Advanced Marine Structures
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Lecture - 5
Spectral Fatigue Analysis

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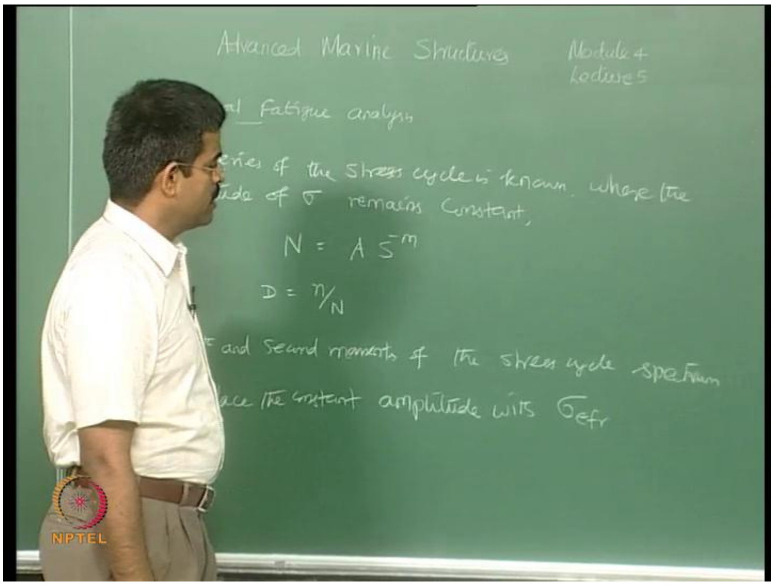

So, we have been looking into the fatigue damage of marine structures.They are initially we said ifthe time seriesof the stress cycle is availablewhere the amplitude of the stress remains constant,thenone can say thatN can be, for the S-N curve is simply A S minus m and the fatigue damage can be simply n by Nand soon. As we know, that the stress cycle with constant amplitude does not remain in marine structures because of the dynamic effects caused on structures.Then we have discussed about the spectral analysis, where we said find out the 0thand the second momentof thestress cycle spectrum.

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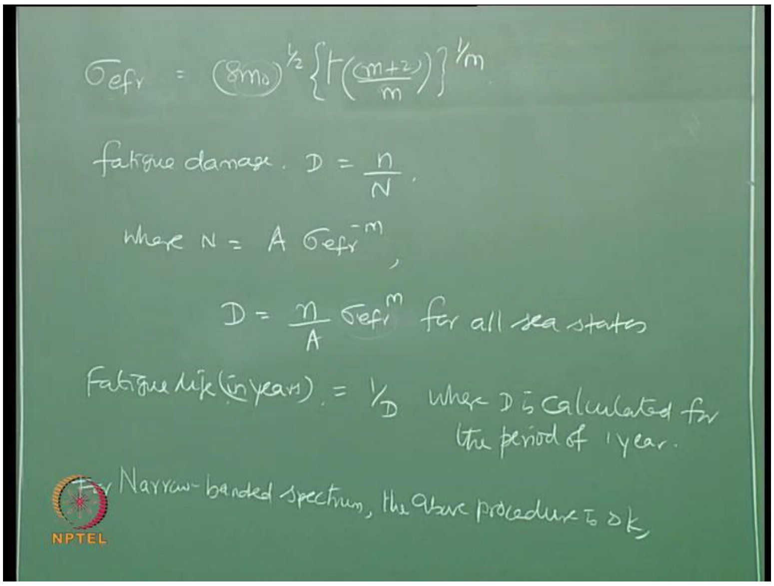

Thenas a simplification, replace the constant amplitude with sigma efr, where efr is the effective fatigue stress range, which is given by a simple expression in the effective fatigue stress range,gamma function ofm plus 2 by 2raised to the power of one minute, mplus 2 by m raised to the power of 1 by m.Once you know this, again you can findthe fatigue damage D is simply n by N, wherein my case nown will be A instead of S. I will use sigma efr to the power minus m. So, in that case D becomes n A…this is for all sea states.

You may wonder, that how this equivalent effective fatigue stress range is representing all sea states, though it is constant amplitude, which is having equivalent value of effective stress range.This is accounting forthe spectral moments, which has been taken from the stress cycle spectrum. So, this single value represents equivalent fatigue stress range, which is now applicable to all the sea states.

Now, if you really wanted to find what we call fatigue lifein years,you can easily find inverse of this,where D is calculatedfor the period of 1year.Now, this is true, this exercise is true when the stress cycle spectrum remains narrow band. So, I should say,for narrow banded spectrum the above procedure is ok, but if it is broad band, then how do we handle this?

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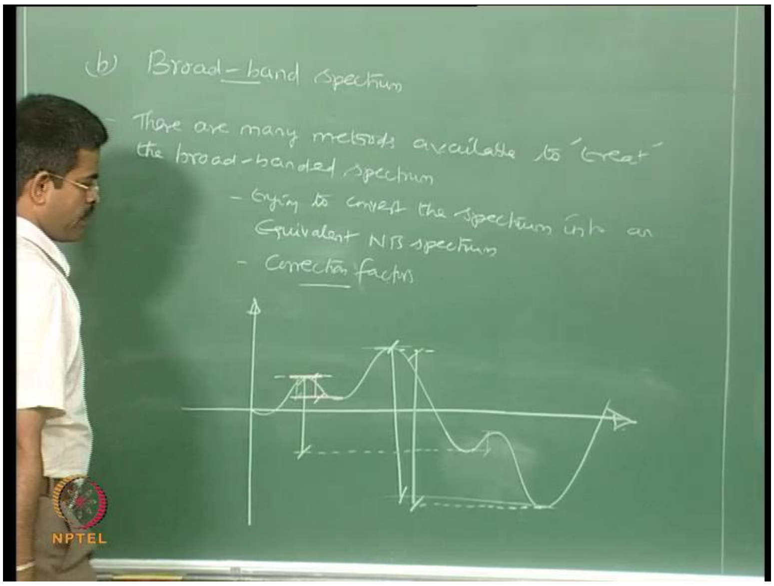

The spectrum, the spectrum is broad banded, then how do we handle this?There are many methods available to treat, I should say,to treatthe broad banded spectrum.Now, what kind of treatment is given?It is trying to convertthe spectrumintoan equivalentnarrow band spectrum. So, we call this as correction factors.

Now, let us see, how they are handled?Let us try to draw a typical broadband spectrum and see what correction factors or what different methods are applicable to treat them into converting them into a narrow band spectrum.Let us see a typical broadband spectrum like this,which I am drawing here.Let us say, this is my typical example of abroadband spectrum.

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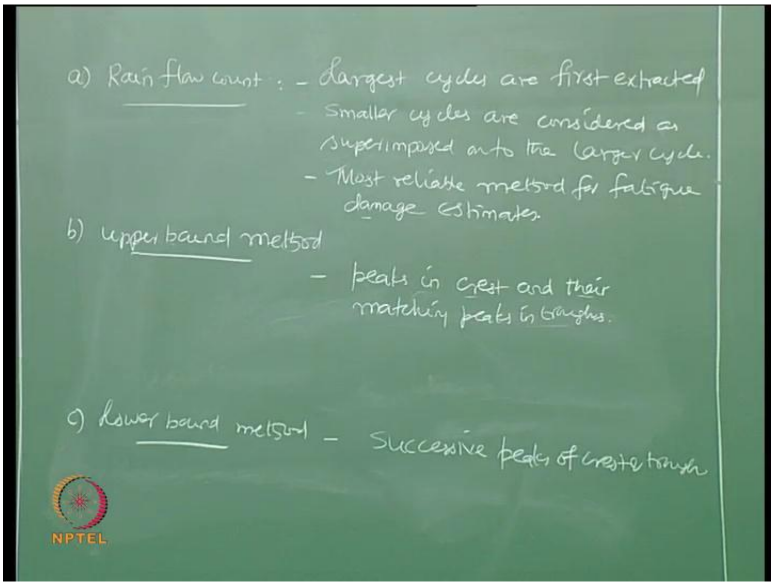

Now, there are three methods available, which is commonly applied in the literature for treating this kind of broadband spectrum.Let us see what are they?First could be the conventional rain flow count.The moment you hear this name rain flow count, you are already remember, that this is the technical, traditional method available and applied to peaks and valleys, which we saw in the time series of the stress cycles, where we used it to find out the fatigue damage estimate.But let us see how I will modify this to use it for a broadband spectrum like this, because there are no specific peaks and valleys.Because for this, there is no valley; for this peak, there is a valley; for this peak, trough, both are trough, etcetera.It is very difficult to identify the successive peaks in valleys. So, let us see how we will handle this.

The second methodis upper boundmethod.The thirdone isthe lower boundmethod.In the rain flow count method, largest cycles are first extracted. So, largest cycles means,let us say,look for the maximum peakand maximum trough, these are first extracted from the figure. Then the smaller cycles are consideredas superimposedonto the larger cycle.This method is considered to be the most reliable techniqueforfatigue damage estimation. So, look at the maximum range. Then subsequently,you can look for the smaller ranges.This is smaller range; this can be another smaller rangeand soon. So, all these are superimposed on the larger range and use that as the rain flow count technique to find out the fatigue damage estimates.

Now, look at the next technique,which is the upper bound method.In this case, look for thepeaksincrestand theirmatching peaksin troughand form a pair.Let usseewhat does it mean? Let us say, in this case I can say, thatthis and this are approximately matching, sothis can beone cycle.And you can say this value and this value,that is, the peak in the crest and the peak in the trough are matching, this can beone case. So, keep on grouping them. So, peaks in the crest and therematching peaks in that trough are grouped and then you can do the cycle counting and do the fatigue damage estimate.

In this case,look for the successivepeaksofcrest and trough, that is, look for this,look for this, these are successive, you can see there is a peak and then there is a trough.Similarly, there is a peak and then there is a trough and soon. So, keep on grouping them. So, it is estimated, that this will give me the lower bound count of the fatigue damage, this will give me the upper bound count of the fatigue damage.This is the most reliable method, which can give me the fatigue damage estimates correctly. So, if you have got the broadband spectrum like this, I can try to group the peaks of the values or the maxima and minima or the peaks of the crest and peaks of the trough in these three manners and then try to estimate the fatigue damage by the following method.

As I said,once I do this, I would like to convert the given broadband spectrum to an equivalent narrow band spectrum because this will help me to use the existing narrow band spectrum equations available to me for fatigue damage. I will apply a correction factor to this suggested by various researchers.

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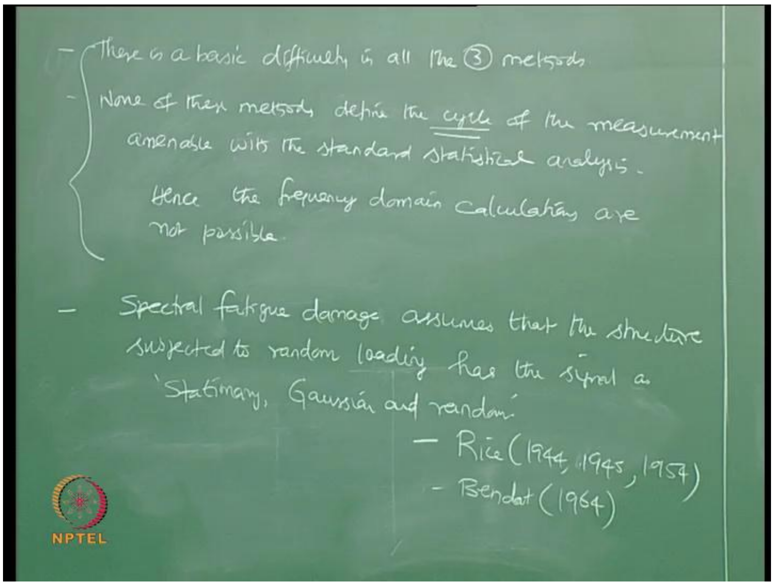

Now, all the three methods have a basic problem. There is a basicdifficultyin all the three methods.None of these methodshas definition of cycle because as I said, keep on picking up the largest peak in the crest and the largest peak in the trough. So, you are picking only the maximum values at the amplitudes, but the cycle is not concerned.You can see in this example, the cycle is well spread. So, none of these methods has definition.Let us put it like this, none of these methods definethe cycleof the measurementamenable,amenable,with the standardstatistical analysis.Hence,the frequency contentis not possiblebecause the cycle or the time cycle is not defined.We pick up only the amplitudes; we do not have any control on the time cycle.Therefore, how to handle this problem?

So, now, the spectral fatigue damageassumesthatthe structure subjected to the random loadinghas the signalas stationaryGaussian and random.This is a very famous assumption, which has been established byRiceand Bendatin early 60s.

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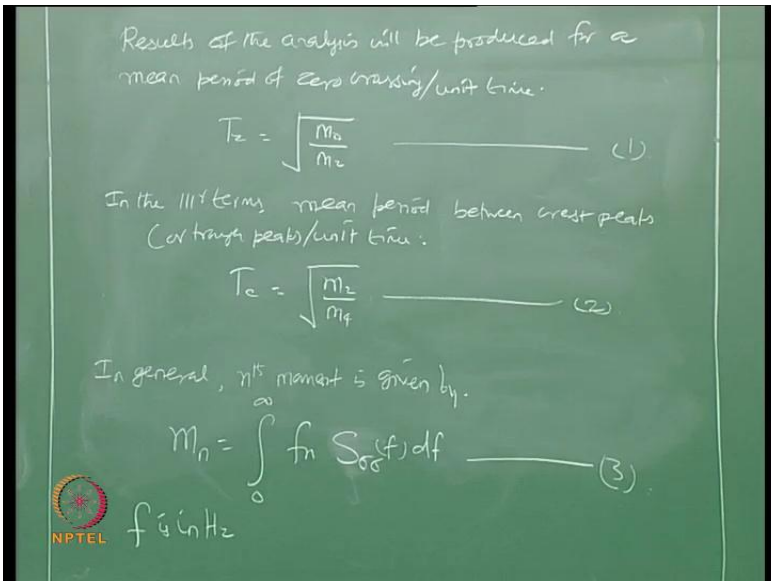

According to this,the results of the analysiswill be producedfor a mean period ofzero crossingper unit time.We already have a classical definition for zero period mean crossing per unit time, which we said is equal tom naught.I will call this equation number 1.

Now, in the similar termsthe mean periodof,crossings ofmean period betweencrest peaksor trough peaksper unit timeis given by T suffix e, I am using crest here, root of m 2 by m 4.These are true when the given narrow band (( )) stationary is Gaussian and pure random in nature, I call this equation number 2.

Now, I have 0th moment, we computed for the stress cycle spectrum,2nd moment,then4th movement.In general, we already have an equation tocompute the nth movement.The nth momentis given bym n 0 to infinity f n, sorry,s sigma sigma fdf as a general equation. So, you can find 0th moment,2nd moment and 4th moment using this expression for a given spectrum, where f is in Hertz and s sigma sigma is the y-axis of the spectrumhere, f is in Hertz.

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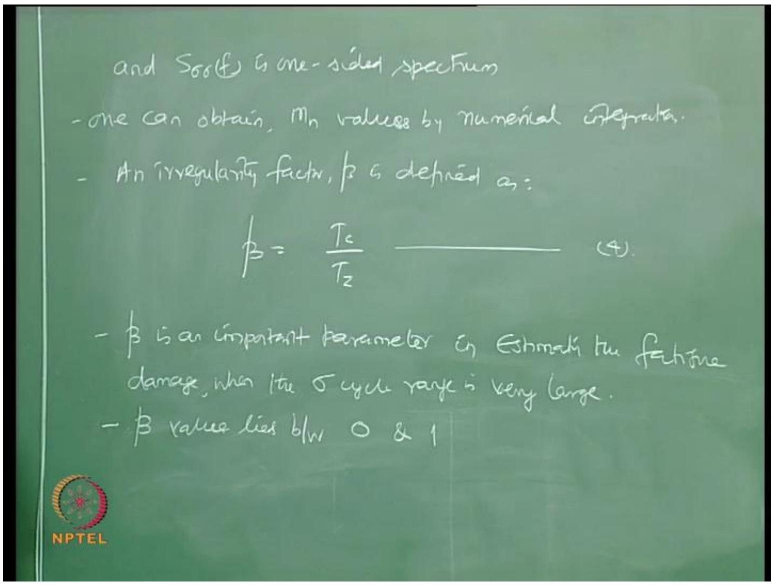

Ands sigma sigma f isone-sided spectrum,that is, the half is not there. So,one can obtainm n valuesby,by, also bynumerical integration.If you are not able to integrate it for a given domain you can use numerical integration technique and find m n values, forn varies for 0, 1, 2, 3, 4, etcetera.Having defined 0, mean crossing period T z and the mean period between the crest peaks or the trough peaks as T c, I define now an irregularity factor,beta is now defined,beta is an irregularity factor,is a ratio ofthesetwo periods,equation number 4.

Beta is a very important parameter in estimating the fatigue damagewhenthe stress cycle rangeis very large.The values of beta lies between0and 1, let us see what 0 means and what 1 means.As beta approaches 1, the irregularity factor approaches 1,what will happen? T z will become T c.

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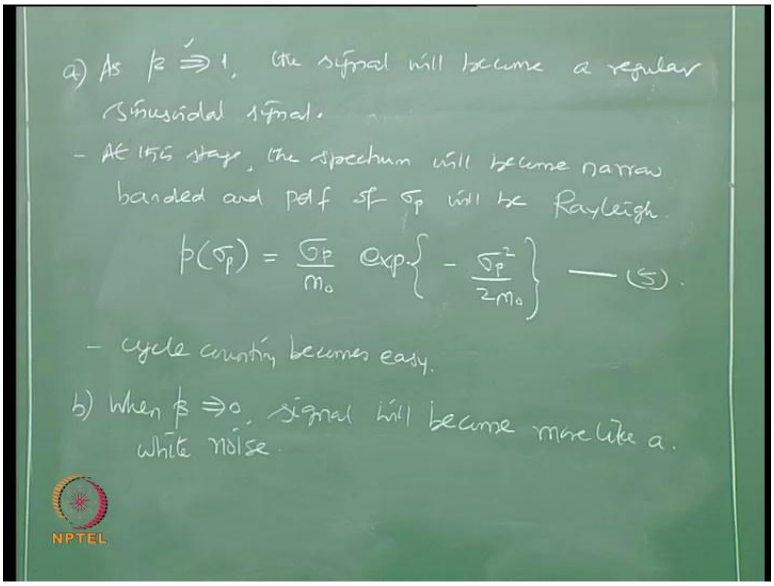

As beta approaches 1, unity, the signal will becomea regular sinusoidalsignal. So, at this waveor at this stagethe spectrumwill become narrow bandedand the probability density function of this stress range will become Rayleigh distribution. So, I can write the probability distribution function of this stress range of the peak. So, I should say, pdf of sigma p peaks, I am talking only about the peaks.It is now a standard equation of Rayleigh distribution, which is sigma p by m naughtexponential ofminus sigma p squareby 2 m naught,equation number 5.So, when beta approaches 1, the cycle counting becomes simple, is it not?The basic problem what we had in the broadband spectrum is cycle counting.We are not having a difficulty in identifying the peaks and troughs; that we have already done.

Now, how to count the cycle, because I cannot use it in the frequency domain, because cycle counting is not clear? So, when beta becomes 1, by introducing an irregularity factor, as you see here,is a ratio of the time periods between crest or the troughs, with that of zero mean crossing periods of the stress cycle.The signal, the signal will get converted to a regular sinusoidal signal.For that the spectrum will become narrow banded and the cycle counting becomes easy.When beta approaches 0, I can say this case a. So, let us say, when beta approaches 0, is not 0, it is approaching 0, close to zero signal, will become more like a white noise.

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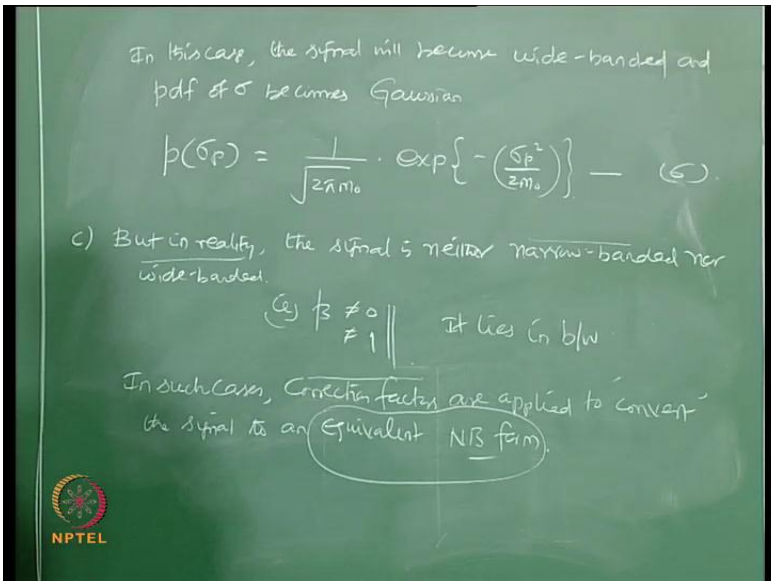

In this case, the signalwill become wide bandor broadband and the probability density function of stress becomesGaussian.It will follow Gaussian distribution, not Rayleigh.In that case, probability distribution of function of the peak will be given by a different equation, which is standard Gaussian equation 2 pi m naught exponential of minus sigma p square by a by 2 m naught.Let us use bracket here also because there is a negative sign here.So, let us say, equation number six.But in realitythe signal is neither narrow bandednor wide banded,that is,beta is not equal to 0, not equal to 1, soit lies in between.In that case how do we handle this?

If case a and b are valid, then you can easily use probability density function directly. I can easily find out thefatigue damage, spectral fatigue damage, but when beta is I mean when it is neither narrow banded nor wide banded how do we handle this problem?In such cases,correction factors are appliedto convert, I should say,the signalto anequivalentnarrow band form. So, people can convert this into an equal and narrow band form.

Now,one can be happy to know, now once I convert this narrow band form all my equations available for narrow band form, which we discussed in the last lecture, can be directly applied. So, I can easily find out the equivalent fatigue stress range sigma efr and try to find out the fatigue damage by the equation, which we discussed in the last lectures or the fatigue live estimates. So, now, I must have a correction factor applied to the existing signal to convert that in equivalent narrow band, sothat an ideal equation is here.

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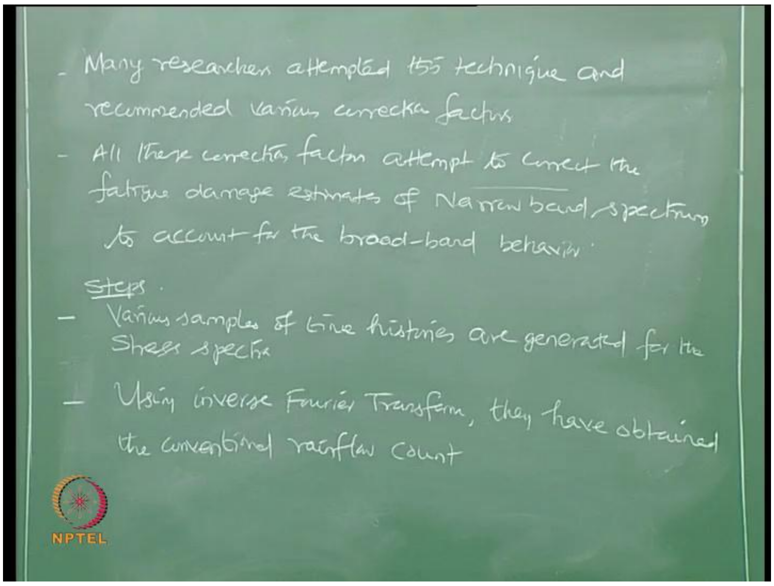

Now, many researchers attempted thisattempted this technique and recommendedvarious correction factors. So, all these correction factors,all these correction factors attempt to correct the,correct the fatigue damage estimatesof narrow bandspectrumto account forthe broadband. So, the correction is not applied on broadband, the correction is applied on the calculations made on the narrow band, sothat correction will take care of the conversion of the broadband to narrow band, right. So, very briefly, we will notget into the methodology of this, these are empirical relationships,let us quickly understand how they have been donebefore we look into what they have done, various samples oftime histories are generated.

These are the steps what people have followed in arriving into correction factors, various samples of time history are generated for the stress spectrumfrom the time history using inverse Fourier transforms.They have obtained the conventional rain flow count, they have obtained the conventional rain flow count and based on which they have given the correction factors to predict the fatigue damage.There are many of them, but we will see only few of them.

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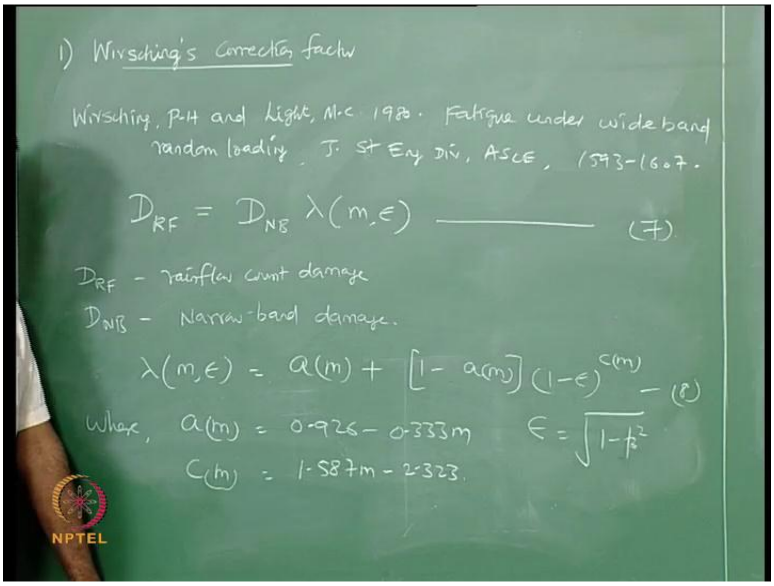

The firstone is given byWirsching's, named after him,correction factor,WirschingP-H and light,M-C, 1980,fatigue underwide band random loading,journal ofstructural engineering division ASCE, 1593-1607, that is the reference where this Wirsching's correction factor is being borrowed.He says thatthe damage estimate based on the rain flow count, the rain flow count is modified by an equation, which is given for the damage estimate for the narrow band multiplied by a factor lambda, which is a function of m and epsilon.

Can you give the equation number here?6…

Student: No sir,7sir.

7, yeah,that is what,whereas D RF is the rain flow count damage, D RF is therain flow count damage, D NB is the damage estimated, is based on narrow band damage.Narrow band damage means, damage based on the narrow band equations andlambda m, e.m epsilon is given by an empirical value, which isfunction of a m plus 1 minus a m 1 minus epsilon to the power c of m, equation number 8.Where,a of m is again an empirical equation suggested bythe researchers,0.926 minus 0.333 m.

I will always request the viewers to look into all these original equations from the paper,though I am trying to reproduce here for the benefit of the viewers for you getfirst hand all the equations, but still you must always look into the original reference and check whether they are all right.a m… C of m…

Where m is the slope of the S-N curvethat we already know, 1.587 mminus 2.323and epsilon is root of 1 minus beta square and what is beta?No reliability, reliability is the last(( )). Correction. No, no…Beta,turn back, turn back, we just now discussed beta,what is beta called?Fatigue damage. What?

It is called adjustment factor,we are trying to apply this. So, epsilon is 1 minus beta square, beta is a ratio of proportion of T c versus T z, and a m and c are empirical values given. So, I know the lambda, Icorrected for different m slope of the curve and try to find the modified damage of rain flow count when I know the damage from the narrow band this equation is already known to me in the last lecture.

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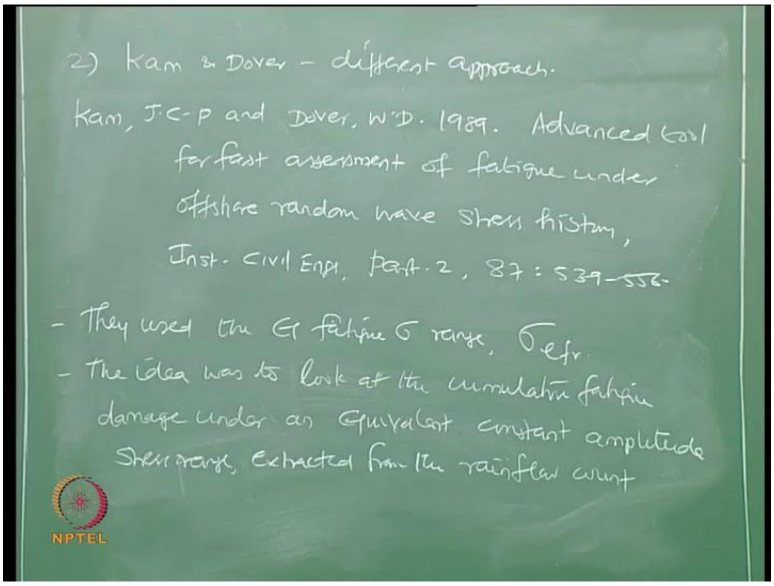

Alternatively, another researcher made a different solution.Kam and Dovergive a different approach.Institution of civil engineers, Part 2, 87,539-556.So, Kam and Dover made a different approach for this problem.They usedthe equivalent fatiguestress range,which is stressefr.The whole idea was to conceiveorto look atthe cumulative fatigue damageunder an equivalentconstant amplitudestress cycleor I should say, stress rangeextracted fromthe rain flow count.

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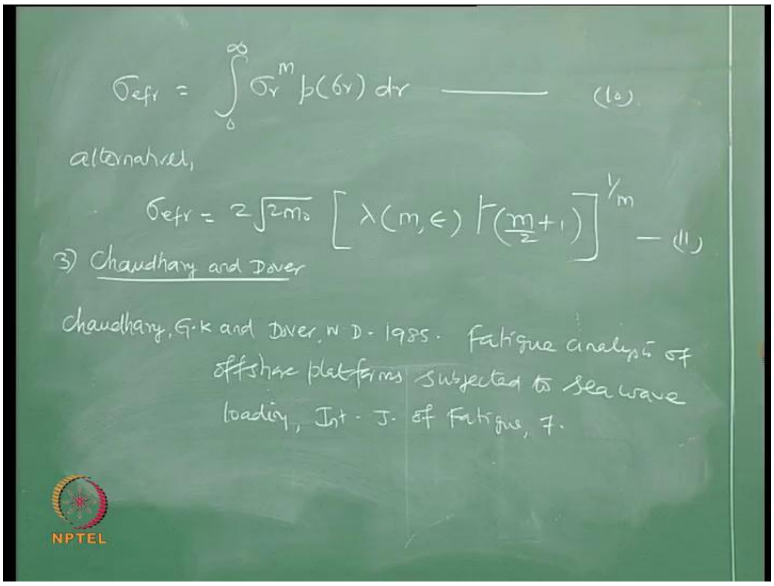

He says sigma efr is integral of 0 to infinity sigma, the stress range, the power m, the probability density function of sigma rdr, equation number 10, because those constants were 9, a of m and c of m were 9.Alternatively,sigma efrcan also be given by,that is nothing but 8 m naught root, 8 m naught root.We already have thislambda m epsilon is a same correction factor withthe gamma function m by 2 plus 1raise to the power of 1 by m.The third equation was proposed by Choudhary and Dover…The constants were 9, this is 10 and 11.Fatigue analysis of offshore platforms subjected to sea wave loadinginternational journal of fatigue, 7.

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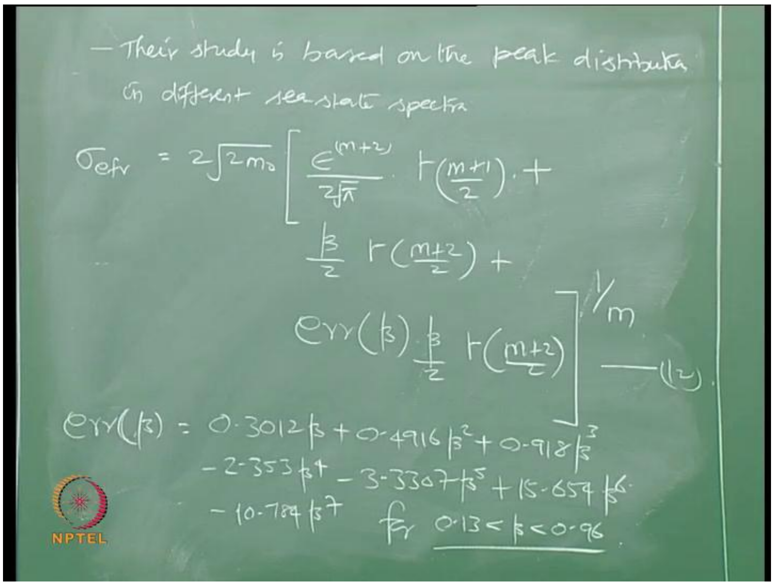

Their study is based on the distribution of the peak,peak distribution in different sea state spectrum.They say, the equivalent fatiguestress range can be given by 2 root m naught epsilon m plus 2 by, 2 pi,2 root pi,gamma of m by 2 plus 1.This is m plus 1 by 2plus beta by 2 of gamma of m plus 2 by 2 plus error function of beta,beta by 2 of gamma of m plus by 2, the whole ratio raise to power of 1 by m. So, they have modified the expression for the equivalent fatigue stress range with an error function, whereas epsilon is as same as you had in the last equationas 1 minus beta square. So, the error function of beta is given by the empirical relationship again,0.3012 beta…

The value of beta varies from0.13 to 0.96 that is the range,which is very, very valid for an offshore structure.So, in this lecture we have discussed how a broadband or wide band spectrum of the stress cycle time history can be handled to estimate the fatigue damage by converting this or by applying a correction factor, which is called an adjustment factor, to the values of the damage estimate made by narrow band spectrum calculations to account for the wide bandedness or the broadband nature of this spectrum in the fatigue damage estimation.

In reality, because in reality, the spectrum is neither narrow banded nor wide banded, it is a mixture. So, we apply a correction factor or an adjustment factor, which is applied on to the calculations and we have seen three approximate procedures suggested by different researchers in the literature, which will help us to estimate the equivalent fatigue damage of broadband spectrum based on the estimates done on the narrow band spectrum whose equations are already available to us in the last lecture.

In the next lecture we will discuss about the continuation of this and talk about more important parameter for stress concentration factors and their link with fatigue damage,how are they linked?Any questions here?

Thank you.