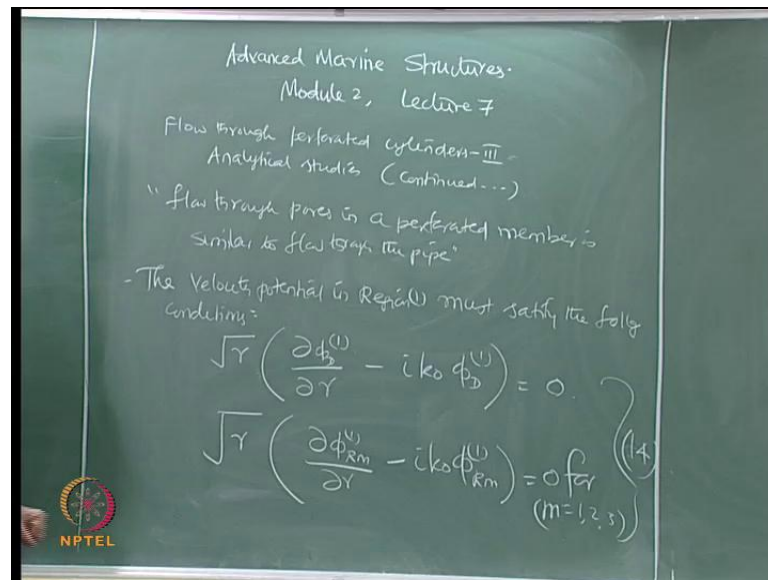


**Advanced Marine Structures**  
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**Lecture - 7**  
**Flow through Perforated Members – III**  
**Analytical studies**

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In the last lecture, we established a basic fact that flow through pores in a perforated member is similar to flow through the pipe, because there has been a close analogy, which we established between the porosity parameter and that of the Reynolds number. Considering this fact as acceptable, the velocity potential in region 1, that is, outside boundary must satisfy the following conditions. Root  $r$ ... This is the equation number... Carefully, try to understand this singular convention, the superscript brackets all stands for regions 1 to 13. I am talking about the region 1 here; so I am using 1 here. In case of the radiation potential, I have also another count  $M$ , which is standing for the number of modes, 1 to 13.

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Diffracted potential in all the regions will be given by :

$$\phi_{Dm}^{(1)} = -\frac{igH}{2\omega} \sum_{n=0}^{\infty} A_{mn} T_{mn}^b(y) \cos\{\alpha_n(z+H)\} \cos m\theta \quad (15a)$$


$$\phi_{Dm}^{(2)} = -\frac{igH}{2\omega} \sum_{n=0}^{\infty} B_{mn} S_{mn}^D(y) \cos\{\beta_n(z+z_2)\} \cos m\theta \quad (15b)$$

$$\phi_{Dm}^{(3)} = -\frac{igH}{2\omega} \sum_{n=0}^{\infty} C_{mn} R_{mn}^D(y) \cos\{\gamma_n(z+h)\} \cos m\theta \quad (15c)$$


Now, the diffraction potential in all the regions will be given by... There is similarity in between these three, so I am writing them simultaneously for you to easily remember. You can see here, this is for region 1, so I am using argument  $z$  plus  $H$ .  $H$  is the water depth. Here, I will use  $z$  plus  $z_2$ . Let us have the multipliers different here, this is  $\beta_n$ , this is  $\gamma_n$ . Again, plus  $H \cos m \theta$ ,  $\cos m \theta$ ,  $\cos m \theta$ , I call them as equations 15a, 15b and 15c. So, the arguments can easily understand, they are related to the boundaries of region 1 to 13.

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- Where  $T_{mn}^b$  is a Hankel function  $H_m(k_n y)$  of first kind and order  $m$ , for  $n=0$
- for  $n \geq 1$ , it is given by modified Bessel function  $K_m(k_n y)$  of second kind and order  $m$
- wave number,  $k_n$  is the  $n$ th real root of the Eqn.  $\omega^2 + g k_n \tan(k_n d) = 0$  — (16)



In this argument, Hankel's function  $T_{mn}^D$  of  $D$ , which is  $H_m$  with an argument of  $k_0 r$ , it is a Hankel function. This is of first kind and order  $m$  for  $n$  equals 0; all the superscripts  $D$  stands for diffraction. Now, for  $n$  greater than equal to 1, then this is given by a modified Bessel's function, which is, say,  $k_m, k_n r$ , which is of the second kind and order  $m$ .

Now, in this argument, in both the cases you need to find the wave number is the positive real roots of the equation, call this equation number 16. So, solve this equation, pick up only the positive real roots. And, that will give you the values of  $k_n$ , which have to use in the Hankel's function, in the Bessel's function for evaluating them. No more argument of  $S_{mn}$  comes; we have discussed  $T_{mn}$ -s.

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$$S_{mn}^D = \gamma^m \quad (f_{n=0})$$

$$S_{mn}^D = I_m(\beta_n \gamma) \quad (f_{n \geq 1}) \quad (17)$$

$$R_{mn}^D = \gamma^m \quad (f_{n=0})$$

$$R_{mn}^D = I_m(\beta_n \gamma) \quad (f_{n \geq 1}) \quad (18)$$

$S_{mn}$  of  $D$  for  $n$  is equal to 0 and  $n$  greater than 1 is simply equal to  $r^m$ . This is  $r$  and this is  $I_m$  of  $\beta_n r$ ; you can say equation number 17. Similarly,  $R_{mn}$  of  $D$  for  $n$  equal to 0, for  $n$  greater than equal to 1, are given by  $R_m$  of  $\beta_n r$  of  $r$ .

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Where  $I_m$  is the modified Bessel function of first kind of order  $m$ .

Further,

$$\left. \begin{aligned} \alpha_0 &= i k_0 \\ \alpha_n &= k_n \quad \text{for } n=1 \\ \beta_n &= \frac{n\pi}{z^2-1} \quad \text{for } n > 0 \\ \gamma_n &= \frac{n\pi}{(h-b)} \quad \text{for } n > 0 \end{aligned} \right\} (19)$$

Where,  $I_m$  is the modified Bessel function, of first kind, of order  $m$ . Further, we also need the  $\alpha$ . In this case,  $\alpha$ ,  $\beta$  and  $\gamma$ , these are all given.  $\alpha_0$  is  $i k_0$ ;  $\alpha_n$  is  $k_n$  for  $n$  equal to 1;  $\beta_n$ ,  $n\pi$  by  $z^2$  minus  $z$  1. You can see, this is the region we are operating, the porous one, is it not. And  $\gamma_n$ , is it not, this is region for region 3,  $n h$  minus  $b$  that is the region 3. These are all for  $n$  greater than 0. I put all of them as equation number 19. Now, we have seen the diffraction potential on difference regions 1, 2 and 3.

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Radiation potential in all the 3 regions.

$$\left. \begin{aligned} \phi_{KM}^{(1)} &= -\frac{igH}{2\omega} \sum_{n=0}^{\infty} A_n^R T_{mn}^R(y) \cos\{\alpha_n(z+h)\} \cos m\theta \\ \phi_{KM}^{(2)} &= -\frac{igH}{2\omega} \sum_{n=0}^{\infty} B_n^K S_{mn}^R(y) \cos\{\beta_n(z+z_2)\} \\ &\quad + f_{2m}(y,z) \cos m\theta \\ \phi_{KM}^{(3)} &= -\frac{igH}{2\omega} \sum_{n=0}^{\infty} C_n^K R_{mn}^R(y) \cos\{\gamma_n(z+h)\} \\ &\quad + f_{3m}(y,z) \cos m\theta \end{aligned} \right\} (20)$$

Now, we will talk about radiation potential in all the three regions. We have had a similar expression,  $\pi R_m^1, \pi R_m^2, \pi R_m^3$ .  $R$  stands for radiation and  $m$  stands for the specific mode and the superscript 1, 2 and 3 stands for regions 1, 2 and 3. This is  $2\omega$ , it is not  $z$ . I think you agree, this, it is  $2\omega$  not  $z$ , it may appear to be  $z$ . Please correct if you have written as  $z$ ,  $z$  is this, these all are limits of the boundaries. In my writing it may look similar, please be careful, this is  $2\omega$  summation, similarly  $A_m, B_m, C_m, T_m$ .

I call them as equation number 20. Let me check this equation once again. This is A... Now, you can see these equations, these equations are more dependent. You can see here, these equations are more dependent.

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These eqn are mode-dependent which satisfies the BC of solid and perforated region.

The fn  $f_{2m}(r, z) = 0$  for  $m = 1, 2, 3$ .

$f_{3m}(r, z) = 0$  for  $m = 1$

(a)  $f_{31}(r, z) = 0$

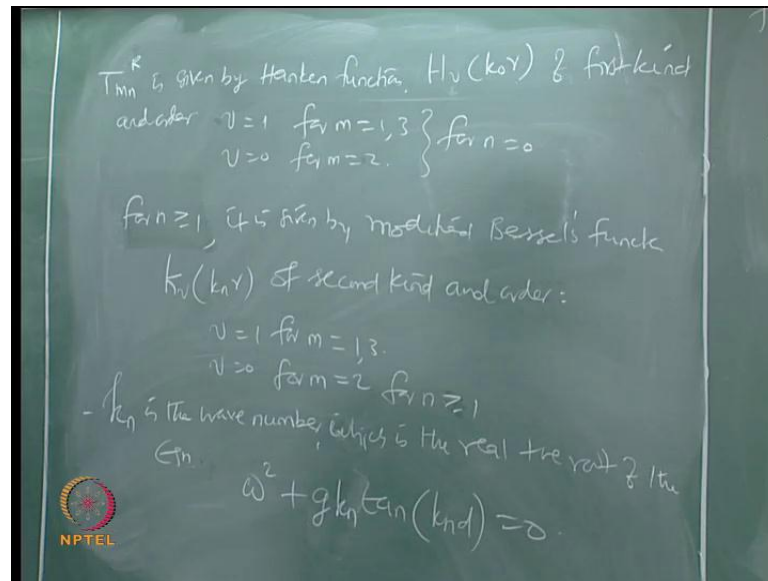
$f_{32} = \frac{(z+h)^2}{2(h-b)} - \frac{r^2}{2(h-b)}$  for  $m=2$

$f_{33} = -\frac{r(z+h)^2}{2(h-b)} - \frac{r^3}{8(h-b)}$  for  $m=3$ .

(21)

So, these equations are more dependent, which satisfy the boundary conditions of solid and perforated regions. Now, the function  $f_{2m}(r, z)$  is 0 for all the  $(( ))$ . So, this has been written for you to complete it in the form of the equation, whereas  $f_{3m}(r, z)$  is 0 for  $m$  equals 1, that is,  $f_{31}$  of  $r$   $z$  0,  $f_{32}$  and  $f_{33}$ , it is  $h$  minus  $b$ , we are talking about the second region, this is for  $m$  is equal to 2 and for  $m$  is equal to 3. Now, this multiplier here,  $r$   $z$  plus  $h$  whole square by  $h$  minus  $b$  minus  $r$  cube by 8 of  $h$  minus  $b$  for  $m$  is equal to 3, I call all these equations as equation 21. Let me check these equations.

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So, in the equation earlier, that is in equation 20 you will see  $T_{mn}$  of  $R$  is given by the Hankel function,  $H_v(k_0 r)$  of first kind and order  $v$  equal to 1. This is the order equals 1 for  $m$  equals 1 and 3;  $v$  equals 0 for  $m$  equals 2 and for  $n$  equals 0. Let us write it here, for  $n$  equals 0. It is given by the modified Bessel's function, which is  $k_v$  of  $k_n$  of  $r$  that is the argument of the Bessel's function. This is of the second kind and order  $v$  equals 1 for  $m$  equals 1 and 3,  $v$  equals 0 for  $m$  equals 2, for  $n$  equal to 1.

This is small correction; this value is for  $n$  equals 0. For  $n$  greater than equal to 1, this is given by the modified Bessel's function. So, as usual we know, that  $k_m$  in the argument, here is the wave number, which is the real positive root of the equation. Can you give me the equation number, which is similar to the diffraction potential solution for the equation number?  $k_n$  is the root of this specific equation, I said. I write down the equation here,  $g k_n \tan(k_n d) = 0$ .

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$$\begin{aligned}
 S_{mn}^R &= \gamma^m \quad \text{for } n=0 \\
 S_{mn}^R &= I_m(\beta_n \gamma) \quad \text{for } n \geq 1 \\
 R_{mn}^R &= \gamma^m \quad \text{for } n=0 \\
 R_{mn}^R &= I_m(\beta_n \gamma) \quad \text{for } n \geq 1
 \end{aligned} \quad (22)$$

Let us see the arguments,  $S_{mn}$  and  $R_{mn}$ , simply, for  $n$  equals 0. I call this as equation number, what is the number?

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where  $I_m$  is the modified Bessel function of first kind and order  $m$ .

$$\begin{aligned}
 \alpha_0 &= i k_0 \\
 \alpha_n &= k_n \quad \text{for } n=1 \\
 \beta_n &= \frac{n\pi}{(z_2 - z_1)} \\
 \gamma_n &= \frac{n\pi}{(a - b)}
 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \text{for } n \geq 0 \quad (23)$$

Where  $I_m$  as the modified Bessel function of first kind and order  $m$ , let us just call  $I_m$ . There are some alphas and betas required in your arguments.  $i k$  naught, is  $\alpha_n$ , sorry, there is no  $m$  here, these are simple numbers, that is, the region of the boundary, that is, the region of this boundary. I call this equation number 23.



So, we have set of equations for understanding the diffraction potentials in the region 1 to 13 and radiation potentials region 1 to 13. The diffraction potentials are more dependent and you should be able to evaluate them for different modes of  $m$  and different regions 1 to 13, equation, using equations 20, 21, 22 and 23 respectively. And the coefficients of  $a_n$ ,  $b_n$  and  $c_n$  of  $D$  and  $r$  are unknowns because we do not know these coefficients, remaining all in the equation of set of 20 and set of 17, which is diffraction and radiation potentials on different regions are explicitly explained except, that I would like to now know the coefficients, which are called unknown coefficients on this equation, which is  $a$ ,  $b$  and  $c$  for diffraction and for radiation separately.

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The Unknown Coeffts

$\{A_{mn}^D, B_{mn}^D, C_{mn}^D\}$  and  $\{A_{mn}^R, B_{mn}^R, C_{mn}^R\}$

$\phi_D^{(1)} = \phi_D^{(3)}$  on  $r=a, -h \leq z \leq -b$  - (24)

$\frac{\partial \phi_D^{(1)}}{\partial r} = \frac{\partial \phi_D^{(3)}}{\partial r}$  on  $r=a, -h \leq z \leq -b$  - (25)

$\phi_{Rm}^{(1)} = \phi_{Rm}^{(3)}$  on  $r=a, -h \leq z \leq -b$  - (26)

$\frac{\partial \phi_{Rm}^{(1)}}{\partial r} = \frac{\partial \phi_{Rm}^{(3)}}{\partial r}$  on  $r=a, -h \leq z \leq -b$  - (27)

all are valid for  $m=1,2,3$

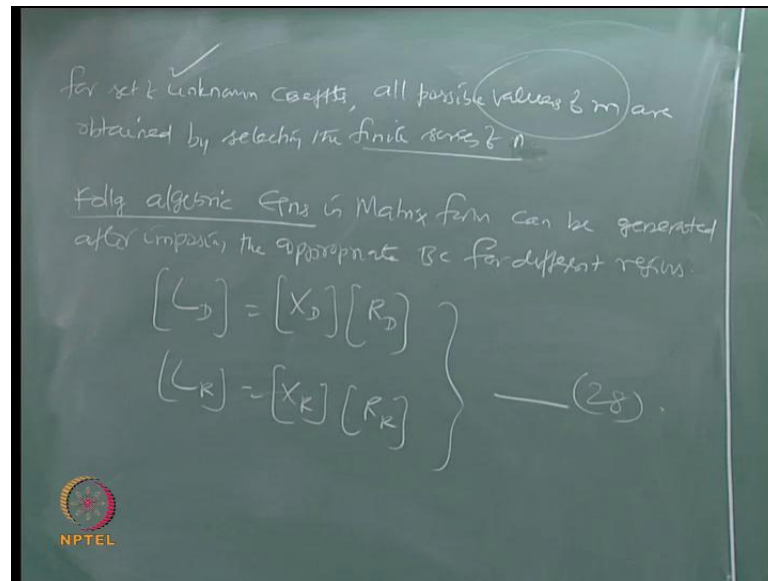
Now, let us see the unknown coefficients, what are these unknown coefficients?  $A_{mn}^D, B_{mn}^D, C_{mn}^D$ , these are diffraction potential coefficients and these are radiation potential coefficients. So, let us say  $\phi$  of  $D$  is same as  $\phi$  of region 3. These are all running for regions  $D$ , stands for diffraction potentials. How come we say 1 and 3 are same? 1 is outer region and 3 is a region below the perforated region. On a specific application,  $r$  equals  $a$  and  $z$  varies minus  $h$  and minus  $b$ ; equation number 23.

Student: 24 sir.

So, now,  $r$  equals  $a$ , and the variation is also true, it is an open domain. Again, similarly,  $\phi_{Rm}^{(1)}$  is  $\phi_{Rm}^{(3)}$ , that is the radiation potential on  $m$ -th modes for region 1 and 3  $r$  equals  $a$ . All these expressions are valid for  $m$  equals 1 comma 2 comma 3.



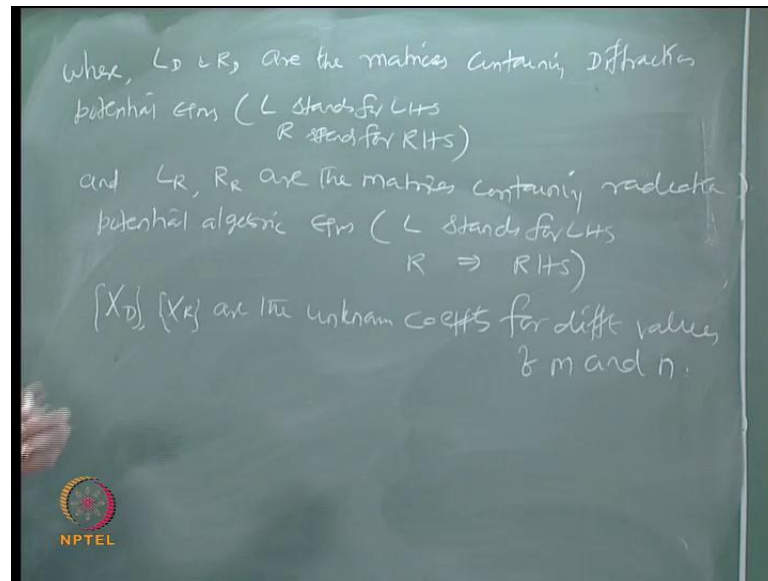
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Now, how will you solve them is, for set of the unknown coefficients, for the set of unknown coefficients. All possible values of  $m$  are obtained by selecting the finite series of  $n$  because you have to select the specific series of  $n$  and try to find out all possible values of these coefficients for different modes: 1, 2 and 3 because the velocity potential in these regions are infinite. So, in that region, for a finite series of  $n$ , for all modes you try to estimate these unknown coefficients. That is how the solution is arrived at. So, the following algebraic equations can be generated in matrix form, can be generated after imposing the appropriate boundary conditions for different regions.

You may understand and agree, that  $a$ ,  $b$  and  $c$  are not crossing the boundaries. If you look at the equation 17 and 20 you will see, that they are applicable only for specific regions of 1, 2 and 3 respectively in diffraction and 1, 2 and 3 respectively in radiation. So, you must impose appropriate boundary condition, which we have already given in equation 2 and 3. What are the boundary conditions for the region 1, region 2 and region 3? Impose them and read them together, apply them together with 17 and 20 sets of the equations and try to get the following algebraic equation, which I am writing here in a matrix form. I call this equation number 28.

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Where...L D and R Dare the matrices containing diffraction potential equations. L stands for the left hand side and R stands for the right hand side of the equation, and L R and R R are the matrices containing radiation potential algebraic equations. Similarly, L stands for the left hand side of the equation and R stands for a right hand side of the equation.

X D and X R are the unknown coefficients for different values of m and n because you have got finite series of n, this equation and possible values of modes in this problem. So, you will get different set of values on diffraction and radiation separate. So, one will be now able to find out after solving this algebraic equations, is nothing but L D inverse of R D and L R inverse of R R. In a L D and L R, R D and R R will be the series of terms, which you get from equation 17 and 20 of substituting their respective boundary conditions and you will get the vector X D and X R solve automatically for different m's and n's the series. So, now you will be able to get to define region of 1, 2 and 3, the diffraction potential and radiation potential in all the three regions independently, 1, 2 and 3 once you know this.

We can apply this on any porous structure and find out because now you know, the velocity potential can easily find out the forces generated by this velocity potential in this respective region 1, 2 and 3 and accordingly summed them in the respective degrees of freedom 1, 2 and 3, which are surge sway, surge even pitch degrees of freedom and do a conventional dynamic analysis and find out the responses. This was attempted in the

previous lecture through a numerical example using star c c m plus and an experimental investigation, which is done on a scaled model of 1 is to 140. So, obviously, you will not, to be able to validate all the three results on a given example problem because these all are different types of approaches. So, obviously, the results may not get validated, that an attempt was made to show you in the last lecture, that how closely the experimental studies and the numerical module are getting validated with a less error of above 5 to 7 percent or 10 percent error.

So, the idea of discussing this particular in this series of lectures and advanced marine structures is, that you have understood, that perforated cover on a given member reduces the forces on different active degrees of freedom that is number one. Number two, it is one of the important and advantageous technique suggested by various researchers of reducing or controlling or limiting the secondary vibrations, caused by vertex (( )) frequencies, is got two advantages. The greatest advantage which one can foresee is, they really want to retrofit a member in a structure in a high stress concentration zones, especially or in particular, near the (( )) because that is the area the corrosion is maximum. So, the material degradation can happen to the maximum there because of differential aeration.

So, one can cover, remember using an outer perforated layer and we can reduce the stresses on the members in addition to reducing the secondary vibrations caused by the vertex induced vibrations of vertex shading frequencies on a fluid flow. So, this ends the lectures on series of lectures on module 2 where we spoke or discussed flow induced vibrations, flow field disturbances caused by members kept in a flow field.

So, the whole idea was in the vision of discussing the force reduction or the structural response factors or parameters. The idea of discussing vertex induced vibration or flow induced structural interaction in terms of hydrodynamic respective is not elaborately covered in these lectures. You are supposed to look into a parallel course where they talk about hydrodynamic aspects of flow induced vibrations. The perspective here was to explain in (( )) point of view what could be the idea of flow induced vibrations in marine structures.

So, this completes module 1 and 2 in total, which has explained you mostly different kinds of problems associated with marine structures and some of the analytical,

experimental and numerical difficulties of programs or problems or examples, which has been discussed in 1 and 2. So, we will move on to the third module later where we will talk about the reliability studies applicable to marine structures.

Thank you.