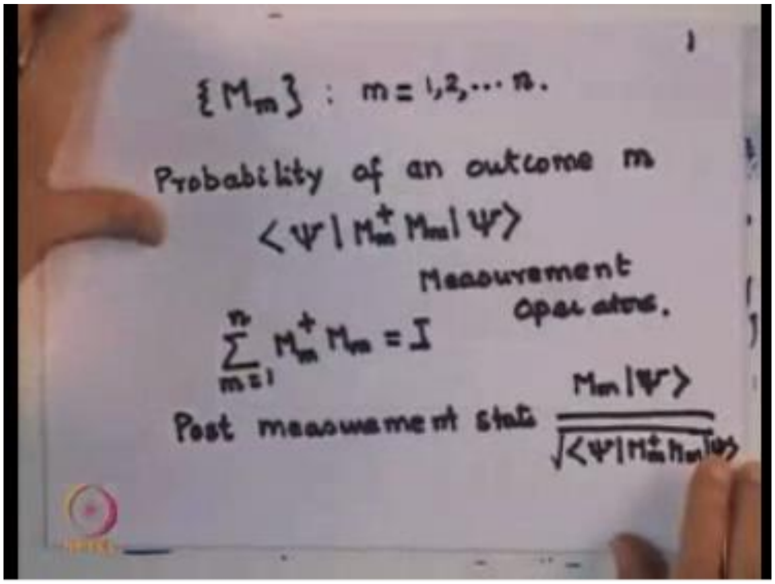

So we have a set of operators which we call as the measurement operators M_m let us put the index as m, where m gives me the possible results supposing there are n number of possible results then m could vary from 1, 2…n. So this is the operator when we take the expectation value in certain steps then I will get a particular result out. Now according to the quantum postulate that we had the probability of an outcome suppose I am talking about a particular outcome m well whatever you want to let us call it just m a specific m is given by this quantity ψ $M_m{}^{\dagger}$ $M_m \psi$ where these are my M_m ,$M_m{}^{\dagger}$ these are my measurement operators.

Now since the only possibilities are theses n values so I must have $\Sigma_{m\,=}{}^{1}n$ of course $M_m{}^{+}$ M_m that must be a resolution of identity operator. The according to the quantum postulate after we have made a measurement and let us suppose we have got a specific M as a result of that measurement the post measurement states is given by and of course it is the measurement operator M_m which as acted on ψ and it is normalized because we are talking about a specific state, so this will then give you $<\psi \mid M_{m*}\, M_m \mid \psi >$.

This is the state to which it will collapse after a measurement has been made. So this is basically our measurement postulate. Now what we do now is the following that we look at what are the various types of measurement postulate. And I will be essentially picking up two off them the most important one is what is known as a projected measurement. Now remember the completeness is there for any type of measurement because your measurement must exhaust, the list of measurement must exhaust all possibilities.

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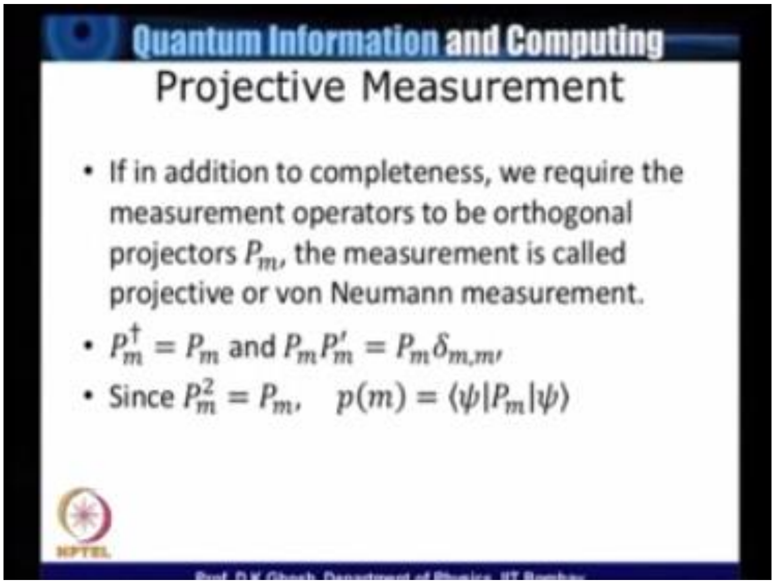

Now suppose I make another demand that these measurement operators are actually orthogonal projectors, remember we talked about the orthogonal projectors on to the one dimensional subs space of the relevant Hilbert space. And so instead of calling it mm and since these are projectors let us call them P_m. So if these are the measurement operators and these we call as a projective also known as von Neumann. Now firstly the von Neumann measurements of course why have hermit history so $P_m{}^+ = P_m$.

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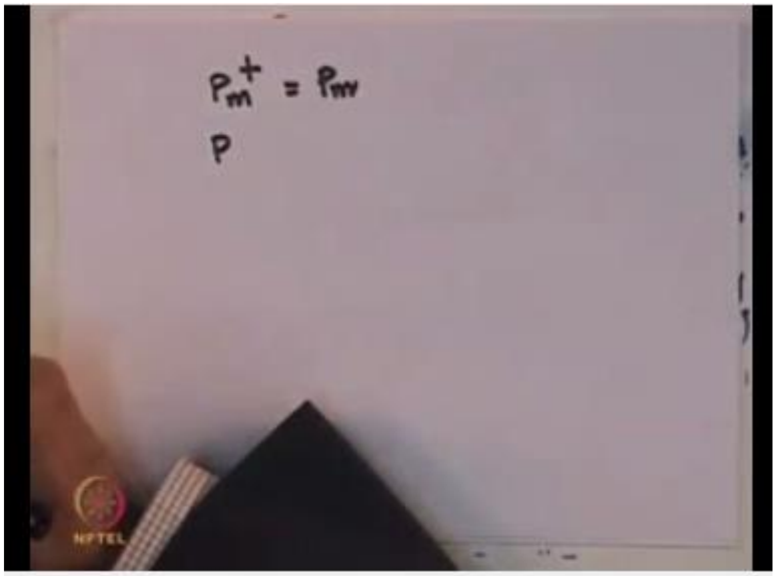

And then I also have, remember that the projection operators are ideal portal operator.

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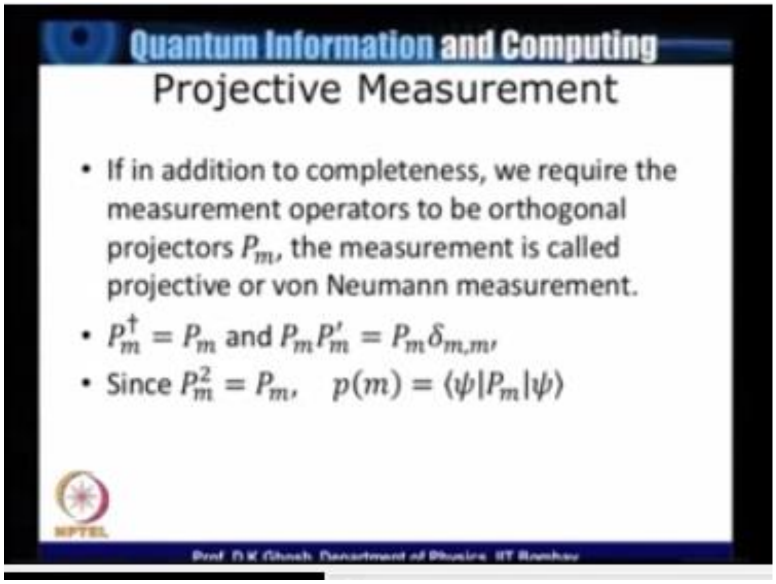

So that Pm^2 is Pm itself the reason is very obvious that if we have once projected it applying a projector a second time is not going to take it out of the space.

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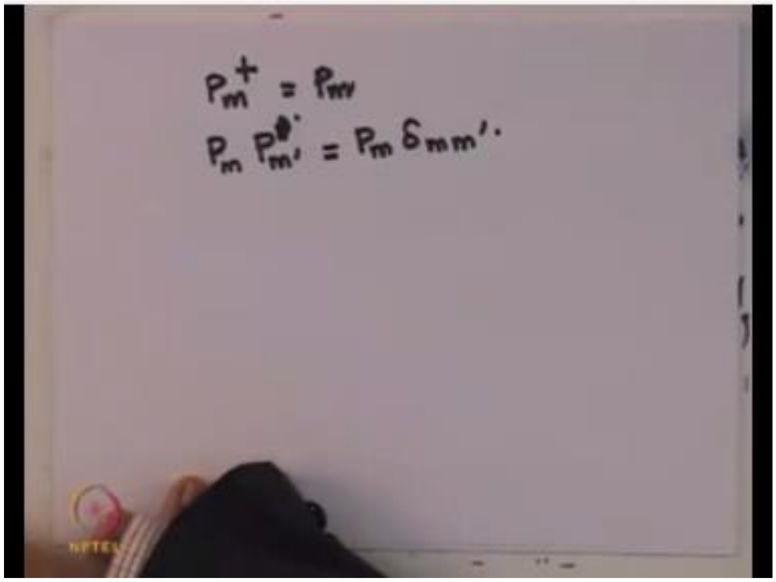

So Pm, Pm` = Pm $\delta_{mm}$` see if m and m` belong to different sub spaces orthogonal sub spaces.

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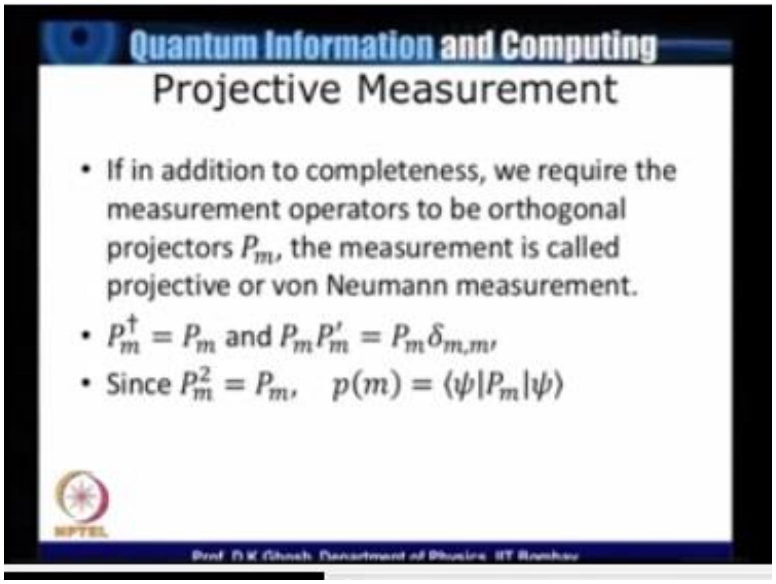

No, no obviously the result possibility equal to 0, now so that along with our definition of what is the probability of the measurement when we had said that the probability of measurement gives me expectation value of $\psi_{mm}{}^{+}$ mm and since in this particular case Pm^2 = Pm.

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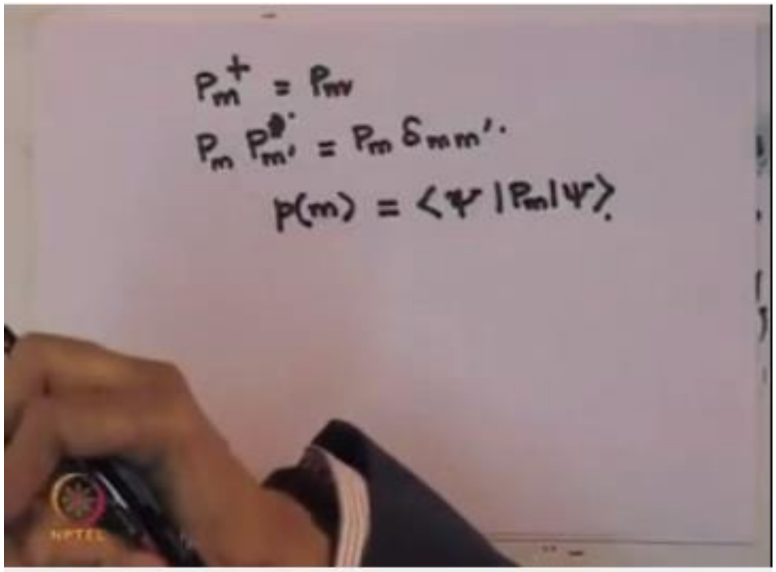

So I get the probability of having a result Pm is given by <ψ|Pm| ψ> so let us look at this specific case because this happens to be the most common.

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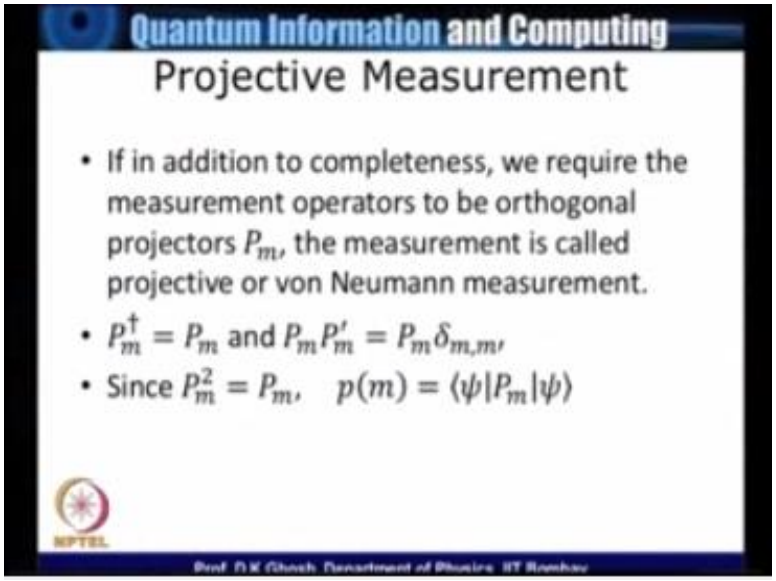

Case that we normally talk about.

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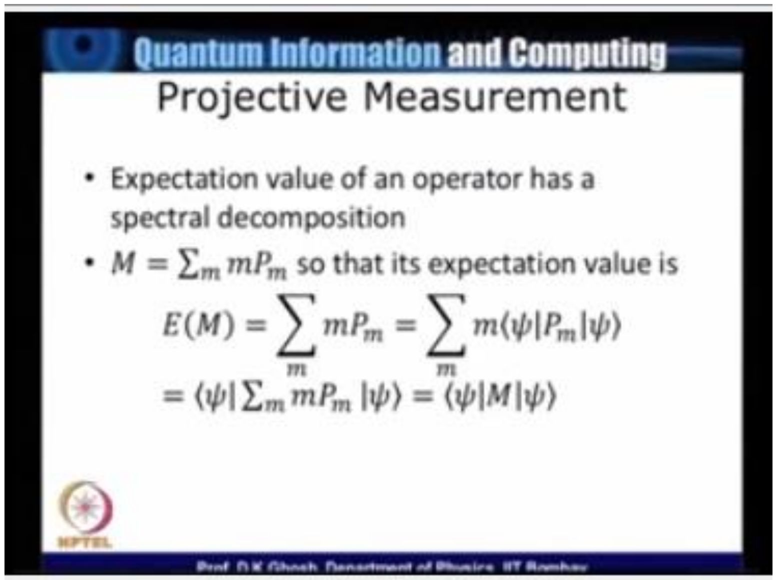

Now suppose I look at an physical operator m.

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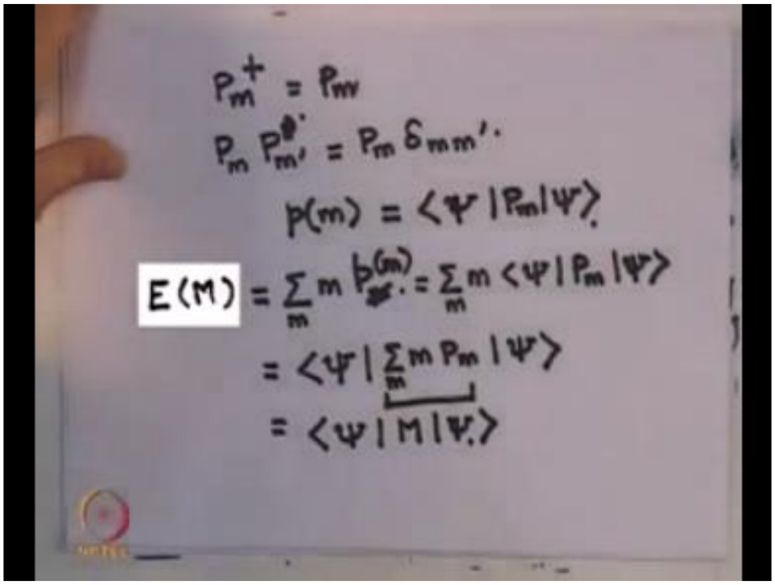

And this m we know as a spectral decomposition m Pm and we have just now said that this is m this is diagonal values and Pm which is their probability is ψ actually I should have written like this |Pm| ψ this is the probability P which I wrote as P bracket, so this quantity is equal to if you re-write it like this <ψ |Σ_m m Pm | ψ> so we will notice that this is nothing but the operator m itself so therefore it is < ψ|M| ψ>. Now to illustrate what this actually means.

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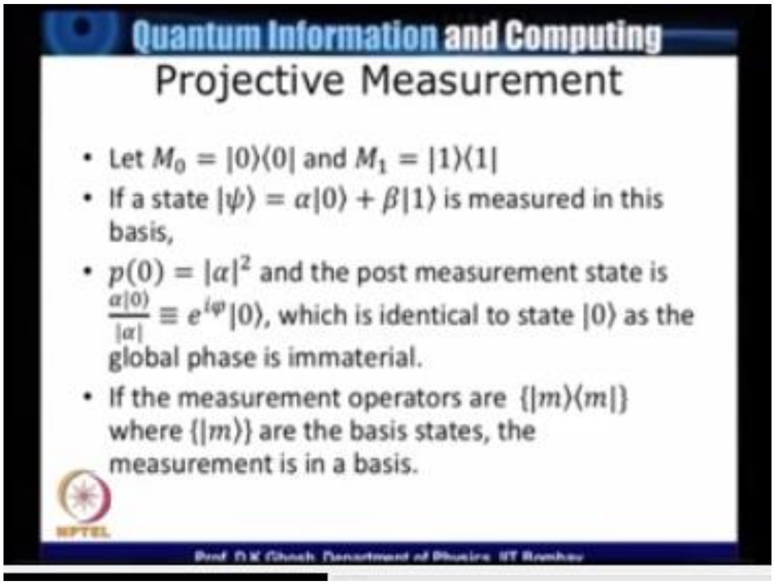

Let me give you a specific example of the projection operators, let me talk about a one qubit state and in that I have of course my basic state as 0 and 1.

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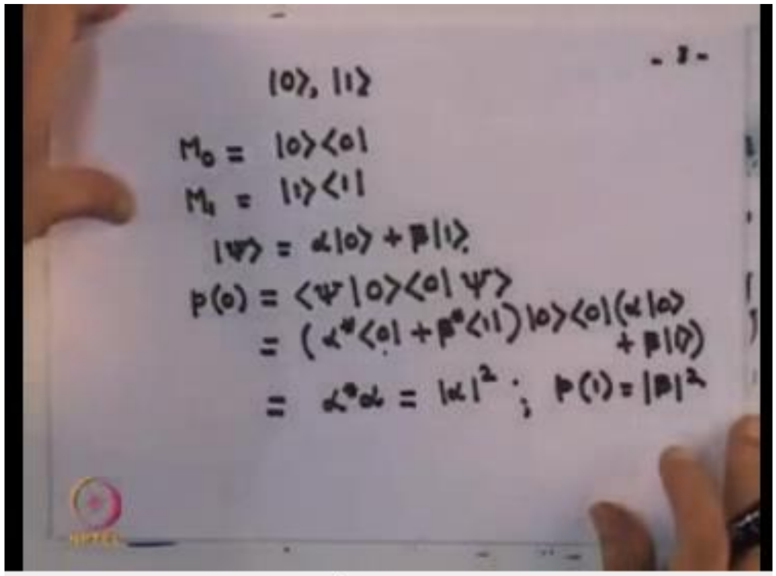

Let me define by measurement operator M_0 as equal to |0><0| and M1 = |1><1| completeness is obvious because 00 + 11 is identity and let me see what this measurement operator gives, when it acts on an arbitrary state ψ = α|0> + β|1> let us look at this, what is the probability of picking up thus value 0, out of the middle, now we just now said this is given by ψ the corresponding projection operator.

Which just happens to be <ψ|0><0| ψ> now you can quickly calculate this because this is the bra's i that we have the cat's i to be given by this, so it is α^* <0| remember αβ's are complex numbers plus β^* <1|> |0><0| and ψ which is again α|0> + β|1>, so we can multiply this through and realize that the only terms that survive here are the product 00 and the so firstly this 00 gives you 1 and the 01 is 0 so if you just multiply it through you will be left with α*α and which is nothing but equal to $|\alpha|^2$ and likewise you can show that the probability of getting the value 1 is $|\beta|^2$.

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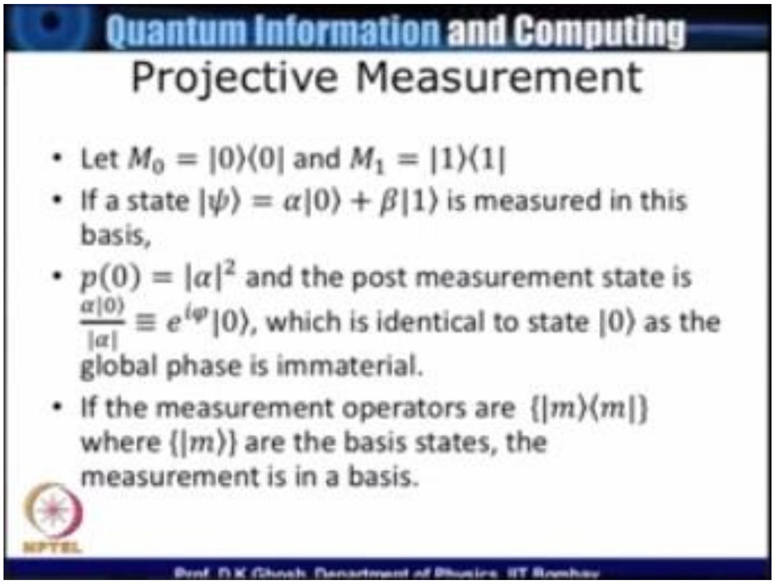

Now suppose, suppose the result of the measurement happens to be 0, in that case.

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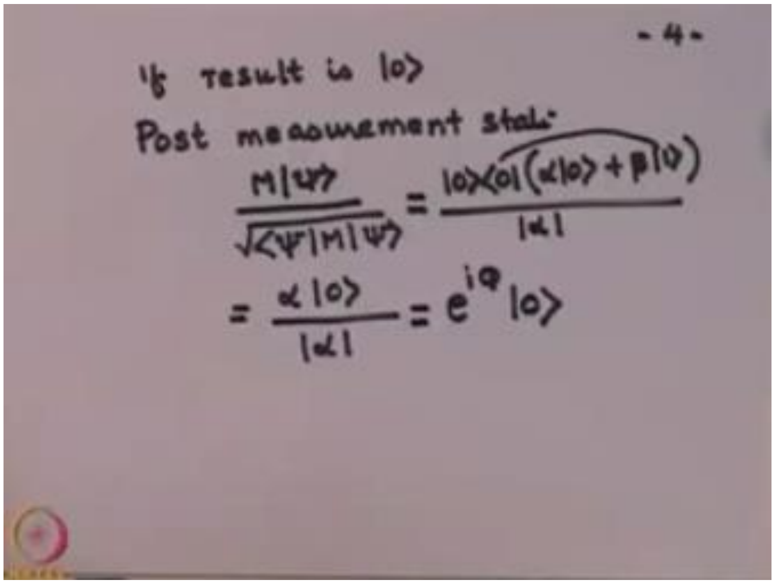

If result is |0> then what is the post measurement state. The post measurement state according to our prescription is then given by remember M acting on |ψ> divided by the probability that is square root of the probability <ψM ψ and this we have seen operators M is corresponding to the state 0 is this, (α|0>+β|1> and this one we have just now calculated to be given by $|\alpha|^2$ square root of that, that is equal to |α|. And, and this 1 obviously this scalar product is 0, so therefore I am simply left with α|0>/|α|.

Now realize that α/|α| is basically a complex phase $e^{i\varphi}$ |0>, now what this is telling us is that if my measurement gives me the result 0, the state 0 the post measurement state will be the same state 0.

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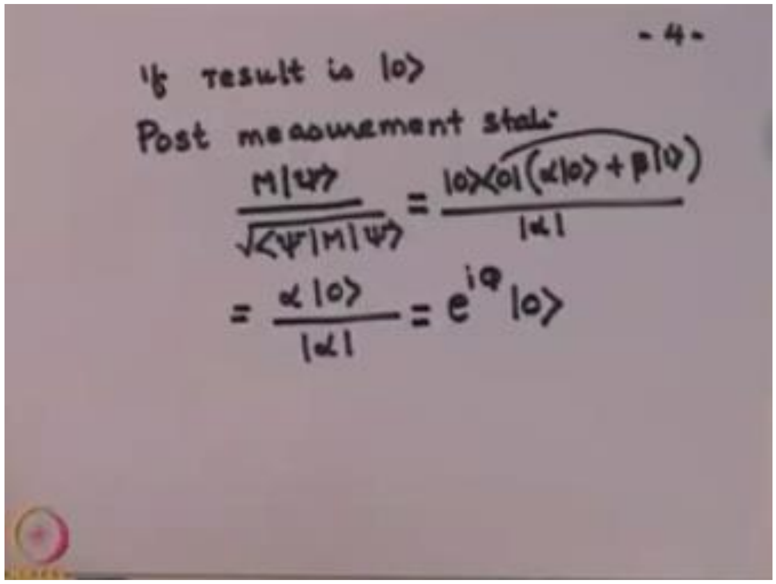

But for a global phase, but as we know that the global phase does not make any difference.

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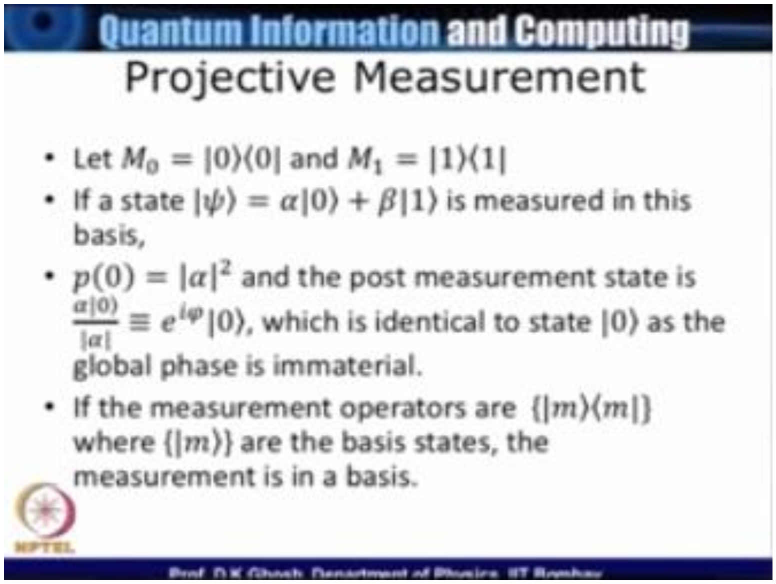

So the state remains φ, the 0 state. The, a very special case, very special case of the projective measurement is when we make the measurement like this example that we give you, where our measurement operators are |m><m|.

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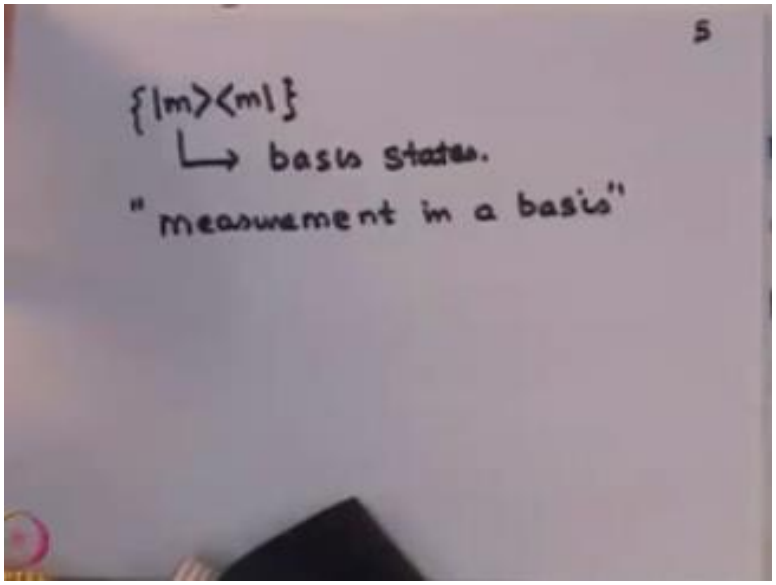

Just now we get the example of 00 and 11, and supposing this is in general true that these are the projection operators, where M's are the basics states, so these are the basis states. Now in that case we say, it is a measurement being done in a basics, all measurements do not have to be done in a basis they could be in general in terms of any orthogonal vector in the Hilbert space.

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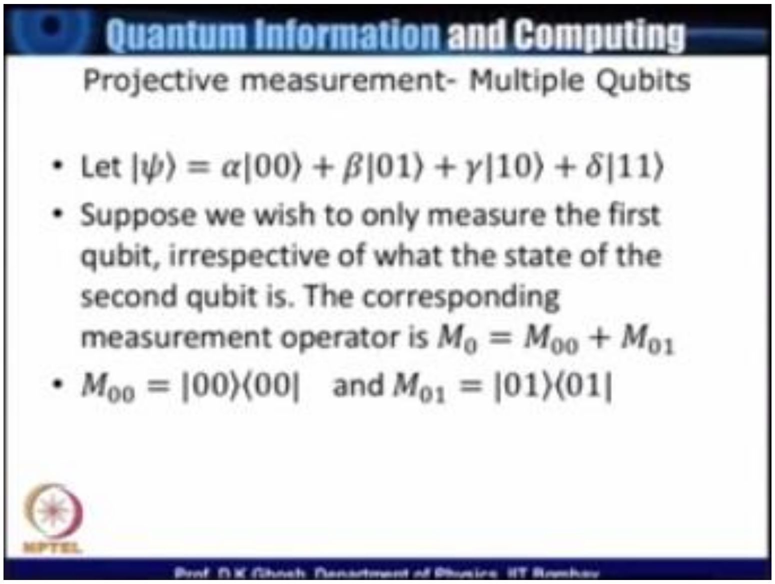

So let me extend this idea to the case of multiple qubits, well we will not really do multiple qubits but once we have taken care of two qubits then the extension to other qubits many qubits should be obvious. But, but before, before I proceed for that, let me start explain how this situation would be supposing instead of measuring in the 0011 basis, I measured it in a different situation, suppose I have the same state.

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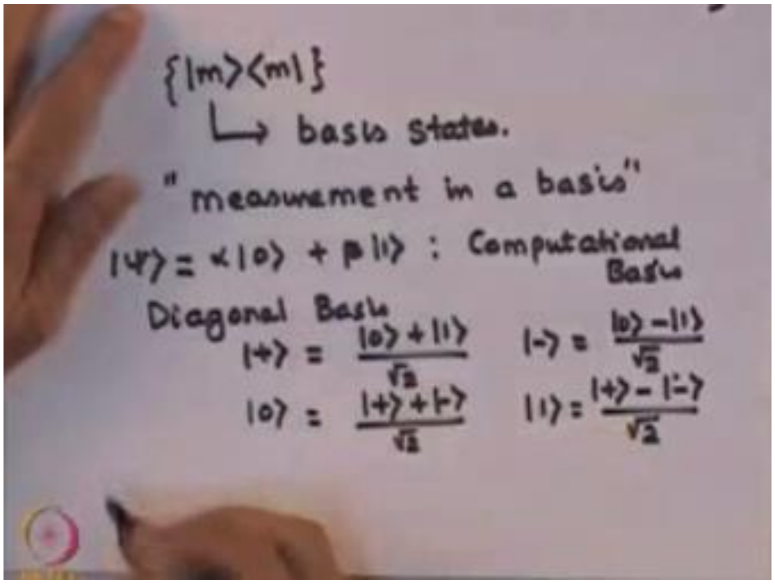

I have $\alpha|0>+\beta|1>$ and we want to measure in a diagonal basis, see this is written what we call as a computational basis. So we have a state which is given in computational basis whose basis states are 0 and 1 and I want to measure it in a diagonal basis whose basis happens to be $|+>$ which is equal to $|0>+|1>/\sqrt{2}$ and $|->$ which is equal to $|0>-|1>/\sqrt{2}$. Now supposing I make a measurement of this state in this basis, now the job that I have to do is to first express by giving state ψ which is expressed in computational basis in the diagonal basis.

And since plus is given like this minus is given like that, I can express $|0>$ as $|+>+|->/\sqrt{2}$ and $|1>$ as $|+>-|->/\sqrt{2}$. Now so I, we express these in terms of +- extra, so that my $\alpha|0>+\beta|1>$ that is my state ψ.

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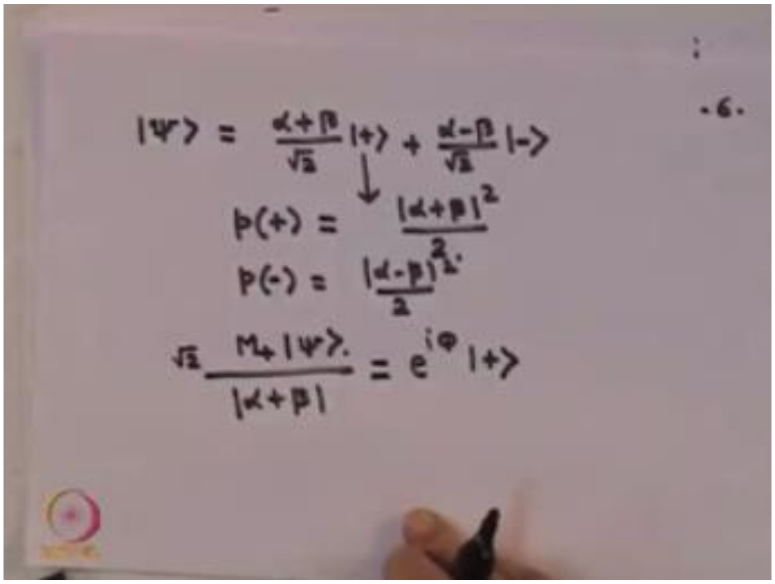

With a trivial algebra we will be written as α+β/√2 |+> + α-β/√2 |-> this is the same thing re-expressed with this. Now clearly, they if I now make a measurement the probability that I get +, the probability of getting a +, is given by $|\alpha+\beta|^2/2$ and probability for getting the result – is given by $|\alpha-\beta|^2/2$. Now suppose my state is + in the same way, I can compute what is the post measurement state you can simply compute M + ψ dividing by the corresponding probability which we have write down there so which is √2 divided by α + ß modulus because this was quire route of the probability and you do the same algebra as we did had here and you will find that this is equal to $e^{I\varphi\ 1}$ which simply tells me that if the result of the measurement.

In the diagonal basis happen to be plus the post measurement state but for a global state will also remain plus. One point you have to realize this at this stage if I made repeated measurement in the same basis since the state have already collapse to a particular state it will continue to be in that so repeated measurement will not make ender. Now let me then make this.

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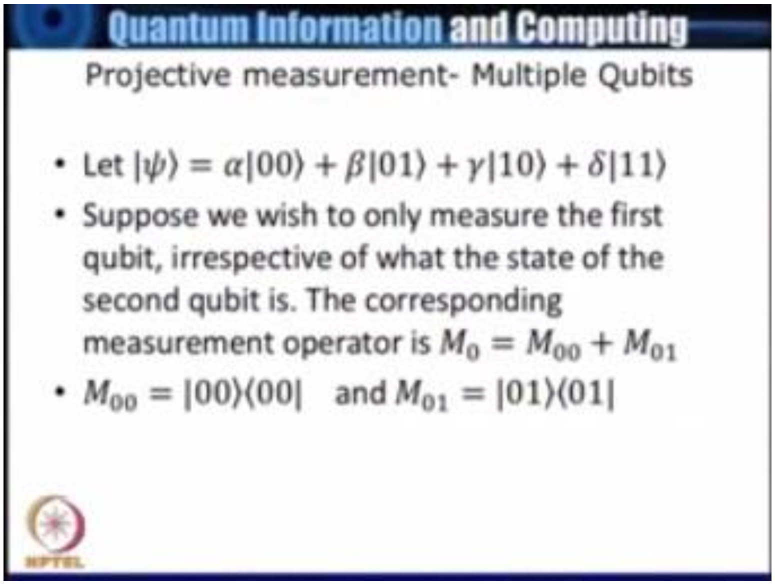

Extension to the projective measurement at this time I will take the case of two qubit which is a first step what generalizing to n qubits so my ψ here is a general to qubit state α 0, 0, + ß 0, 1, + ɣ 1, 0 + δ 1,1 now pliant here is this that suppose I wish to measure only the first qubit now what is this that what is my operator corresponding to that, see the first qubits measurement and suppose I am talking about the measurement operator corresponding to the situation.

Where is the first qubit is 0 respective of quart the second qubit is now then my measurement operator corresponding to the first qubit being 0 is given by measurement operator were the first qubit is 0 second qubit is 0 measurement operator for first qubit is 0 but second qubit is 1 and this one obviously is 0, 0 remember this a two qubit state so it is 0, 0, 0, 0, and this one is 0, 1, 0, 1.

So if you add them up what do you get so we have got 0, 0, 0, 0, + 0, 1, 0, 1, you have remember that when I write two qubits in this way the notation is this is standing for the first qubit. And this one is standing for the second qubit and likewise it is true for the corresponding bracket but you notice one interest thing here when I have written this the bra that kept on the bra for the first qubit in this term is 0 the kept in the bra of the second qubit in this term is also 0 but in this case that kept on the bra for the first qubit is 0 for the second qubit in the second term is 1.

So in other words in both cases my first qubit supposing I have to know change the notice on a little bit and wrote this as 0, 0 okay then we have notice what I am getting is the following so basically what I have done is to say that this is 0, 0 which is this turn and multiplied with 0, 0, + 1, so this is for the second qubit and this is for the first qubit which is the same in both these qubits.

But this is nothing but resolution of idea I completes so therefore this term is 0, 0, let me call it I_2 meaning there by this is the identity operator which acts on the second qubit.

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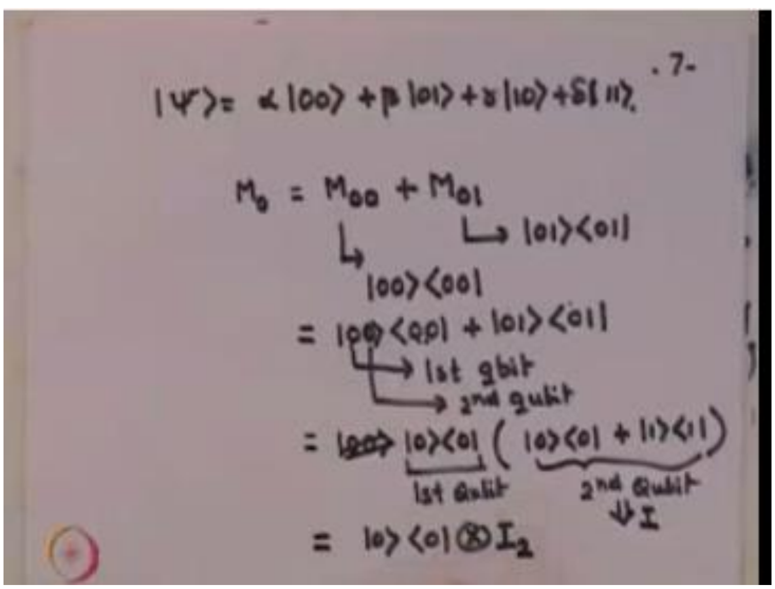

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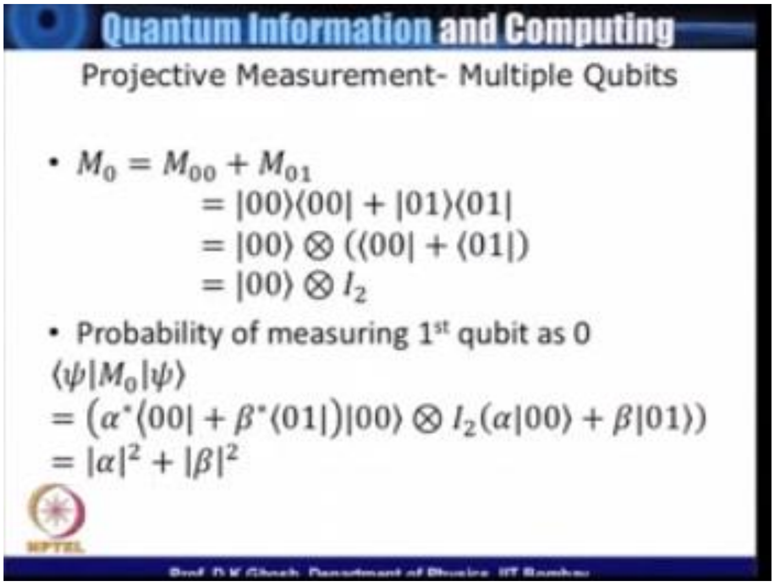

So therefore what is my probability of getting first qubit as 0 the probability of getting my first qubit as 0 is remember the for in this particular case look at what I will get it has been ψ M_0 ψ the sense the identity operator is acting on the state so basically what have got is the following let me write it down. I need ψ M_0 as we wrote it 0 that is only a first qubit and we have already seen that this is ψ 0, 0, ψ I have already given you the ψ there and you remember that this is the operator for the first qubit and only my first two terms of ψ then the α 0, 0, + ß 0, 1, had 0 in the first place.

So therefore that will be the only term that we survey and trigger algebra will see to it that this is actually $\alpha^{2,}$ $ß^2$ now remember that this is I have assumed when I wrote down my first value of ψ I said it is equal to α 0, 0, + ß 0, 1 etc, quire the normalize onwards modules $\alpha^{2,}$ + $ß^2$ + modules $\gamma^2 + \delta^2 = 1$.

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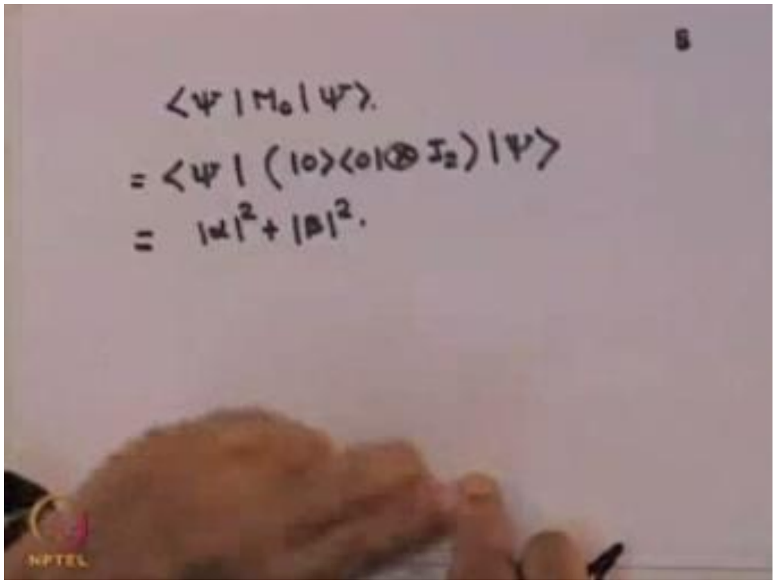

So this is the probability and if you did the corresponding job for the first qubit being equal to 1 you will get this as modules $\gamma^2 + \delta^2$.

So we have talked about one normal or the projective measurement which is a special case of generalize measurement theory the what will do in the next lecture is to tell you how to generalize it for the mixed case how does one take care of one of my measurement for the next case and then go over to another example of the measurement which is need not be projective or the orthogonal.

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NPTEL
Principal Investigator
IIT Bombay

Prof. R.K. Shevgaonkar

Head CDEEP

Prof. V.M. Gadre

Producer

Arun kalwankar

Online Editor
& Digital Video Editor

Tushar Deshpande

Digital Video Cameraman
& Graphic Designer

Amin B Shaikh

Jr. Technical Assistant

Vijay Kedare

Teaching Assistants

Pratik Sathe
Bhargav Sri Venkatesh M.

Sr. Web Designer

Bharati Sakpal

Research Assistant

Riya Surange

Sr. Web Designer

Bharati M. Sarang

Web Designer

Nisha Thakur

Project Attendant

Ravi Paswan
Vinayak Raut

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