Advanced Marine Structures

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Lecture - 8
FOSM and AFOSM Methods of Reliability

So, in the last lecture we have seen what are the different factors, like a statistician, probabilistic engineer or probabilityengineer,then reliability engineer and a marine engineer who all jointly playarole to estimate the reliability or formation of reliability problem in a given space.

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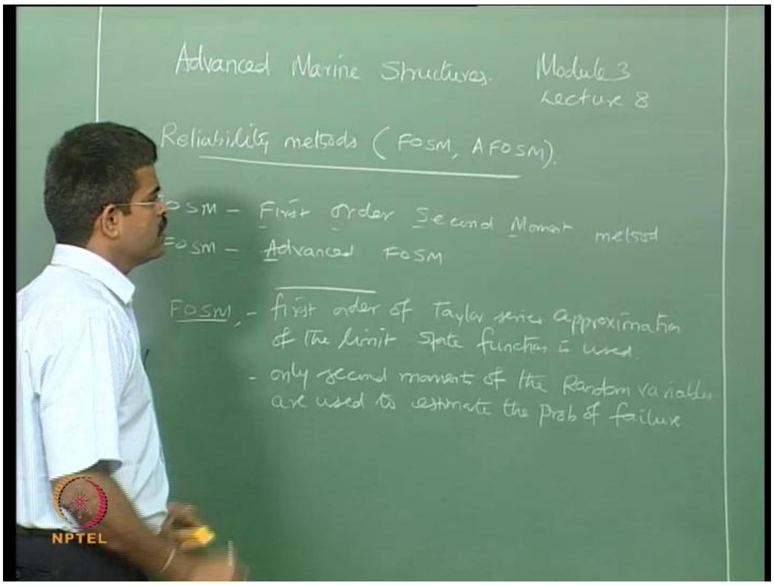

Now, let us quickly see there aretwo important methods by which reliability studies can be carried out easily for a linear limit state function, for a non-linear limit state function, which we call as FOSM and AFOSM.FOSM stands forfirst ordersecond moment method; AFOSM stands for advancedfirst order second moment method.Then first order second moment method, first order of Taylor series expansionor I should say Taylor series approximation not expansion of the limit state functionis used, that is why itis call first order.

So, you approximately formulate the limit state function is in Taylor series, and in that series you can construct only the first order termsand if you do that, then that method of reliability is generally called as FOSM.Further,only the second moments of the random variablesare usedto estimatethe probabilityof failure.

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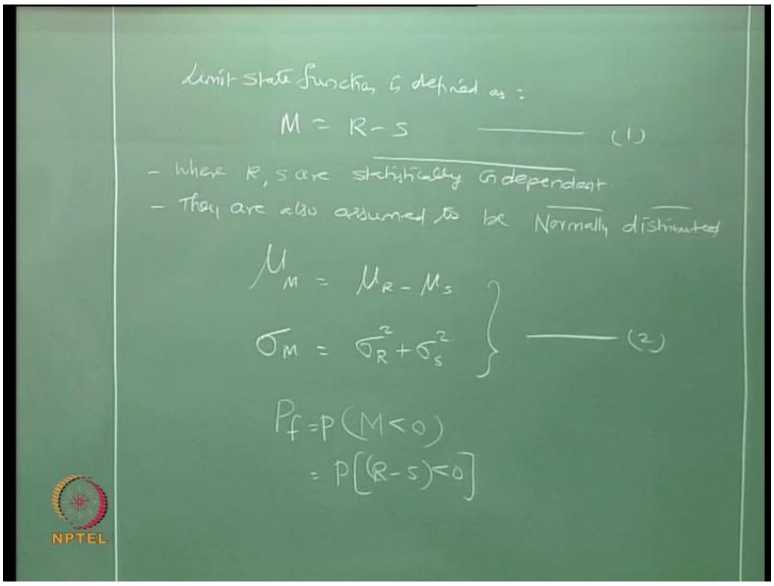

Let us saythe limit state functionis defined asM is equal to R minus S, M is a margin of safety where R and S arestatisticallyindependent.They are also assumed to benormally distributed. So,onecan say mu M and sigma M as mu R minus mu S andsigma R squareplus sigma S square.I call this asequation two.

Therefore, probability of failureis M less than 0,that is,R is lesser than S.What is the probability of M less than 0,which can otherwise, you say probability ofR minus S less than 0.

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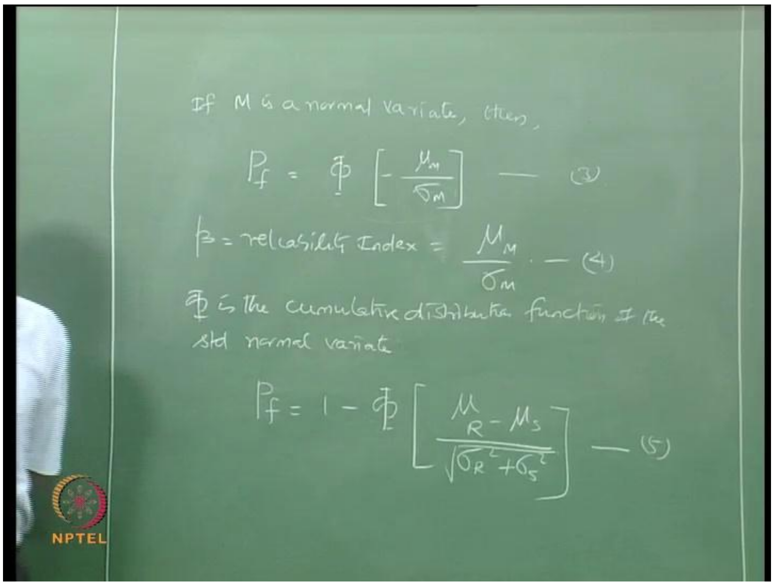

If M is a normal variate,because R and S are normally distributed, thenwe already know thisprobability of failureis simply given by phi ofminus mu M by mu R,sorry, sigma M,equation number 3,where the reliability index betais nothing but mu M by sigma M.And phi in this caseis thecumulativedistribution functionof thestandardnormalvariate.

I can substitute from mu and sigma M,as we saw from the equation number 2.Its probability of failure can be 1 minus phi ofmu R minus mu S by sigma R square plus sigma S square.Phi of any functioncan be 1 minus phi of this.This, of course, is root.Equation number what, sayI call this as 4,this as 5.

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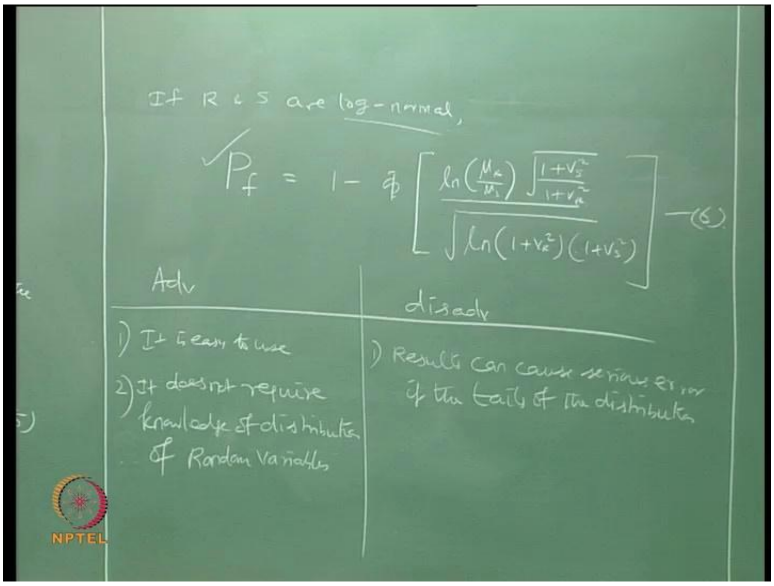

If R and S arelog normal,we have already given in this equation, you can easily tell mewhat is the probability of failure.We have already given in this, this1 minus phi of,when they are log normal distributed,natural logarithm ofmu (( )) by mu s.1 plus V R V S square (( )), this is V S and V R. This is equation number 6.If they are log normal,be on this equation, if this is normal distributed we have this equation.

Now,FOSM method has some advantagesand disadvantages.The advantages are the following.It is easy to usebecause you should know only the mean and standard deviation variance of R and S.You can find the probability of failure directly using thesetwo expressions of 5 and 6, if they are normally distributed or log normal. So, it is easy to use.Most importantly,it does not requireknowledgeof distributionof random variables.Disadvantages areresults can causeserious errorifthe tails of the distributionfunctioncannot be approximatedby normal distribution.

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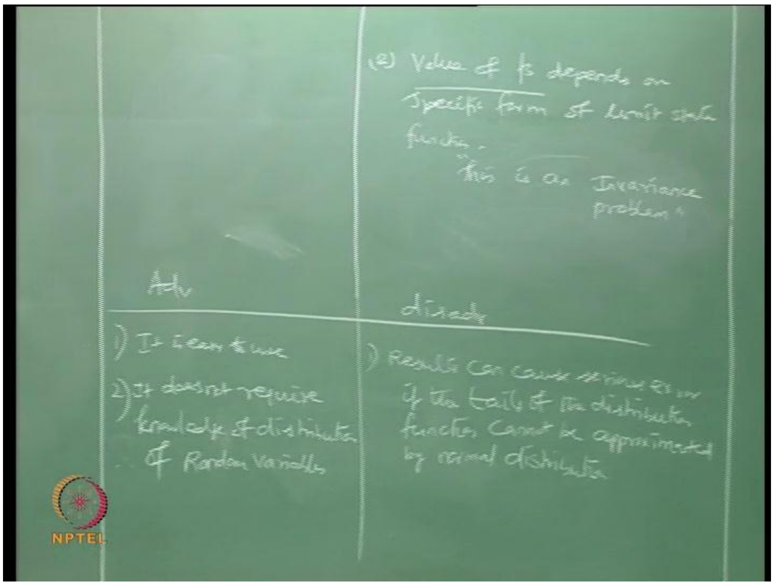

The second disadvantage could be value of reliability indexdepends onspecific formof limit state function.As in this case you have seen that you have considered only first order approximation to illustrate this expansion, sodepends upon specific form; that is very, very important.Why advanced FOSM was introduced?The value of reliability index depends only on the specific form of limit state function;that is very important, it is form dependent.This is what we call asan invariance problem.

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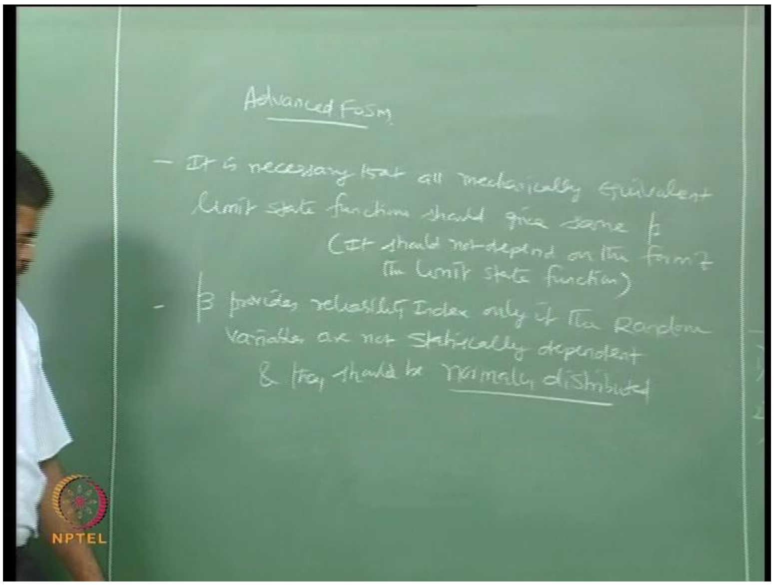

Let us talk about advancedFOSM.We all understand thatit is necessary,thatallmechanically equivalentlimit state functions shouldgivesamebeta reliability index.On the other hand,it should not depend onthe formofthe limit state function.

Now, we all know, that beta provides reliability indexonlyif the random variables are notstatistically depended,that is, if they are independent, then beta gives the reliability value andthey should benormallydistributed.That is why we said, calculating reliability index using FOSM does not require any knowledge of distribution of random variable; itis understood, that random variables arenormally distributed.

If thesetwo conditions are violated, then how to estimate reliability function or the reliability index beta,that is the question. So, that is addressed in advanced FOSM. Hassofer-Lind has given a methodor algorithmbased on which the reliability index can be computed.

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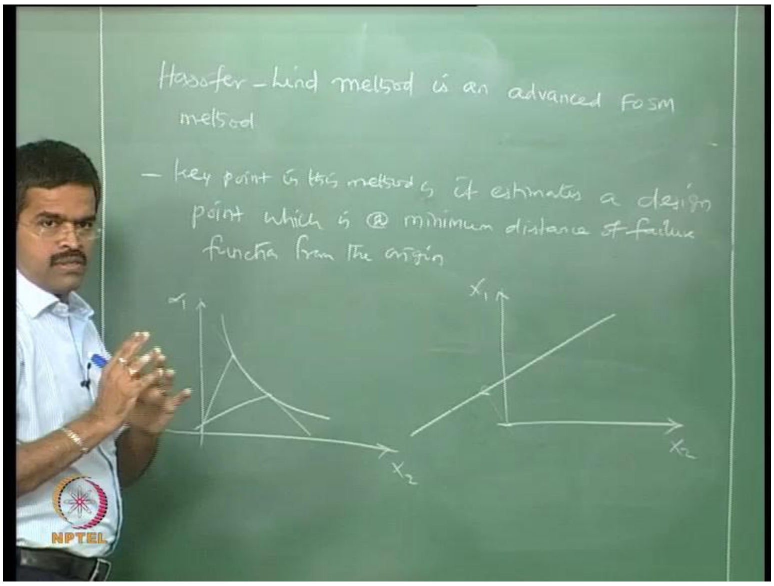

Hassofer-Lind methodis anadvancedFOSM method,which we will discuss.There are some limitations to this method also, we will also discuss them.Now, the key pointin this method is, it estimatesa design point,which isatminimum distanceoffailure functionfrom the origin.I will come to that.

Let us say,I havetwo variables,x 2 and x 1,I have a linear failure function; it is a linear failure function. To this linear failure function,I must find a point on this failure function, sothat the point remains at a minimum distance from origin.So, obviously, I have to draw a perpendicular and this becomes my point and this becomes myminimum distance,I call this as a design point.

Now, if the function is linear you can easily find this.If the function is non-linear for example, then if you take a point here and draw a tangent and say, this is my minimum point.If you take a tangent here, you will say this is my minimum point. So, beta keeps on varying, is it clear.

So, if the function is linearor non-linear between variables x 1 and x 2, how to handle this? So, the whole key issue in Hassofer-Lind method is to estimate a design point, which is located on the failure envelop or the function, which is minimum distance from the origin.Now,onemay ask why it should be a minimum distance, why not maximum?I will answer this question.

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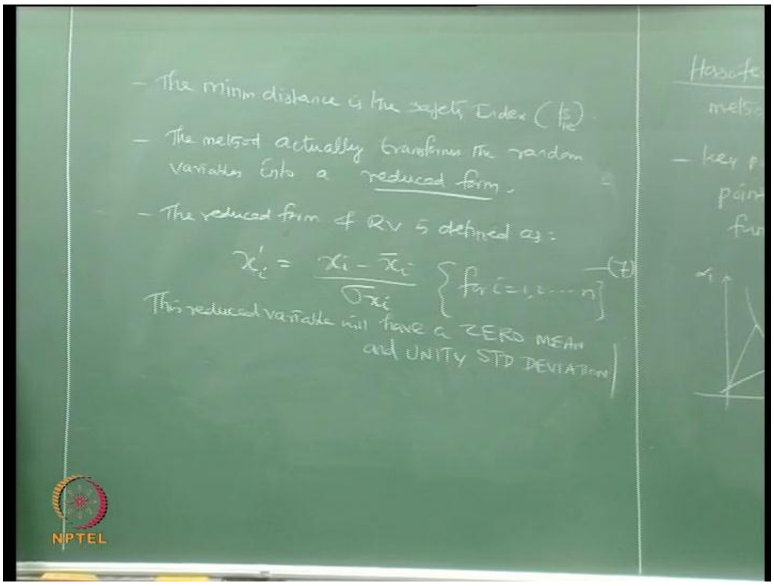

Hassofer-Lind method says,the minimum distanceisthe safety index directlyin this geometric meaning. So, the minimum distance, if it is estimated geometrically, that itself is the safetyindex or beta.To be very specific, we write beta HL,HL stands forHassofer-Lind. Now, the method is very simple,the method actually transformsthe random variablesin to areduced form.I will come to that, what is reduced form?The reduced form of random variableis definedasx dashi-th value,which can bex i minus x bar iby sigma x ifori equals,let me call this equation number, what is the number?

Student: 7sir.

7,see instead of using directly the random variable, which are R and S,I am going to use a modified, a reduced form of the random variable x dash, where x dash will be value given by this expression 7.Now, the interesting part is this reducer formor this reduced.

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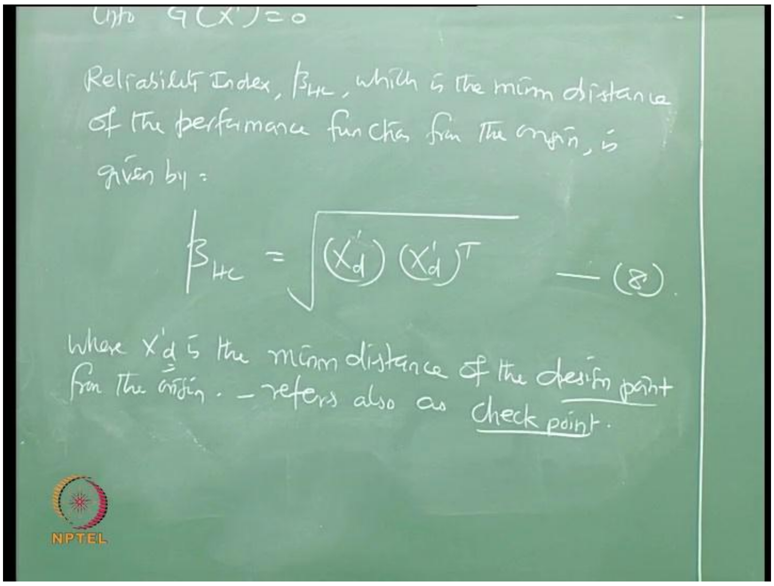

Now, the performance function G of X 0is now convertedinto G of X dashequals 0. I am now transforming the variablefrom x to x dash.Reliability index,which is beta HL,which is the minimum distanceof the performance functionfrom the origin,is given by…Where x dash dis a minimum distance of the design pointfrom the origin. So, d stands forthe design point. Some literature refers thisalso ascheck point, sodesign point or check point.

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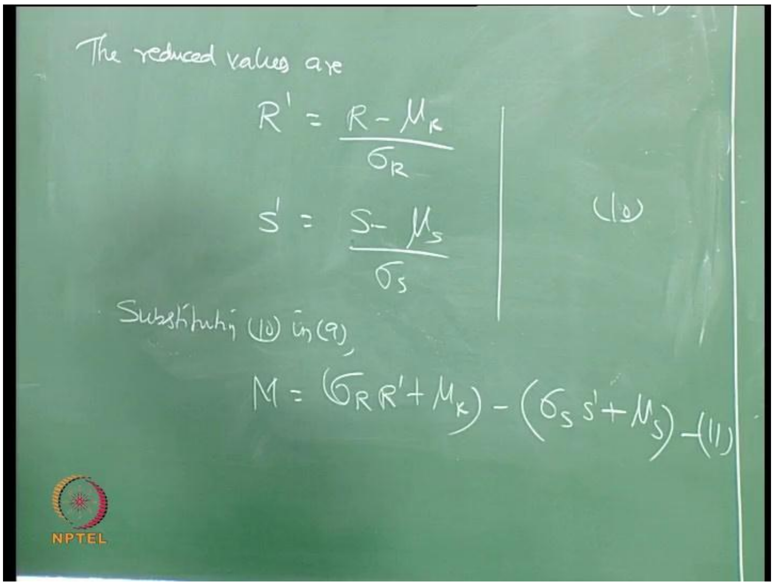

Now, let us taketwo cases.Case 1, when the limit state function is linear,we will see how to handle this. So, let us considerMas R minus S,which is equal to 0.The reduced values areR dashand S dash,which are R minus mu R by sigma R,which is S minus mu S by sigma S,equation 10. So, I must substitute R and S here I have R and S here, rearrange this term,I can rewrite M.Now, substituting 10 in 9, Mcan be rewritten assigma R R dash plus mu R minussigma S S dashplus mu S;call the equation number 11.

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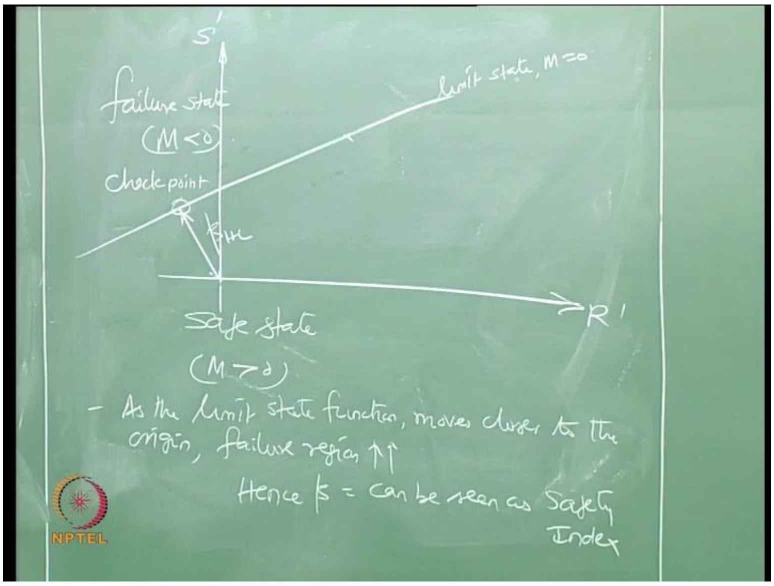

So, in limit state functionthey look like this, which can be R dashand S dash.It is a straight line, it is a straight line, R minus S,this will limit state M is equal to 0.This is mybeta HL and this becomes mycheck point and this is my failure state,where Mless than 0 and this is my safe state,whereMgreater than 0.We can see here, for all values here, for all values, for example, this point we can see, the slope is gentle, is mean, R is greater than S,safe state, whereas in this case, given to be failure state.

Now, you can see from this figure,as the limit state functionmovescloser to the origin,what will happen when this moves, line keeps on moving closed to the origin?The failure state will be kept on increasing,the failure stateor I should say, the failure regionkeeps on increasing. So, there is a level at which the failure state is optimized and that is controlled by the minimum distance from the origin.Hence,betacan be seenassafety index.That is the reason why,the minimum distance is considered as reliability index in Hassofer-Lind technique.Is this clear?

Now, let us move to the…

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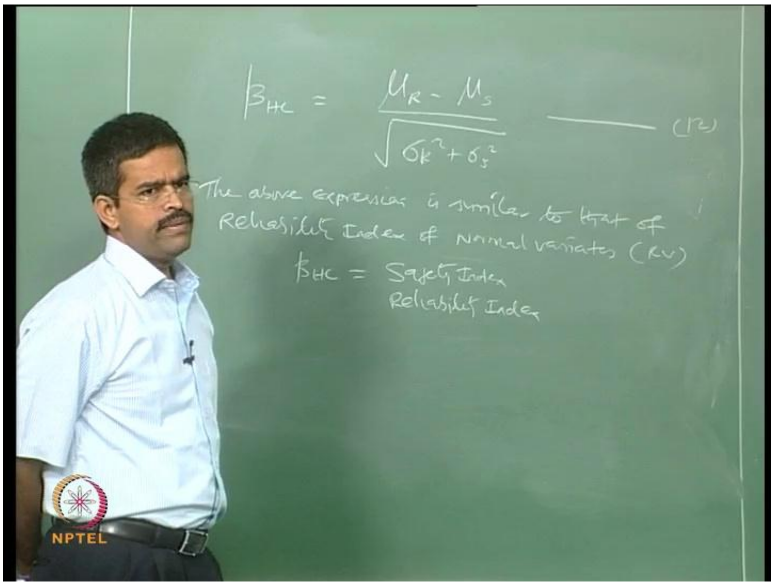

So, in my case,beta HL,beta HL can be simplymu Rminus mu S bysigma R squaresigma S square.What is the equation number? 12,you will see, that this expression is same even in the case of normal variates, is it not?Expression is same, the above expressionis similarto that ofreliability indexofnormal variates, random variates, is it not?That is why, beta HLis also calledsafety index or acceptedreliability index.Remember, beta HL equation has been derived using the geometric calculation from this figure, is not from the formula.

It is a very simple issue, that I want to find the minimum distance between the line from the origin 2-D geometry problem,you can easily find this.I have not shown the mathematical working out of finding on beta, as on geometrically I will get this equation, but this equation is fortunately same as that you have for thenormal variates.Therefore, I can call this minimum distance as reliability index; that is the definition why Hassofer-Lind index is called as a safety index.Now, when the limit state function is non-linear, now it is linear,it is a straight line,when it is non-linear what happens?

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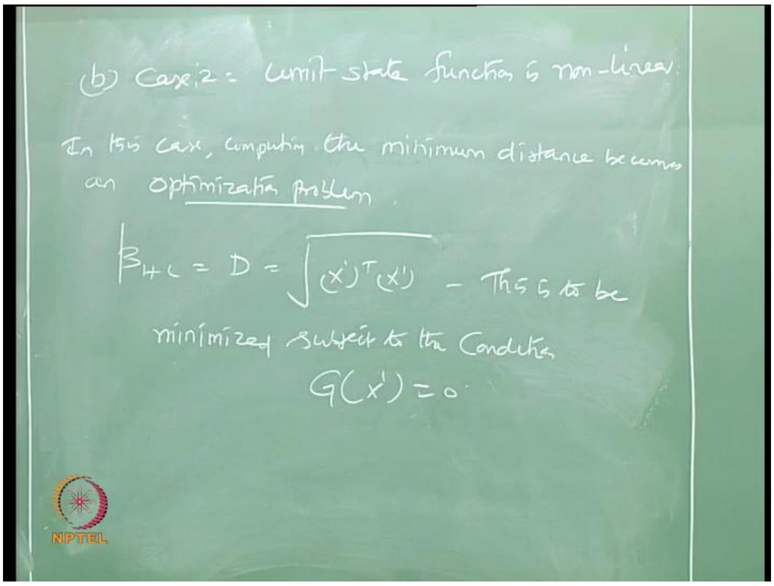

In this case,computing a minimum distancebecomesan optimization problem.So, what you are going to do is,beta HLis nothing but a distance, which is given byx dash transpose x dash,we have got to minimize this,subject to the condition G of X dash is said to 0.

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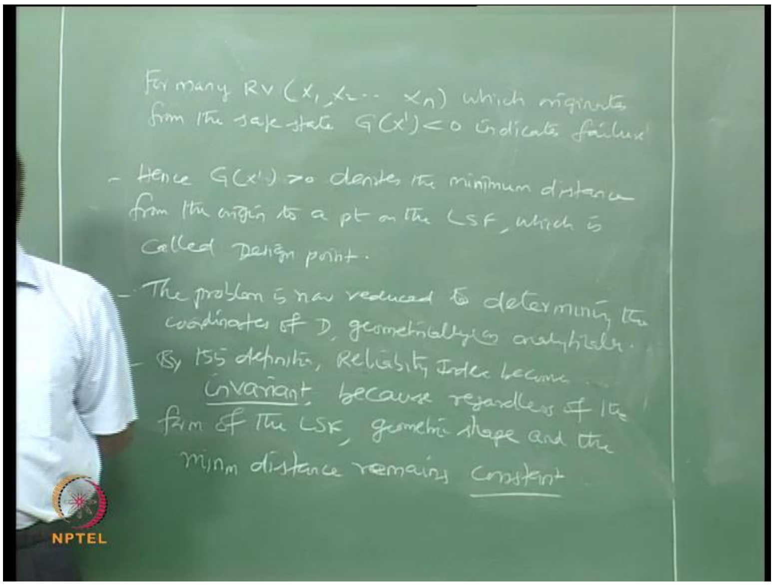

For many random variables,x 1,x 2, x n,which originatesfrom the safe state G of X dashless than 0 indicates failure.Therefore,G of x dash greater than 0 denotesthe minimum distancefrom the originto a pointon the limit state function,which is calleddesign point. So, the problem is reduced todeterminingthe coordinatesof D, design point, geometricallyoranalytically.

So, by this definition,reliability indexbecomesinvariantbecauseregardless of the form of the limit state function the geometric shape,geometric shapeandthe minimum distanceremainsconstanttheone,which was a difficulty in FOSM is now handled by Hassofer-Lind by a different technique.Because now, by obtaining the coordinates of this point D for any form of the geometric shape of the limit state function, the minimum distance will remain constant, has to remain constant.

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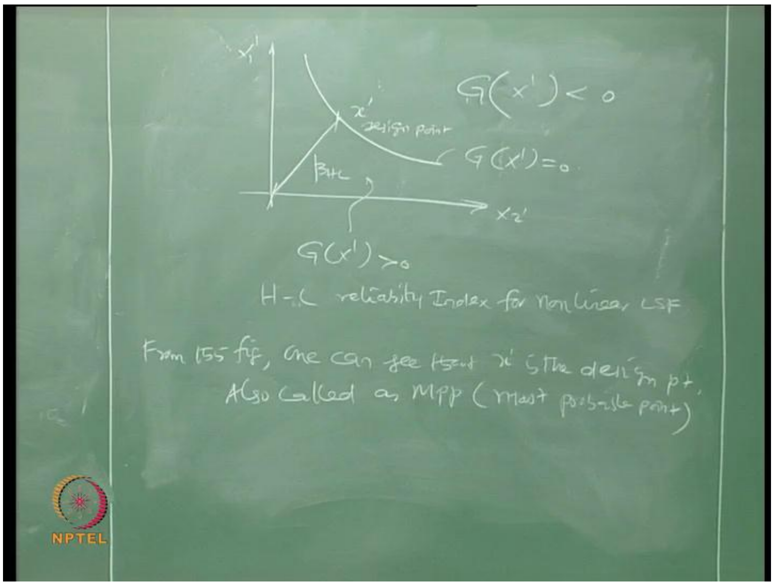

Let us say, the plotof thesetwo variables,x 2 dashand x 1 dashfor a non-linear limit state function looks like this.Let us say this is my design point,I call this point asx dash. So, this is my design pointand this is where G of X dashless than 0 and this is the distance,what I callas beta HL and this is the functionG of x dashequals 0.This is function, there is an area whereG of X dashis greater than 0. So, this isHassofer-Lind reliability indexfor non-linear limit state function. So, from this figureonecan see,thatx dashis the design point, also called asMPP, mostprobablepoint.

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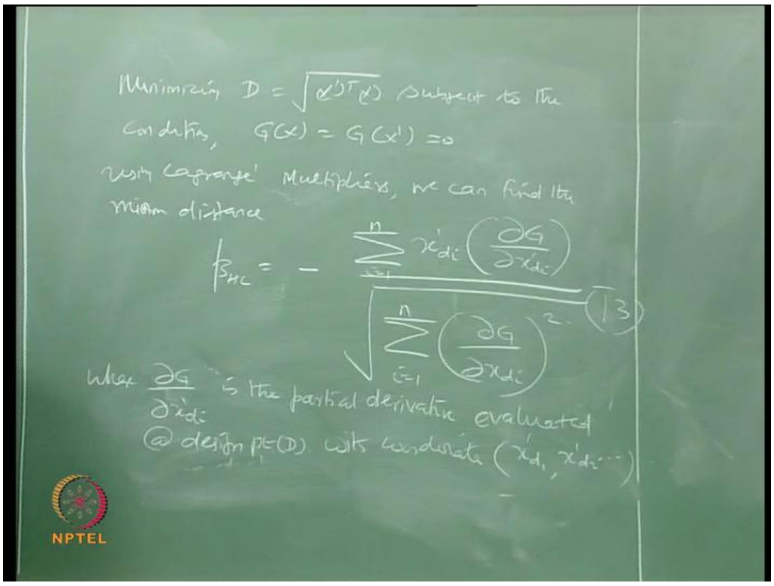

So, minimizingD, which is equal to square root ofx dash and x,subject to the condition G of X,that is,G of x dashshould, is equal to 0 because I am transforming the coordinates from x to x dash;that is what you are doing in advanced methods.UsingLagrange's multiplierswe can findthe minimum distance,beta HL,asminus ofsum ofi(( )) to n x dash di, d stands for the design point,i is a summation.Is what equation number is this?13,wheredou g by dou x dash diis the partial derivative.Why it is partial derivative?The values of x are no more independentlyevaluatedat design pointD with coordinatesx dash d 1,x dash d 2,and soon.These are the coordinates of the design point.

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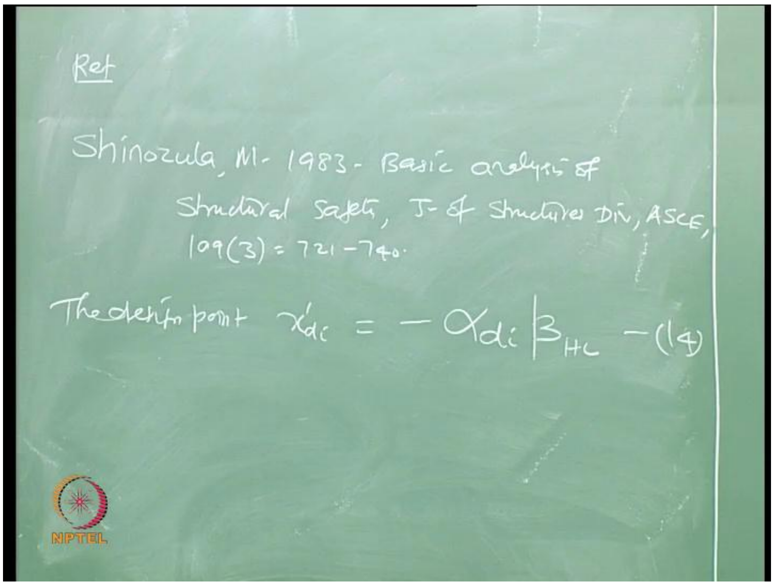

So, the study has got a very interesting reference I want to the give you the reference, Shinozula,M-1983,Basic analysisofstructural safety, journal of structures division ASCE, 109 (3), 721to 740.The design pointx dash di, because I am interested in only finding the design point, is simply given byminus alpha di beta HL;is equation number14.

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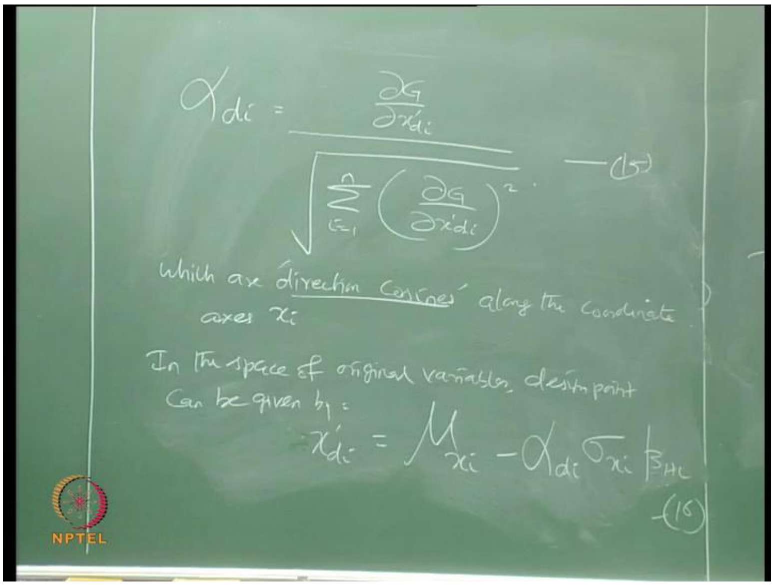

Where alpha dican bedouG by dou x di dash divided by root ofsummation 1 to ndouG bydou x dash disquare, equation number 15,which are direction cosinesalong the coordinate axisx i.So, in the space of original variable,because x dash are all reduced form of variables, in the space oforiginal variables,the design point can be given byx dash di, is mu xi minus alphadi sigma xibeta HL,that is the equation16. Ofcourse, this method has got few limitations and there are improvements suggested on this method, which we are not discussing in the scope of this lecture.

So, in this class, in this lecture we have seentwointeresting cases, where I have picked up the first order second moment method of reliability analysis and showed you how to find out the safety index or reliability index for a given standard variate, which is normally distributed.But we have seen, that limitation for FOSM is, that it depends upon the form of the limit state function.If it is linear, is given different answer; if is it non-linear, different answer.But strictly speaking, it should not then advanced method of (( )) suggested by Hassofer-Lind saying, that generalize this function in a different reduced form, which has got unit standard deviation and 0 mean process and find out the coordinates of thesocalled design point or the check point or maximum probability point on the limit surface.

So, this becomes an optimization problem, you have got to minimize the distance d with respect to the failure function, as we have discussed in the case. So, as the process goes ahead we can easily find the design point coordinate using this expression where beta HL will give.

So, this process,it is an iterative process.Initially, you have got to assume, that the reduced form of variables are all standard variable as mean, as standard deviation, find out from equation 15, or in our case, equation14. The newB HLorbeta HL or you will find the x dash di in terms of beta HL. Then substitute back in the original equation of beta HL to find out the value of beta HL, substitute again back in equation14,keep on getting new x d s. So, you will ultimately get the coordinate of the design point, which does not vary between the first approximation of the standard variatesto the actual real random variables, which are depended on each other. So, the corresponding b HL will give you the safety index of the problem.

So, there are good illustrations and examples available in the literature.Unfortunately, in this lecture I am not able to show you some examples of the workout problems.If time permits, we will see an appendix later in the end of this course, but still as a part of this particular module we are introducing reliability in this scheme of advance marine structures.We have discussed that in length.

So, I wish, that you have understoodthe fundamental concepts of reliability as applied to marine structures, what are the limitations. what are the different levels, what are uncertainties, where are they formed and how are they responsible, how are they interdependent, how do they control the whole design process, why reliability cannot be accurate, how reliability can be a design scheme, what are the limitations, what are the fields of application, what are the players involved in reliability methodologyand what is AFOSM and FOSM, that is the focus of this whole module, which we discussed.

Andif you have any questions, you can always address me to NPTEL,IITMadras,under this scheme.I hope you will go through this lecture once again and try to understand.Of course, there is on advanced course on reliability,especially handling on marine structures, which will come up in the next scheme of NPTEL.You can look for that course where we address only reliability problems and marine structures with classified site examples. So, we have finished three modules, now we havegot one more module, which we will take up in the next class onwards, which will go in to address the (( )) strength of marine structure members.

Thank you very much.