Advanced Marine structures
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Lecture - 29
Plastic capacity of sections
under combined loads – II

So,in this lecture,we willdiscuss about the plastic capacity of section estimates based on the combined loading. In the last lecture we understood that what would be the necessity of understanding the compact limit, in a given section howto estimate the compact limit or howto establish whether the section is compacted or not.And we have understood that the section is established compact limit, then we need not have to consider the buckling effect in the analysis,onecan straight away find the failure by yielding.Otherwise, the failure loads at buckling is much lower thanthat of yielding.Therefore all yourfailure phenomena whichis applied for plastic analysis will not hold good the section is slender or the section buckles.

Sobuckling effect can be ignored, provided the section is compact and you can choose a section such that the compact limit is established.It means b by t ratio, d by t ratio, b 1 by t f, b 1 by t w can all be chosen in such a manner or fabricated in such a manner that the section remains stiff or compact, sothat the buckling effect is not preluding the yielding effect in the material.Now before we move on to estimating the plastic capacity under the combined action of bending and shear, bending and axial,etcetera,let us quickly look at the summary for our interested designerwhat would be the equation which are readily available in the literature, let us look at this.

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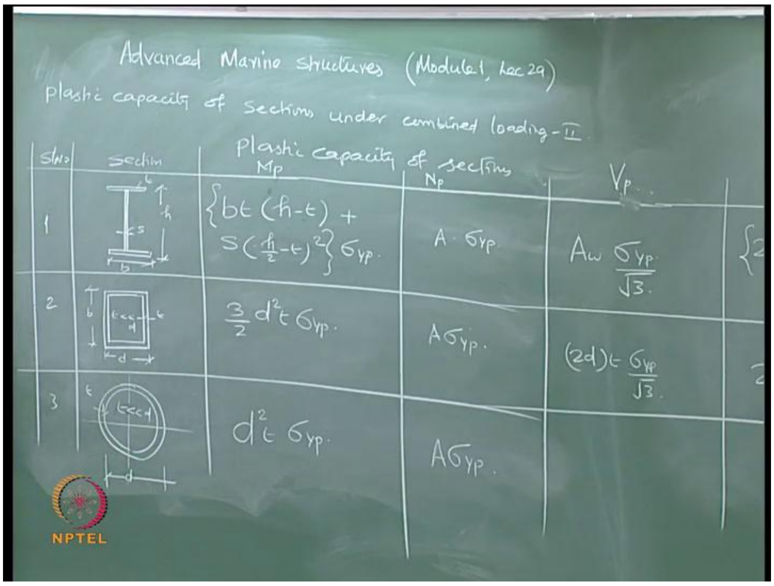

So, let us look into only aboutthree types of sections - I section and box section.All are thin walled sections,even I am also thin-walled,Iam not drawing it here.Let me draw that circle and tubes. The standard dimensions are marked hereand I call this thickness as s and of course this thickness as t. For this section let us say this is my band this is my d and this is tand of course we know t is very less thand,it is a thin-walled section.For this section this is the diameter dand of course thickness of the section is t and we also know that t is much lower than d.So, we are looking for the plastic moment carrying capacity M p,we will also look for the axial load carrying capacity N p,we will also look forthe shear capacity V pand we also look for a torsion capacity T p. It is asummary for our understanding.

So, this isbth minus t pluss intoh by 2 minus t the whole square of sigma Y p will give me my M p.This is Ainto sigma Y p. This is area of the web, this is V p,Iam writing it here. This is shear capacity V p Aw intosigma Y p.Of course for shear, we are applying thevon Mises failure theory, so it isby root3 and for T P it is 2bt square.Shear and torsion areanyway connected to each other; we already explainedthat in the last lecture.And for this box sectionssigma Y p, this is Asigma Y p, this is 2 dt sigma Y p by root 3. This is 2d square t sigma Y p by root3. As for the tube sections are considered,it is d square t sigma Yp.

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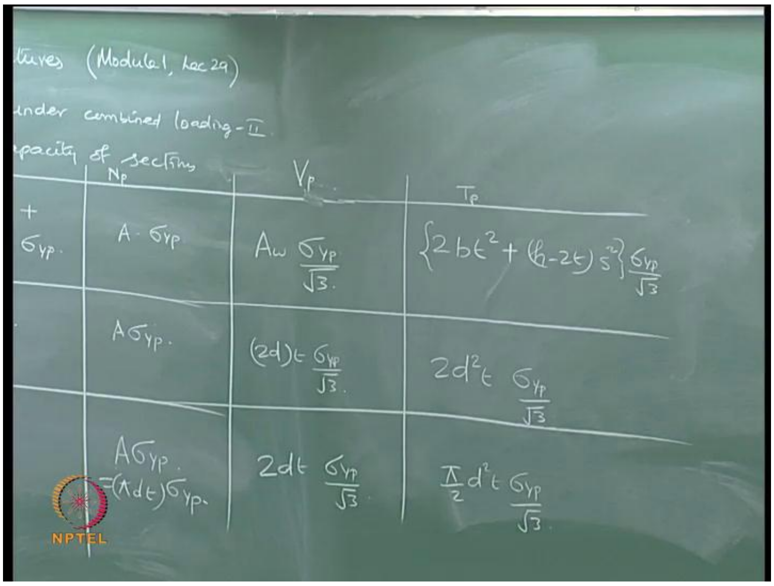

This is A sigma Y p where A is area cross-section. In my case,area of cross-section is actually equal to pid into t,it is not pi d square by 4. The circumferential thicknesswrite it as pi d into t into sigma Y p. So, 2 dtand pi by 2, just now we saw this derivation. So, this gives a comprehensive comparison of most important commonly employed cross-sections for members in marine structures where I have a table which gives me M p, N p,V p, T p,independently.Now, what I am interested is when the bending moment and shear force or bendingmoment axial force are acting together, whatdo they influence on the plasticmoment carrying capacity?When they are acting independently, this is what the table is which we discussed sofar.We will now discuss what is the combined actionof this? Is it clear, can I erase this?

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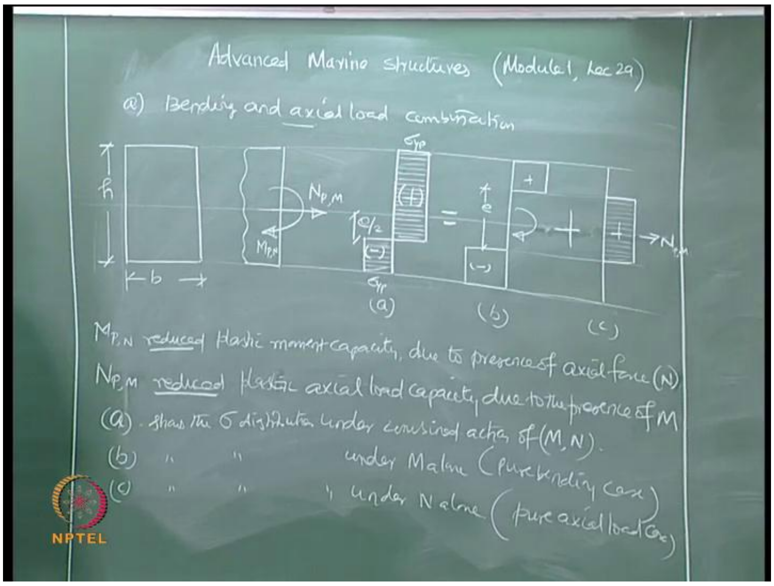

Let us look atbending and axial loadtogether. So, we will take up a rectangular section, we will also take up a I section later, first let us understand this.Let us say the section is having breath as band depth as h.The section is subjected toan N moment which is M p comma N, I will come to what is Mp comma N, also subjected to an axial forcewhich is N pcomma m,Iwill come to that whatit is. NowM p comma Nis the reduced plasticmoment capacity due to the presence ofaxial force N p,we should say Nnot N p and N P comma M is the reduced, the important concept is reduced here, plastic axial load capacitydue to the presence of M. We are looking at that. The stress distribution looks like thisand the combined action it looks like this,sigma Y p,sigma Y p and let me call this distancease by 2.Of course, this is positiveand this is negative.

This is awhich showsthe stress distribution underthe combined actionof M and N.I split this into twoparts, I say this is equal totwo parts.One is because of M alone, and other is because of N alone.I say this isnegative and positive and of course this remains as E by 2 and of course now this becomes E.bis due to bending aloneplus this is positivewhich is N alone,actually this is N P here. So, b indicates the stress distributionunderM alone and c indicates stress distribution underN alone. I am looking for the combined action which is a, but I will derive this in part and parcelof b and c.So, I could call this as pure axial caseand I could call this as pure bending case.M alone means pure bending case.

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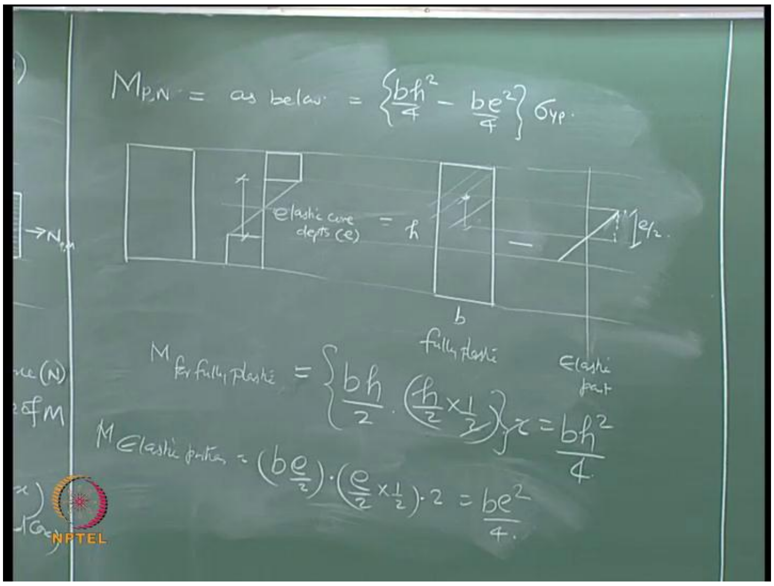

Now I am interested in finding out the pure bending casewhich I call as M p comma N which is this, the pure bending caseis given by a standard equation which we already know because this becomes a depth of elastic core, itcan be computed as below.Though we have done it, quickly we will repeat this.Let us have a rectangular sectionwhich has a distribution like this. We call this as elastic corewe already know this.Now I construct this astwo parts,one is fully plastic which is b and hminusthe elastic part.This is fully plasticminus the elastic part which can be done asM for fully plasticwhich is not true but still is given by this is b and this is h, so bh intoh by 2.

We should sayput it like this,bh by 2 intoh by 2 of half,h by 2 sigmain the centreand half of the rest cg of that particular portion and we have twice of this,the two parts. So, that gives mebh square by 4. Is it not, whereas for the elastic portion,the elastic portion is I can superimpose this here and we already know this is e by 2.We can write this as b into e by 2 into half of thatinto two parts of that, which will give me be square by 4.Therefore, now I can saythis nothing but for this notationwhich can be bh square by 4 minus be square by 4 of sigma yp,we can remove this.

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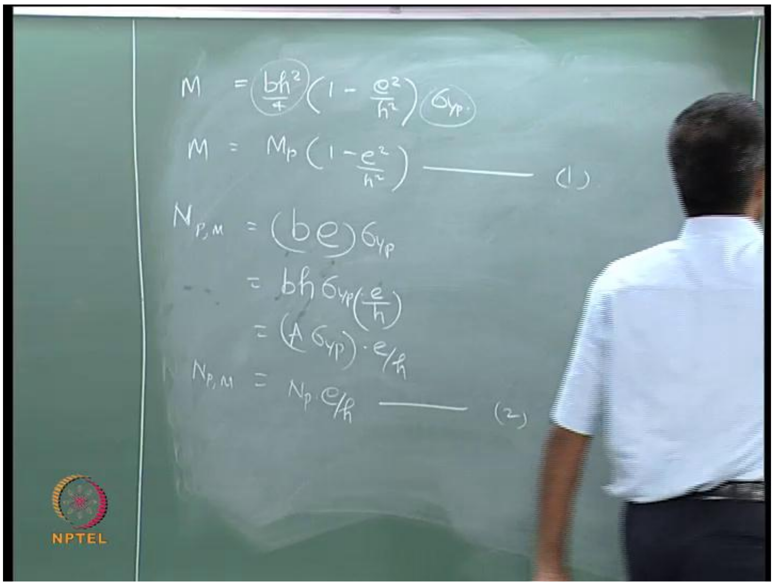

b h square by 4 1 minuse square byh square,this is Mwhich we can call this as into sigma y of course which iscan call as M p 1 minus e square by h square. So, standard relationship because this multiply by this is nothing but your M p,equation number one. N pMwhich is pure accelerate force, in that case for this rectangular section could be simply b into e into sigma Y p.If you look at the drawing or the figure one which we made earlier,the elastic part will have a depth of v by 2 and e by 2 which is e, the breath is b and sigma Y p;which I can rewrite this asb h sigma Y p into e by h,we can write like this, b into h is a whole area of the section. So, I can say this asa into sigma Y pof e by h, a into sigma y pis N pinto e by h.This is N p,call this as equation number two. Now combineone and two.

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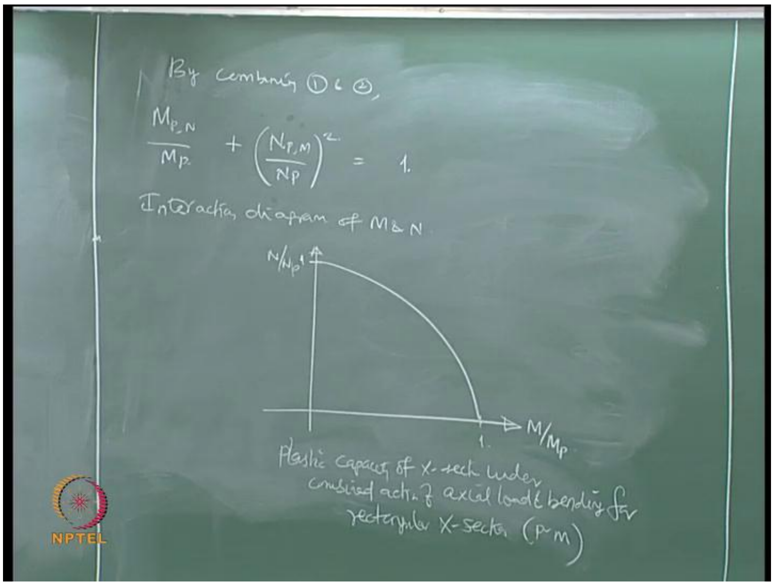

Simply by combining one and two,we are now studying the effect of bending and axial force together, plastic capacity. So, this is M p N, that is what we are addressing. So, I should say M p N by M p which will give me 1 minus e square by h squareplus N pMby N pthe whole squarewill give me e square by h square. When I add these two, I will get 1.This is called p-M interaction diagram; this is called axial force and bending.You can try to plot this, it will look like this. If I try to plotM by M pand N by N pand if this value is 1, this value is 1,look like this. So, I should say plasticcapacityof the cross-section undercombinedactionof axial loadand bending for a rectangular cross-sectionwhich we call as famously p-M interaction diagram. Here pdoes not stand for plastic, this p means axial force in M S bending;bending moment in equationis here.Having said this,let us extend this concept to an Isection.

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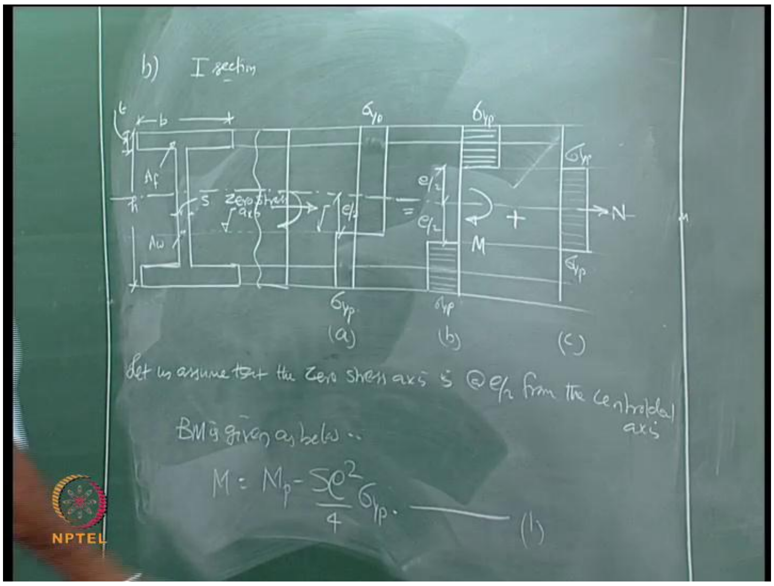

Same way we can do I section because I section is nothing but sum of rectangles of members, is it not. We have seen for one rectangle, we are now going to see sum of rectangles.Let us do for I section.I call this ass and this of course as tand this as band this as h.We call this asarea of the web and I call this as area of the flange.A f stands for area of the flange and Aw stands for the area of the web.Let us say I have astress distribution,this is subjected tobending and axial force. So, the stress distribution goes like this.By the way what is this axis called where I am marking the stress 0, what is this axis called? Zero stress axis or equal area axis. Let us say this is sigma Y p, this is alsosigma Y p and centroid of the section is somewhere hereand this distance is e by 2 as you had earlier same manner.

So, I would say nowthis section is equal to where, this is also equal to e by 2, thisis also equal to e by 2 subjected tomoment onlyplus wherever these two values are there, I have an additional section on this stress distribution because these are all sigma y pand become like this, so sigma Y p here.So, already this are same as meaning as a, b, c,a is the combined action, b is the bending alone and N is the axial load alone. So, this is what we callas zero stress axis. So, this is zero here.Let us assumethatthe zero stress axisis at e by 2 from the centroidal axisas shown in the figure. Bending moment of the sectionis given as below,M is going to be M pminus, same equation I am using which we did for a rectangle,there we said it is be square by 4. Now I will say Se square by 4 because b in this case is S of sigma y p is equation one.

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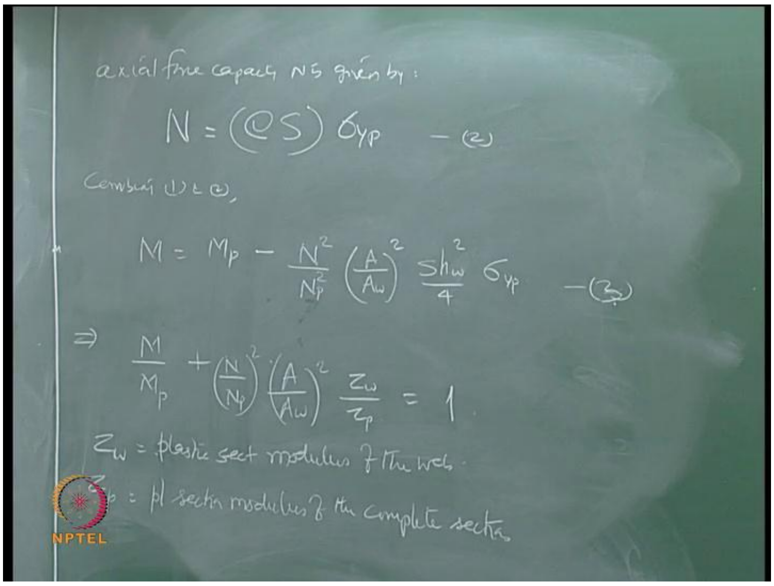

Axial force Nis given bywhat will be the value of N? Axial force N has to come only from the webandthat is the N part.N part is not acting on the flanges,N is only purely on the web. Sowhat would be the value, area of the web into sigma Y. So, I should sayes into sigma Y p,e is this valueactually,e by 2 e by 2 and S is the thickness of that portion, soequation two.Already we have an expression for N pwhat we derived in the last application of a rectangle. So, I am going to rewrite using these two equations one and two. Socombining one and two, I will slightly modify this, see how we are doing it. Thisis how the standard form given in the literature. So, we are doing it like this.

M is given as M p minus, so I have es sigma Y p in terms of n here. I am going to substitute that here because I have es sigma Y p here. Substitute that here and do somemathematical manipulation. So, after doing thatI write this as N squareby N psquare. N palready we have,it is nothing but a into sigma Y p. So, A by Aweb alone,this is for the entire sectioninto Sh omega square by 4 of sigma Y p.You can substitute back for N pseparately and you will see automatically you will land up in 1 minus 2 or 1 and 2 combining you will get the same equation of 1 and 2 here.This is how it is being expressed in the literature.So, I am giving exactly the same equation here.

So, simplifying further e can say M by M p,I can call this equation number three.Two is here. So, rewriting threeM by M p plusN by N psquareof A by AwsquareZ w by Z p, Z is the section modulus of the web alone and Z p is the section modulus of the entire sectionis given as one,where Z w is the plastic section modulusof the weband Z p is the plastic section modulusof the complete section. I will remove this; I would like to retain this. Now I can remove the figure,no problem.

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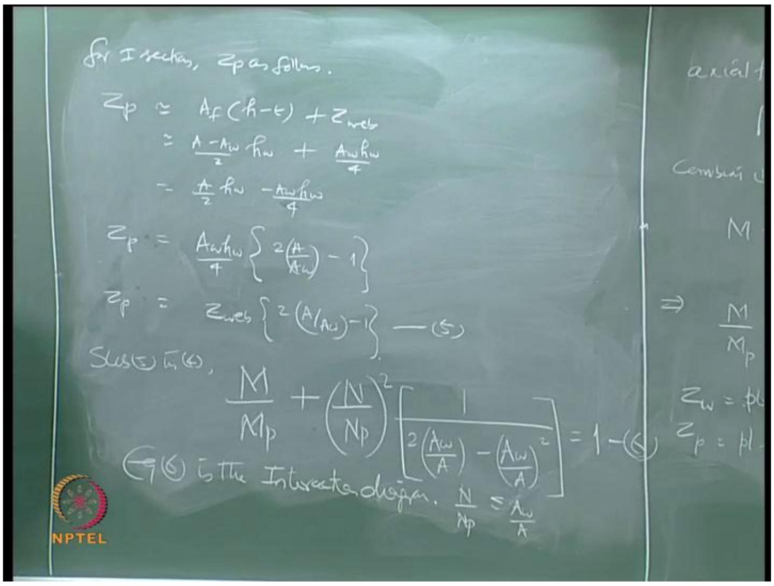

Sofor I sections,you can compute Z p as follows,that is section modulus of the entire section. Plastic section modulus of the complete section as Z p. So, Z pis approximately equal to because we are doing some adjustments. It is area of the flangeh minus t where h is the overall depth and t is the thickness of the flange alone plusZ of the web. So, area of the flange can be simplyA minus A w by 2 because there are two flanges into h w;h minus t will give you h w plus Z of the webA wh w by 4,that is web alone. Now I can say,there is A w s w by 2 here by 4 here, I can simplify rewrite this and saying that,it will becomeA by 2 of h wminus A w h wby 4. So, I can express Z pnow as Z w ofthat isA w h w by 4 2Aby A wminus 1,A w h w by 4 we already know it is Z of the web,2A by A wminus 1, that is Zp.

I already have the Z p value and Z w value in this equation here which I call as equation numberfour. I call this equation numberfive.Now substitutingfivein four, I can rewrite four like this which is Mby M p plus Nby N psquare1 bytwice of area web by A minusarea web by A the whole square equal to 1.Now equation six is the interaction diagram.Let me check this equation again,M by M p N by N psquare 1 by A web by

Aminus A web byA square is equal to 1. This is true for a specific condition whereN by N pis much lower thanA w byA.

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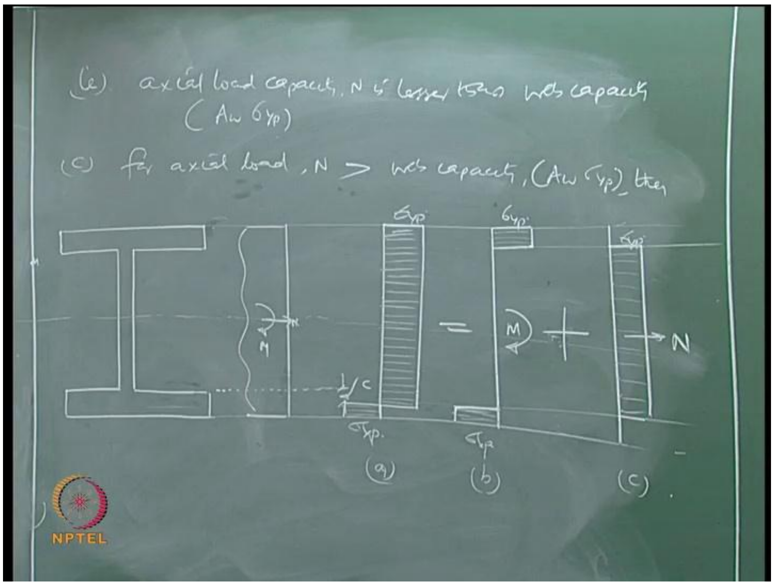

That isaxial load capacityNislesser than the web capacity.Web capacity is nothing but A w into sigmaY p.If it is true, this becomes interaction diagram, if it is not true foraxial load capacitygreater than the web capacitythat is A w sigma Y p,thenthe equation goes slightly different.Let us derive that again. Stress diagram goes like this.Now the difference between the zero axis stress and the flange,this value is called asc the offset between the top of the bottom flange to that of the zero stress axis is given by an offset by name c,that is the combined action when the axial load carrying capacity is higher than the web capacity,it is very simple.

If the axial load carryingcapacity is lower than the web capacity, obviously, the zero stress axislies in the web. Since the axial load capacity is much more than the web capacity, it comes to the flange. It comes to the flangenow; itis coming to the flangesomewhere here. So, this can be now said asa pure case of stress rectangles or stress distribution diagrams ofplus. So, this is case a combined action, this is pure bending, and this is pure axial,this is N, and this is M, this is plus.

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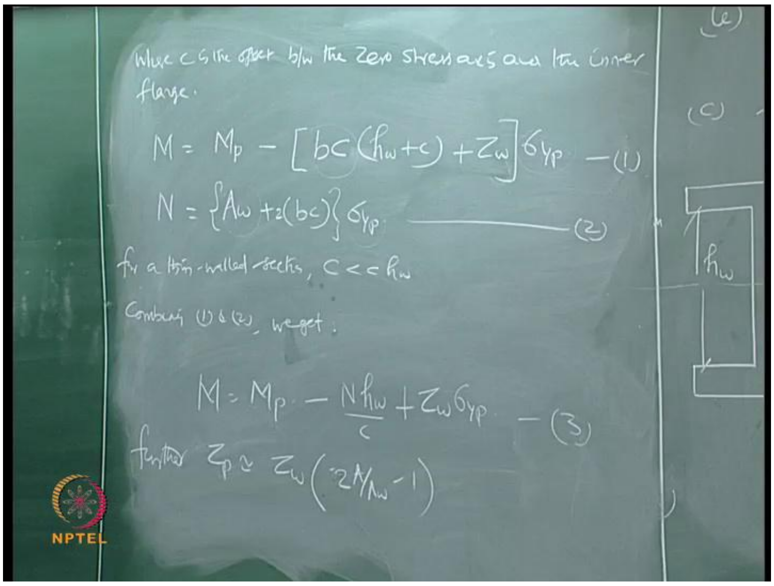

So, c wherec is the offsetbetweenzero stress axisand the inner flangeas shown in the figure. So, in this case nowM will be equal to M p minus as we did the last caseb c because this is c and the width at that is b. So, b c into h w plus cbecause this is h w height of the web, there is c added to it now,h w plus c of course plussection modulus of the web aloneintosigma Y p.And the axial force Nwhich is taken from the figure c is nothing but the area of the web.Now web alone is not there, you have got some part of the flange also. So, that is going to be b into c twice, there are upper and lower both right, ofsigma. So, for a thin-walled sectionc is much lower than h w, c is very very small compared to h, this is h w.c is much lower than h w.

So, c square term and all will go away from here,we can neglect them. Socombining the above I can call this equation number one again, I can call this as equation number two again or combiningone and twowe getM is equal to M p becauseyou have b c, sigma Y p,you have got A w, sigma Y p which areall present in the equation above I am justsubmitting them and then rearranging them.So, M is equal to M p minusN of h w by c plusZ wsigma Y p,I call this equation number three. FurtherZ p in this section is approximately equal to Zw of 2A by A W minus 1,that is what we have seen in the last derivation for I section, we have seen this.So substituting back here,I now generate the interaction diagram.

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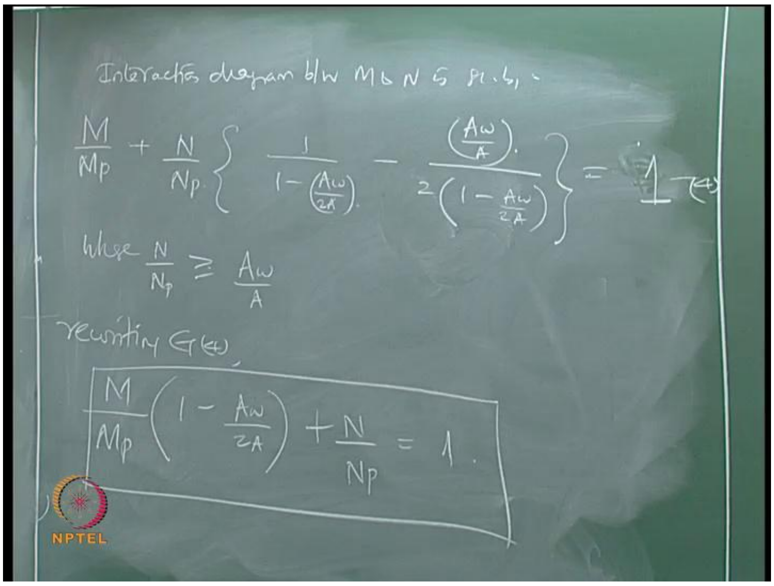

So now, the interaction diagrambetween M and N is given by M by M pplus N by N p of1 by 1 minus A w by 2A,this is 2A hereminus A w by Aby1 minusA w by 2 A of twice of this.Let me write this slightly in a different manner,minus AW by Atwice of1 minusA Wby2 A. Let me check, itis equal to 1.That is the interaction diagram now where in my caseN by N pis much greater than or equal toA w by A;that is the second case. Previous case was the otherway.I can rewrite this equation back again,rewriting equation four,M by M p of because I have got common terms I can rearrange them,1 minus A w by 2 A plus N by N pas1, that becomes the interaction diagram.I will plot this; then we will stop here.

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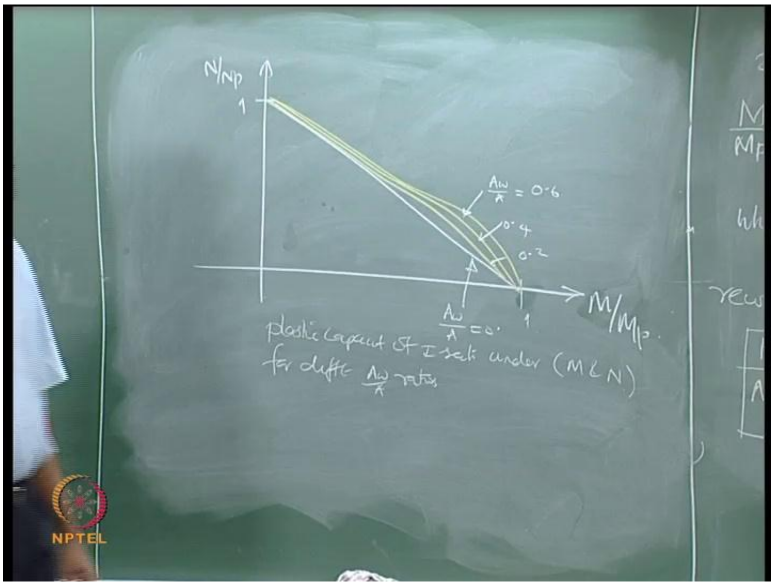

So if you try to plot this, the interaction diagram looks like this. Thisis my M by M p,this is my N by N p,this is my value at one and one,so obviouslyat A w by 2 A or A w by A when it is zero,this becomes a linear curve. So, it gets a straight line. I can call this as A w by Ais 0. For any other value the curve goes, I think I can do it in a different color.The curve goes and this bulges out. Then parallel one bulge out,parallelone bulge out for different values ofA w by A of 0.6,0.4, 0.2 and soon. So, this is my plasticcapacityof I sectionsunder combined action ofM and N fordifferentA wby A ratios.In next class we will look at the box section and the tubular sections. Then we will move on to plastic capacity of plates. We will talk about collision problems, that will conclude the module one.Any difficulty, any doubt here.

Thank you.