NPTEL

NATIONAL PROGRAMME ON
TECHNOLOGY ENHANCED LEARNING

IIT BOMBAY

CDEEP
IIT BOMBAY

Quantum Information and Computing

Prof. D. K. Ghosh
Department of Physics IIT Bombay

Modul No. 02

Lecture No.10

Super Dense Coding

In the last lecture we had talked about quantum teleportation the basic principle behind teleportation was as follows. We assume that we have two people called Alice and Bob as I told you earlier that quantum computing has these characters as its standard bearers. Alice has with her a quantum state which is α0 +β1 with α and β complex in general which he wants to send to Bob with whom, she also shares an entangled pair which we took to the bell state.

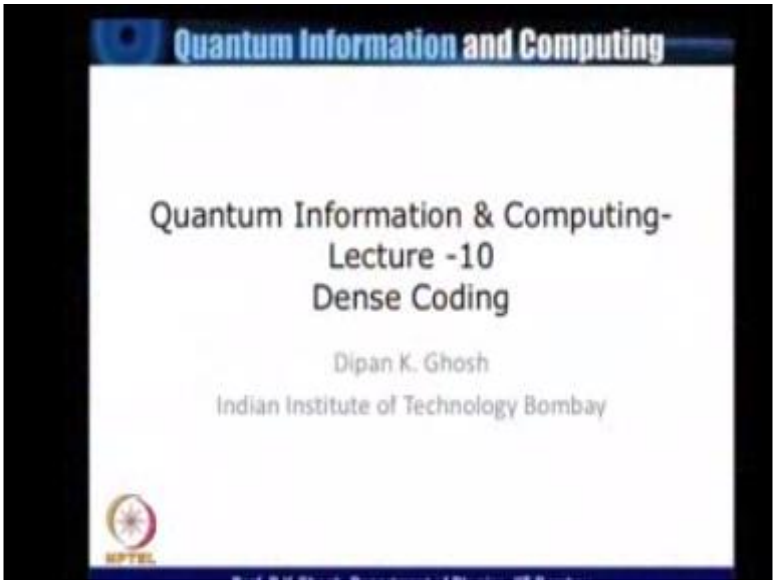
Then I found that because of the restrictions imposed by quantum no cloning theorem such a transportation of a quantum state from one place to another is not possible. And this can be done in a different way that is if Bob has enough information about this state he could reconstruct such a state at his point. So what Alice does is to make her part of the integrity qubit interact with this unknown state α 0 +β1 and does certain operations and based on her measurement the Bob takes certain actions which we have discussed in our last lecture which enables Bob to reconstitute that state at his end.

There are two questions which we answer towards the end one is that since we never talked about the, how far these two people Alice and Bob are located is this in principle a faster-than-light communication which we know violates the dictates of special theory of relativity. The point is that irrespective of what the status there is Alice has to still communicate the result of our measurement to Bob by a classical channel. So therefore the information transfer if any has to take place following the dictates of relativity. So there is no faster-than-light communication in this case.

The second point was since Bob has a copy of the same state which Alice wanted to send him is this a cloning which we know is not permitted the answer again is no, because if you look at Alice's end what has happened to the state that he has now collapsed to either a state 0 or a state 1. And what Bob has done is to reconstitute α0 +β1 state in place of the state that he had of using the information that Alice has given.

So there has not been any cloning, so that was quantum teleportation which we will see has its role to play in when we talk about quantum cryptography at a much later part of this course. Now what we are going to do today is to look at in some sense something which is inverse of this process, but slightly different which is known as dense coding also known as super dense coding.

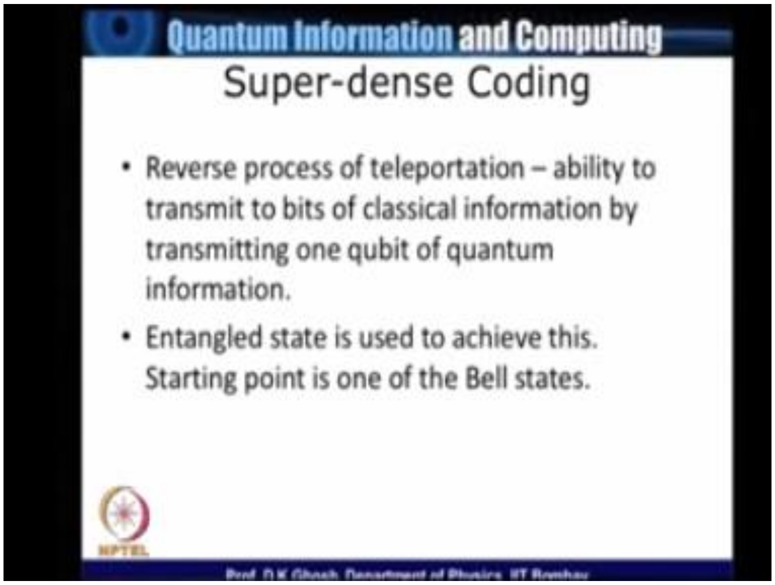
(Refer Slide Time: 03:51)



Depending upon what language you want to use. So let us look at what does this dense coding implies first, then we will work out how it work. See the idea of dense coding is the Alice wants to send in this case, two bits of classical information. So therefore, Alice wants to say let us say 00, 01, 10, or 11. Now Alice is to do that using a quantum state to send this information remember that in the quantum teleportation she had sent classical information through a classical channel and a quantum state was prepared.

Now the purpose is essentially to send a quantum bit to Bob and enable Bob to have classical information, the difference also is that she will be sending one bit of quantum information and Bob should be able to extract two bits of classical information and hence the name code coding or dense coding. So let us look at how does it actually work.

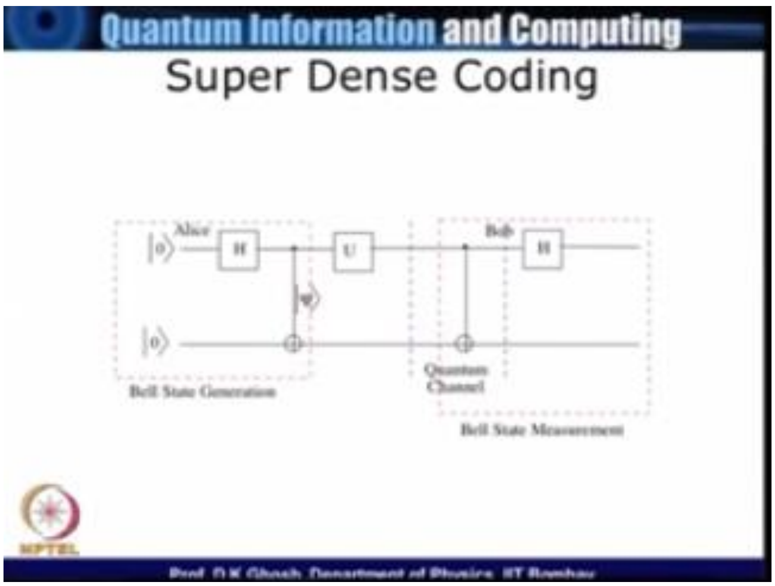
(Refer Slide Time: 05:16)



So once again we use a an entangled state to achieve this. So we will take the starting point to be one of the bell states and the one cubit is with Alice and the second qubit is with Bob. Now in this process what Alice will do will be to have certain operations on her part of the qubit they having done that she will send this qubit, the modified qubit if you like to Bob through a quantum channel.

And Bob on receiving this qubit see he has now this modified bit from Alice and the original part of the qubit that he had said as a part of the builder. Now he will do certain measurements on his qubit and based on that he will be able to infer what classical two bits of information that Alice wanted to sign up and let us see how it work.

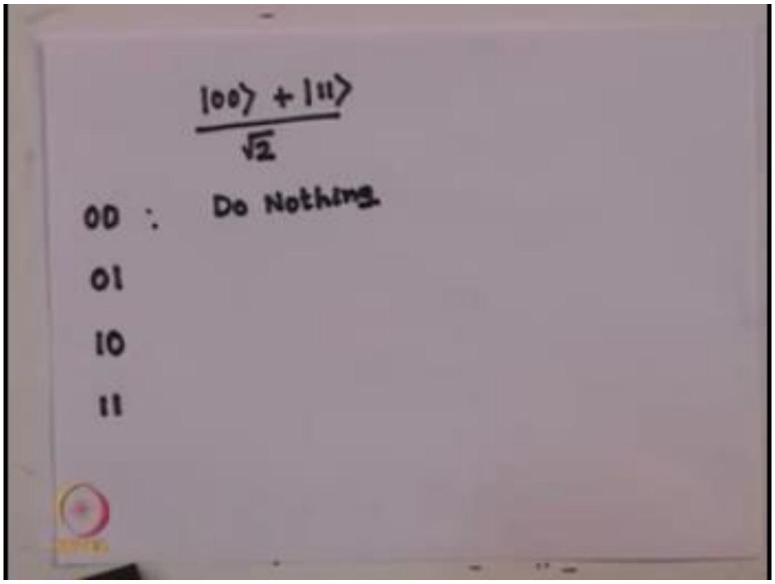
(Refer Slide Time: 06:25)



So this is a schematics of the type of circuit that we have, the left-hand part of the circuit which is enclosed in that red dotted rectangle is really not a part of our process because that is simply telling you the, how a bell state is generated actually. So you start with the two states 00 and then you apply a Hadamard followed by a CNOT that is the way to generate the bell state. So therefore, the left-hand side part is simply a bell state generation.

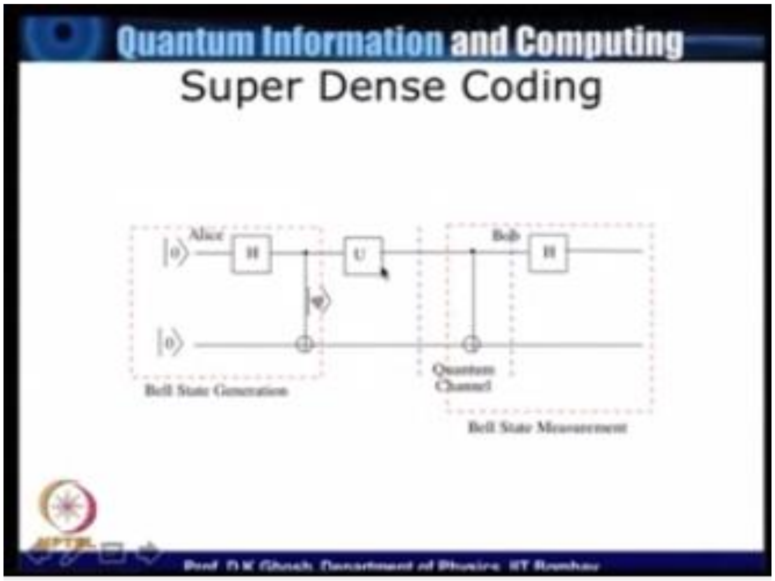
Now what happens is this that once Alice and Bob have got their bits entangled. So I have assumed that we have taken the state 00 +11.

(Refer Slide Time: 07:21)



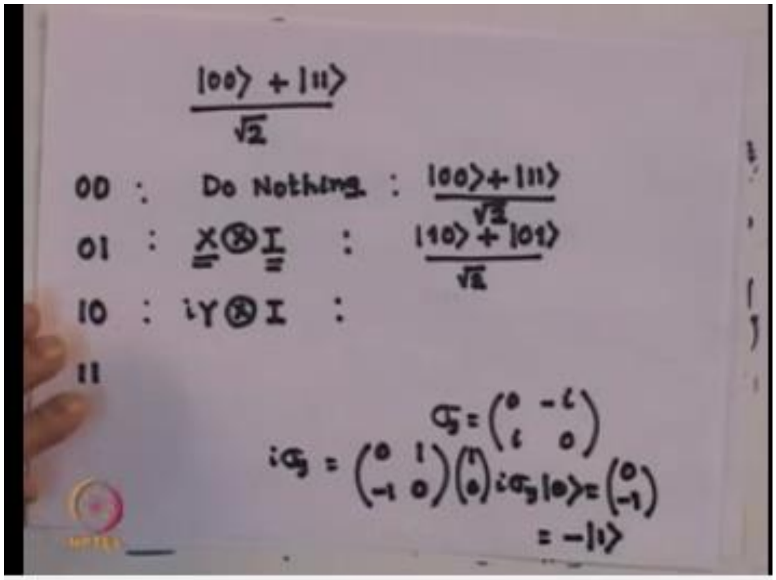
So this is the entangled state that Alice and Bob share. Now the idea of the coding is this, Alice wants to send 00, 01, 10, or 11 to Bob. Now based on what she wants to send see will be doing some unitary operation on this bell state. Now remember she can only modify the states the qubit which is with her part of the qubit see cannot do anything to the other qubit. Now in this particular case that, what Alice will do if she wants to send 00 she will actually do nothing.

(Refer Slide Time: 08:30)



Now so let us return back to this state again, so we have the picture again so this stands for the unitary operation which Alice will perform the depending upon what she wants to achieve. So at this point I have said that if she is interested in sending the state 00 she will do nothing.

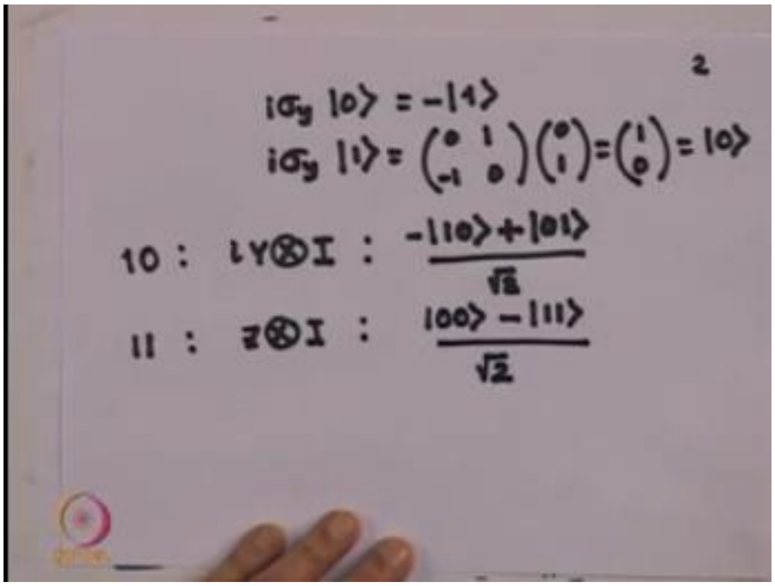
(Refer Slide Time: 08:55)



Now suppose she wants to have 01, then what she will do will be to perform. Now let me write it down X x I meaning thereby on her qubit X is applied on Bob's qubit nothing is applied because she cannot apply anything on Bob's qubit. So let us see what will this give me. So if she does that, so therefore this results in whatever state we had 00 +11/√2 and this will result in remember what X does is to interchange X 0 and 1.

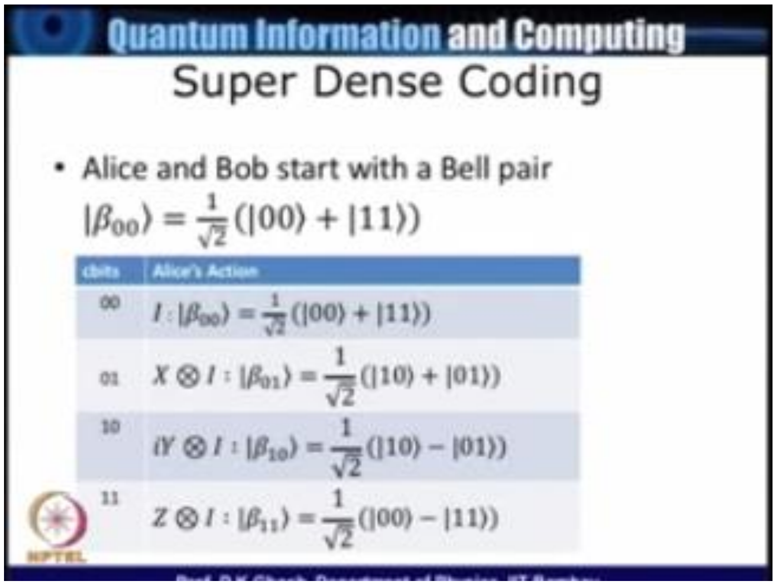
So therefore, it will give me 10 +01/√2. Now suppose she wants to send 10 there what she will do will be to send use an operation Y gate iy of course cross I, but note the following that the matrix σy is 0, – i, i, 0. So therefore, if I write down I times σy, so that gives me 0, 1, -1, 0 so as a result I times σy acting on the state 0 it gives me, well I operate this on 10, so this is 0 -1 which is equal to - of state 1 and i times σy I will return back to this list.

(Refer Slide Time: 11:00)



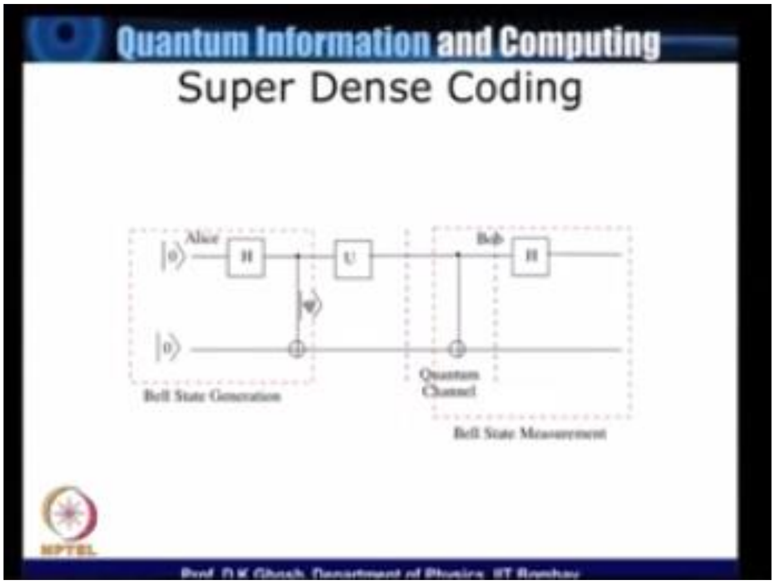
So i times σy acting on state 0 gives you -1 and i times σy acting on state 1 is 0, 1, -1, 0 acting on 01 which is equal to 10 which is nothing but 0. So what we said is that if she wants to send state 10 she will apply iy x i on that state. So that was, so what it will do is I had a 00, 0 becomes -1 so it is -10 and 1 becomes 0 so i get 01/√2. So that is then, and if she wants to send 11 then what she will do is to apply z x i so z x i will give you okay, remember that what z does is to change the sign of 1 so therefore on her qubit it I get a 00 – 11/√2.

(Refer Slide Time: 12:35)

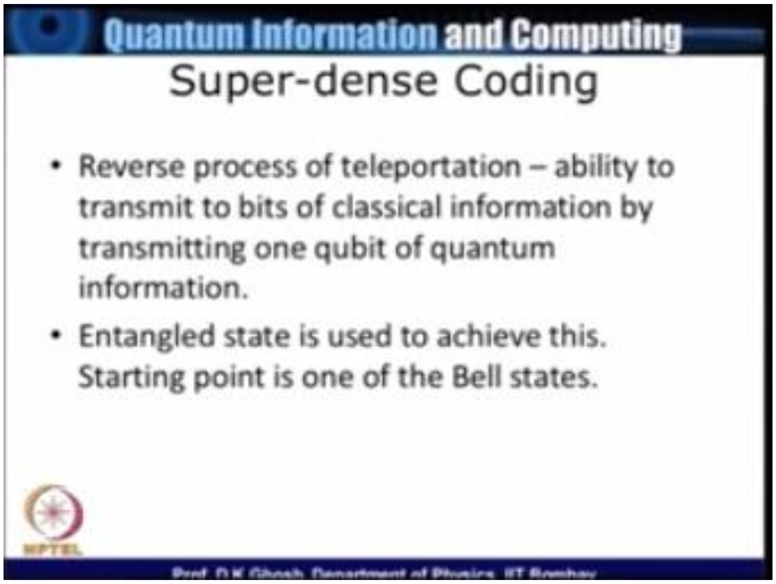


So that is the next slide gives you the full picture of what happens I think the third one needs a - sign change but that as we know overall sign [indiscernible][00:12:45]. So this is exactly what happens when Alice does something to help you with okay, having done that. So you notice what we have got basically are the four bell states which are generated depending on what classical information Alice wants to send to Bob.

(Refer Slide Time: 13:44)

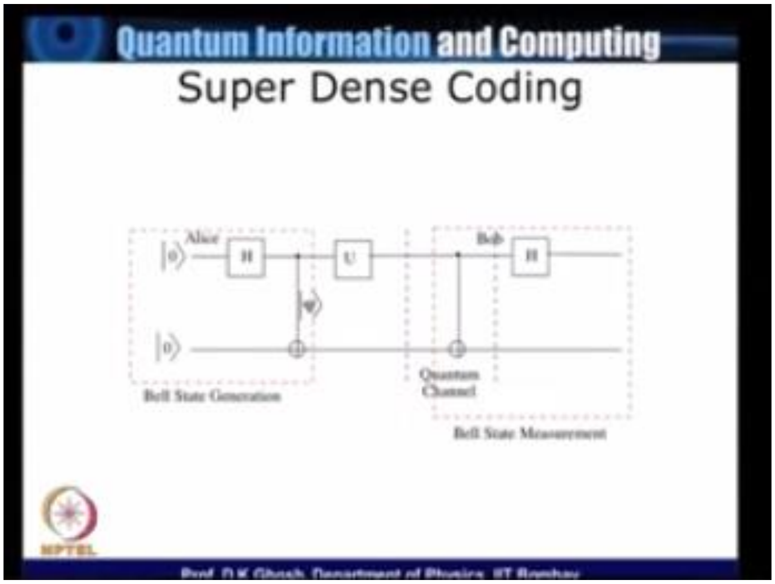


(Refer Slide Time: 13:24)



So now that Bob has received that let us once again look at our the diagram.

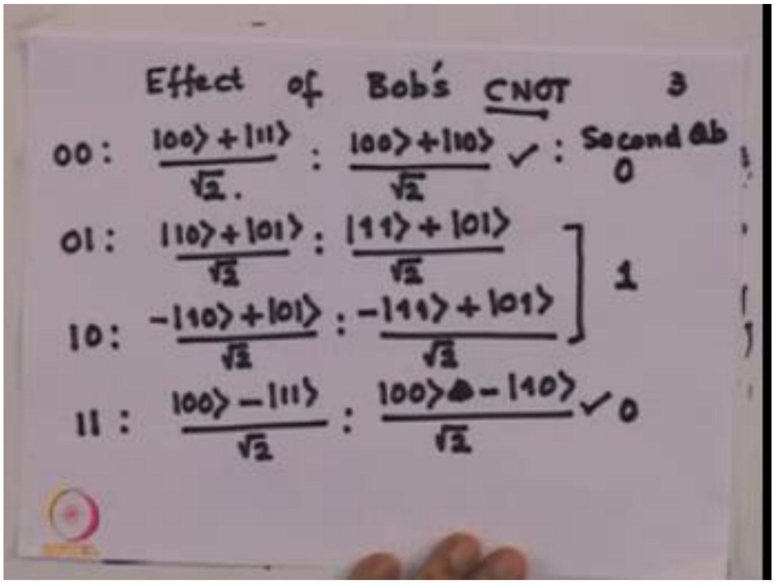
(Refer Slide Time: 13:33)



So now the next part you look at the right-hand side there is a red rectangle which tells you that these are the measurements that Bob is doing. Now what did Bob have, what Bob has the four different bell states, and these have been generated by Alice by doing something to her qubit alone depending upon what information she wants Bob to have. Now what Bob does now is to use a CNOT operator first.

Now let us look at how does that work, so what we have said is using the Alice's bit as the control and Bob uses as CNOT. So let us look at what is the effect of CNOT Bob's CNOT.

(Refer Slide Time: 14:36)

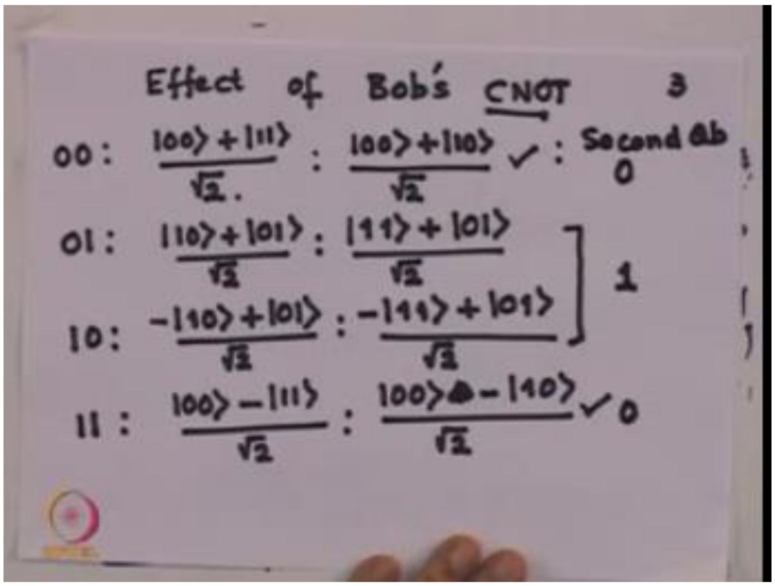


So when Bob applies the CNOT what will happen is let us look at again what Bob has received. So Alice wants to send 00 so we have got 00 +11/√2 when Bob applies CNOT because it depends upon what is Alice's bit here so it will be 00 and + 10/√2. So this is what Bob has got after he has applied a CNOT operation. When Alice wanted to send 01 we had seen earlier that what Bob received is actually 10 + 01/√2 and if you apply a CNOT what you get is because this is 1 so I get 11 + 01/√2.

If she wanted to send 10 what we got is -10 + 01/√2 which gives me -11 + 01/√2 and finally, if she wanted to send 11 Bob received 00 -11/√2 and which then would become 00 + -10/√2. This is the application of CNOT gate results now. Now notice very interesting thing that after Bob's CNOT operation using the first qubit as the control and the second qubit as the target, you notice the second qubit is in this case as well as in this case happens to be 0. And in these two cases it happens to be 1, so let me make a note here the second qubit is 0 here, second qubit is 0 here in these cases it is equal to 1.

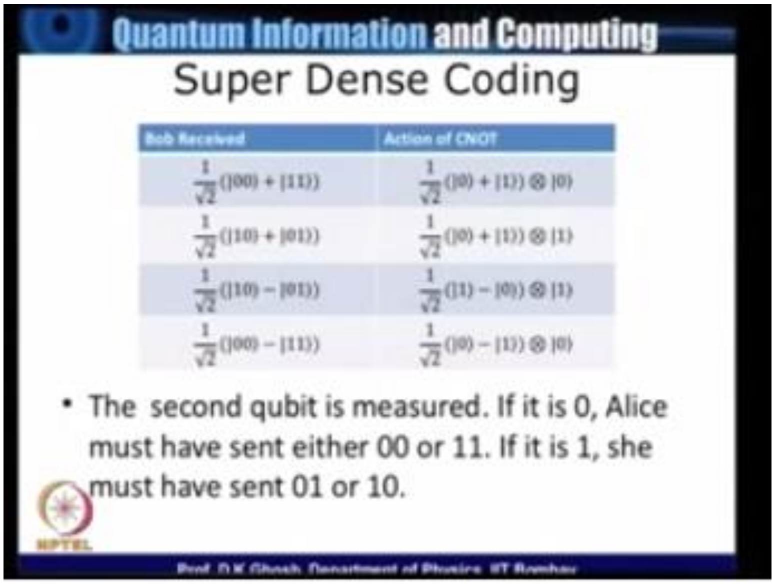
So at that stage what Bob does is to measure the second qubit. Now when Bob measures the second qubit Bob will get either a 0 or a 1 with equal probability. Now if Bob got, look at this picture again.

(Refer Slide Time: 17:48)



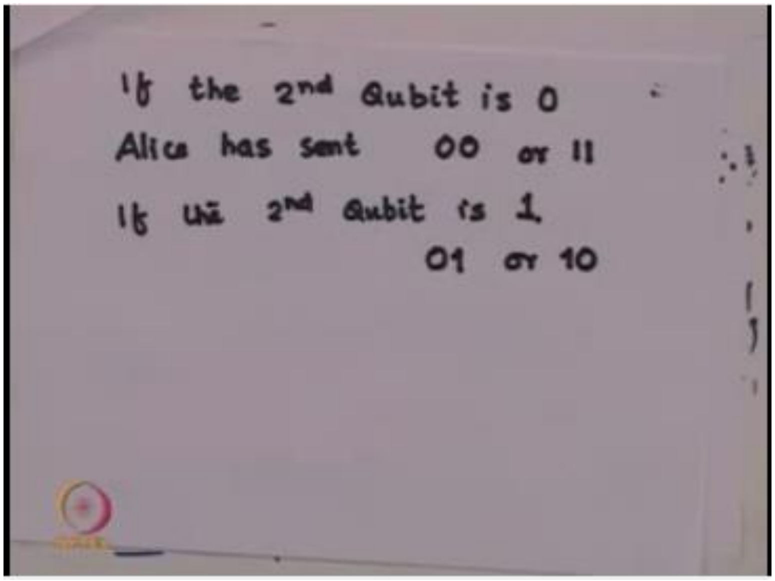
This is the same thing which is there on the slide also.

(Refer Slide Time: 17:51)



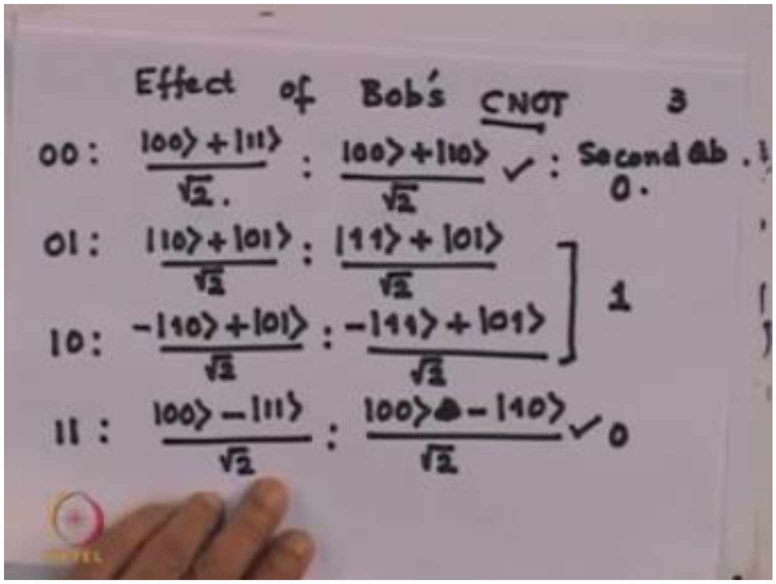
If you look at that picture of what happens when Bob applied a CNOT so in second qubit is measured and the bit is 0.

(Refer Slide Time: 18:07)



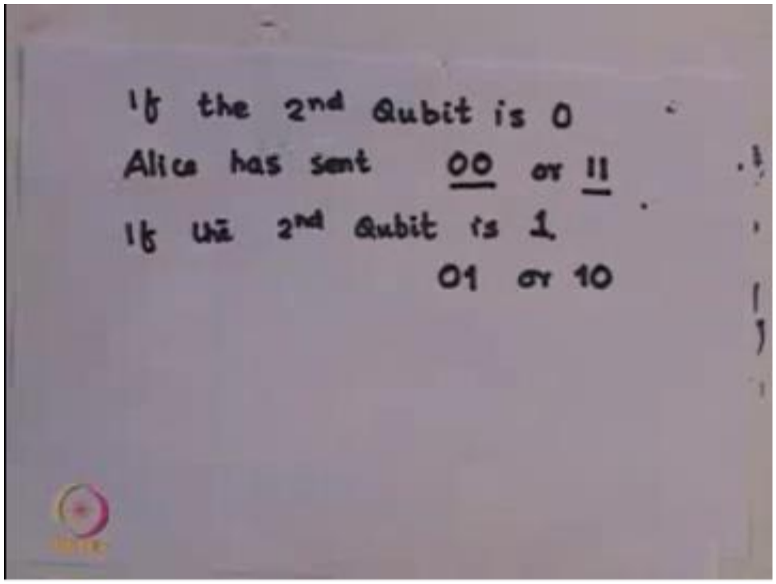
So if the second qubit is 0, Alice must have sent either 00 or 11, is important and if the second qubit is one Alice must have sent either 01 or 10. So this is the intimation that we get from measurement of second qubit. Now second qubit has collapsed, but the first qubit is still in a linear combination of states corresponding to the situation where the second qubit is 0 in the first case or second qubit it is 1 in the second case. So let us look at what that happens here.

(Refer Slide Time: 19:16)



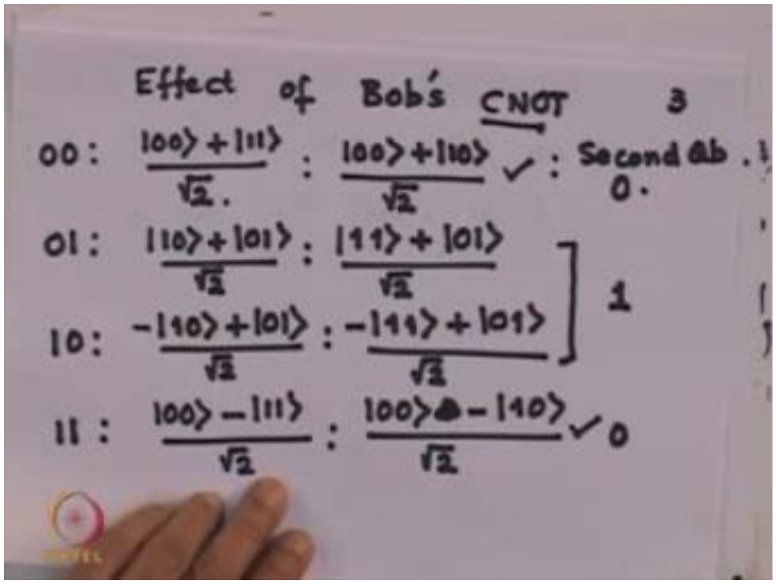
Now let me return come back here again to this state, look at when the second qubit is 0 the first qubit in the first case is 0 +1/√2 in this case it is 0 -1/√2.

(Refer Slide Time: 19:33)



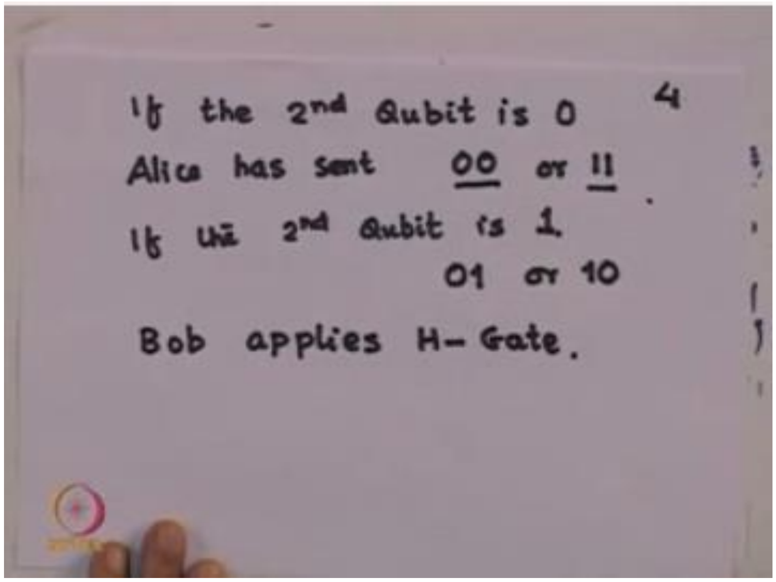
So therefore whether the case is this or the case is this there is a difference in the first qubit because this is 0+1/√2 and this is 0-1/√2.

(Refer Slide Time: 19:49)



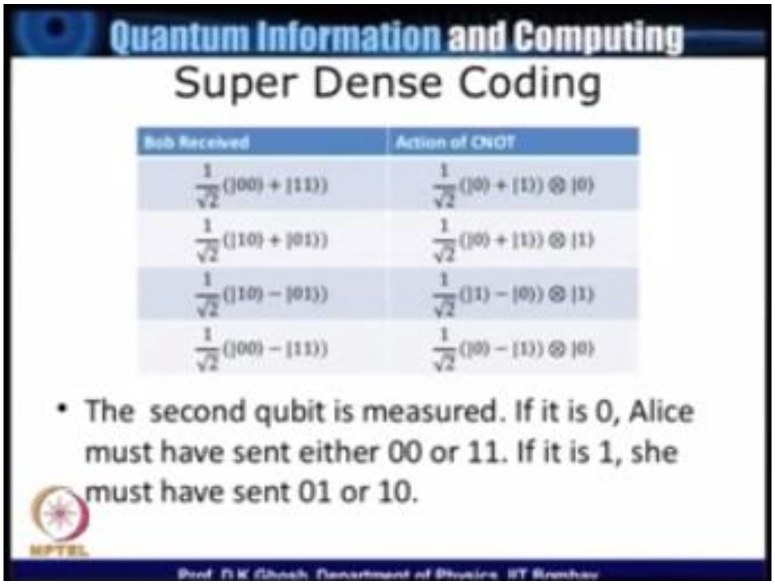
And likewise if the second qubit is 1, when the second qubit is 1 the first qubit in this case is 0 +1/√2 this is 0-1/√2.

(Refer Slide Time: 20:00)



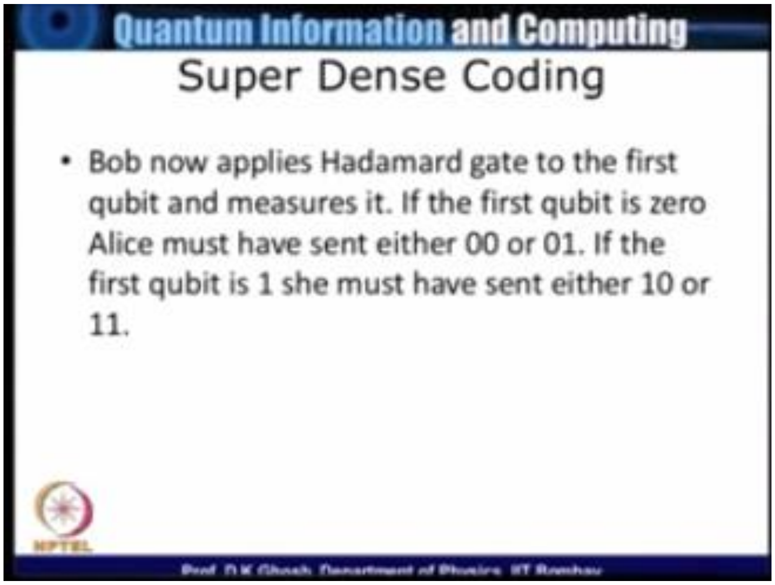
So the first qubit is different in the two cases. Now what Bob does now, Bob applies now a Hadamard gate. So look at the result of the Hadamard gate from the side.

(Refer Slide Time: 20:26)



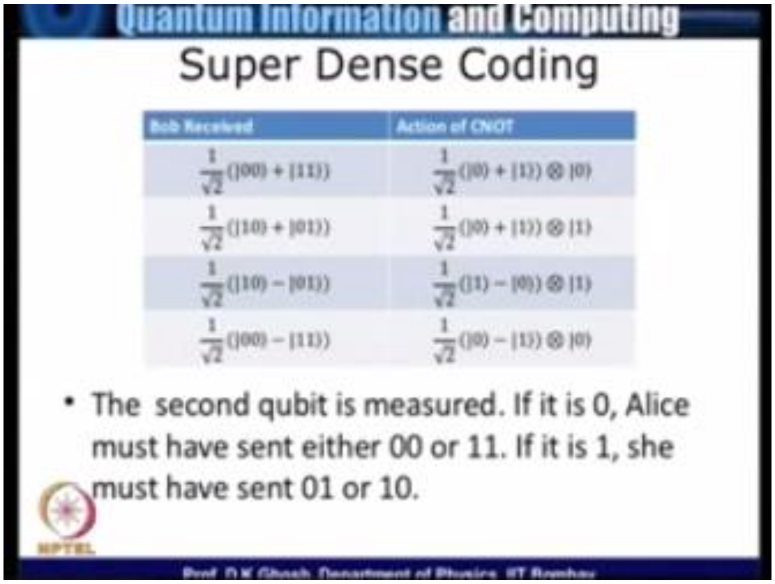
So in this particular case when I had the after the CNOT so this is the written as a factor. So if this is 0 it is 0 +1/√2 if it is or it is 0 -1/√2. So if you apply Hadamard on the first qubit you will get this to be 0 and this to become 1. So notice that what Bob will now do will be to make a measurement of the first qubit. Now when Bob makes a measurement of the first qubit okay.

(Refer Slide Time: 21:05)



In the first qubit is 0 okay.

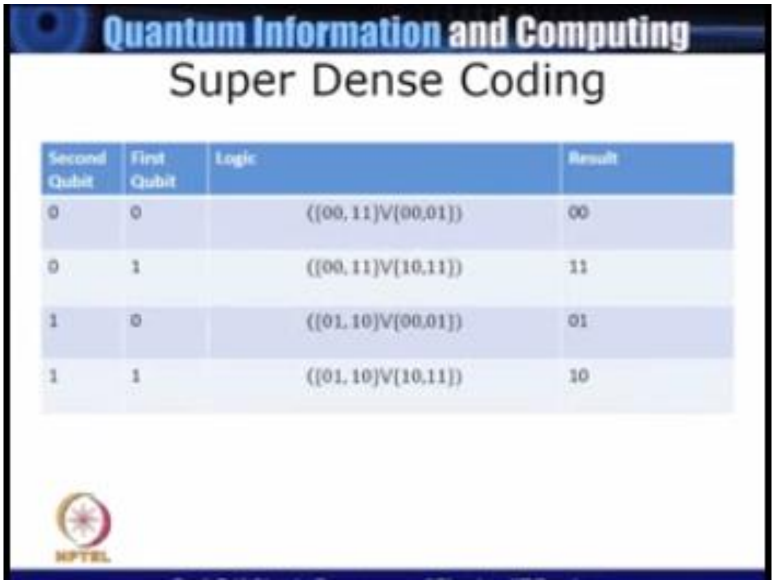
(Refer Slide Time: 21:13)



Then look at the situation back from the table again. So look at the situation in the first qubit. So the first qubit becomes 0 or application of Hadamard here, first qubit becomes 0 on application of Hadamard here. So therefore, if first qubit is 0 Bob will confirm in for that Alice must have sent either 00 or 01, but then he had a second qubit measurement Alice must have sent either 00 or 11.

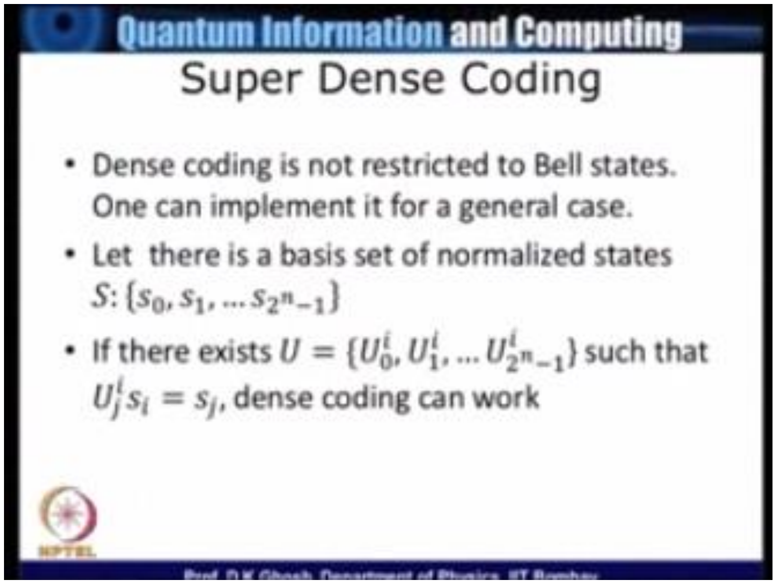
So therefore a combination of the measurement of the first and the second qubit will enable Bob to infer what exactly was the state sent.

(Refer Slide Time: 22:00)



Because this is the complete logic, so if the second qubit is 0 first qubit is 1 then it is an intersection of 0, 0, 1, 1 and 0, 0, 0, 1 so obviously the qubits bit sent must have be 00. So this is the idea of super-dense coding. Now I would like to point out that though we have used bell state as an illustration, the super dense coding is not necessarily restricted to bell state where does super dense coding work, the super disclosing work because of the following things.

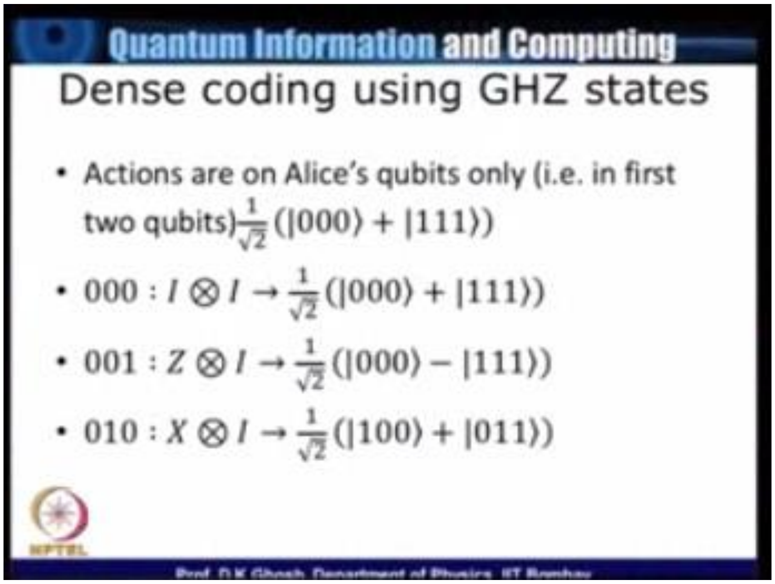
(Refer Slide Time: 22.46)



That suppose I have a basis set of normalized states S which contains let us say 2^n basis states and which I have called as S_0, S_1 to S_2^{n-1}. Now suppose there exists a unitary operation which is such that, that if I apply that there is a unitary operation corresponding to every qubit such that the UJ of I acting on ψ=SJ. Remember this is just a formal way of stating what we did with the bell state.

What we did with the bell state is to say that suppose I wanted to send 00 I start with the bell state. Now if I do wanted to send 01, but I am allowed to do an operation only on the first qubit. So therefore, I do certain operations on the first qubit to convert that state into another state which belongs to that set namely in that case the set of the four bell states. So by doing an unitary operation on the first qubit I converted the bell state into one of the four bell states. Now if this is the situation in general then my dense coding or super dense coding as it is sometimes called will work just to give you an example.

(Refer Slide Time: 24:36)



You can check the following I just tell you how it actually works and then you can do this exercise at home. So I will take what is known as the GHZ state which is a 3 qubit state which is 000 +111. Now in this case what Alice will try to do, will be Alice has two cubits and Bob has the last qubit. Now what Alice will try to do is to send two qubits of quantum information to Bob, hoping that the Bob will be able to get one of the eight possibilities namely 000, 001, 010, etc., the next page has the complete list.

And then Bob by making a suitable measurement which you can try to work out will be able to guess what is the 3 qubit three bits of classical information that Alice wanted to send him. So for instance, we start with GHZ state supposing Alice wanted to send look at the last element 010, so Alice does an X x I remember Alice cannot touch Bob's bit. So what does X do, so X as we know interchanges okay, X will interchange 1 and 0.

So therefore, this is what Alice wants to do and this is what she will get. And then she will send the whole thing to Bob, Bob will do certain operations which are not telling you here and based on which Bob will be able to conclude what bits Alice wanted to send him, what classical bits

Alice wanted him to have. So the idea of super dense coding was that by sending a smaller number of quantum bits Alice is able to communicate to Bob a larger number of classical bits of information.

The point to notice that they must share an intended state beforehand and Alice should be able to perform certain operations only on her qubit leaving Bob's bits untouched. Having completed it she would send them by a quantum channel to Bob, Bob will perform certain operations using use certain logics based on the result of the measurements she will have on these qubits that he has after Alice has sent them and he will be able to conclude what classical bits Alice wanted him to have.

NATIONAL PROGRAMME ON TECHNOLOGY
ENHANCED LEARNING
(NPTEL)

NPTEL
Principal Investigator
IIT Bombay

Prof. R.K. Shevgaonkar

Head CDEEP

Prof. V.M. Gadre

Producer

Arun kalwankar

Online Editor
& Digital Video Editor

Tushar Deshpande

Digital Video Cameraman
& Graphic Designer

Amin B Shaikh

Jr. Technical Assistant

Vijay Kedare

Teaching Assistants

Pratik Sathe
Bhargav Sri Venkatesh M.

Sr. Web Designer

Bharati Sakpal

Research Assistant

Riya Surange

Sr. Web Designer

Bharati M. Sarang

Web Designer

Nisha Thakur

Project Attendant

Ravi Paswan
Vinayak Raut

NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING (NPTEL)