Welding Metallurgy
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Lecture – 23
Temperature Distribution in Welding

Welcome to the lecture on temperature distribution in welding. So we will be extending you know from the previous lecture where we were towards giving getting the expression for the temperature distribution in welding process and we were talking about that condition where there is a moving heat source. So as we discussed that as there is moving heat source.

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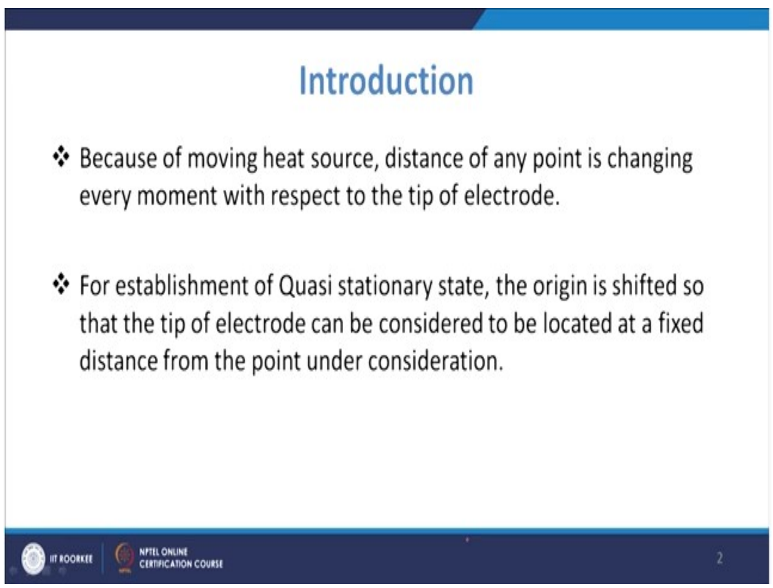

So distance of any point as it is changing with respect to the tip of electrode and for establishment of Quasi stationary state what we do is originally shifted, so that the tip of electrode can be considered to be located at a fixed distance from the point under consideration. So that way you are changing that and what we saw in our you know earlier lecture.

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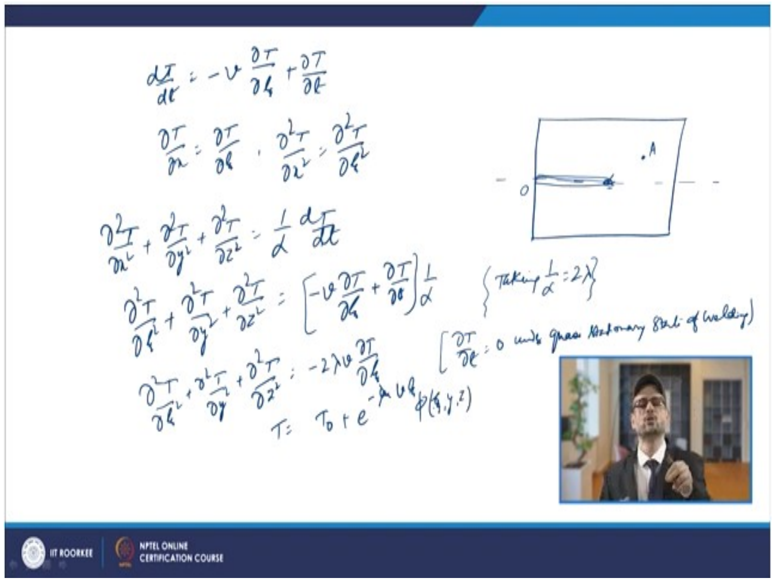

That you had we are taking this you know plate large plate and in that this is your direction of welding and this is your you know origin and this is the point suppose some point A is there which is x, y, z but what we do is you know we define one parameter zeta that is x-vt and accordingly we got you know so each since x is ζ-vt so ζ is x-vt. So you can have the expressions for $\frac{\partial \zeta}{\partial x}$, $\frac{\partial^2 \zeta}{\partial x^2}$ and all that.

So what we saw now in this case this is your so this is the origin and this is how it is moving at and at any point say your this is the point suppose where the tip of the electrode is there. So that is why we take this point as the in this direction we take in place of x we take as ζ. So what we you know got in the earlier case we got $\frac{\partial T}{\partial t}$t as -$v\frac{\partial T}{\partial z}$and you know +$\frac{\partial T}{\partial t}$.

Similarly, we got also $\frac{\partial T}{\partial z}$is $\frac{\partial T}{\partial \zeta}$and we also got $\frac{\partial^2 T}{\partial x^2}$as $\frac{\partial^2 T}{\partial \zeta^2}$. So now we have the you know we had seen the Fourier's equation and what we had seen there that it is $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$. Now this is basically we derived it to be 1/α of $\frac{\partial T}{\partial t}$. So we can put these expressions into this place.

And then we can see that $\frac{\partial^2 T}{\partial x^2}$is $\frac{\partial^2 T}{\partial \zeta^2}$. Similarly, these y and z values will be same because there is no change into it and we are getting the $\frac{\partial T}{\partial t}$value as -v $\frac{\partial T}{\partial \zeta}$. So you will have -v $\frac{\partial T}{\partial \zeta}$and + you know $\frac{\partial T}{\partial t}$and then this is multiplied by $\frac{1}{\alpha}$term. So that is the alpha is thermal diffusivity $\frac{K}{\rho . C_p}$. So that is what we are getting.

Now for convenience what we take we are taking so we are taking for convenience $\frac{1}{\alpha}$as 2λ. So if you take this $\frac{1}{\alpha}$as 2λ in that case we can write the expressions as $\frac{\partial^2 T}{\partial \zeta^2}$+ $\frac{\partial^2 T}{\partial y^2}$ + $\frac{\partial^2 T}{\partial z^2}$. So it will be -2λv. So this is α λ so $\frac{1}{\alpha}$is 2λ so it will be -2λv and then it will be $\frac{\partial T}{\partial \zeta}$.

Now you know when the quasi-stationary state is established. If we are talking about the quasi stationary state in that case you know temperature at any point on and instance does not change with time. So in that case $\frac{\partial T}{\partial t}$ will be 0 because it is not changing with you know time under that quasi stationary state. So under you know quasi stationary state you know quasi state of welding.

So this is the equation which you know takes in this form you know taking this v into account. Now for you know solving this equation what we do is so this equation is basically the three dimensional differential equation for the you know quasi steady state of welding. When your heat source is moving with a velocity v. Now in this case you can you know handle this equation better if you take the you know T so T is replaced as T_0 + $e^{-\lambda v \zeta} . \varnothing(\zeta, y, z)$.So this way so this is $\varnothing$ x will be basically replaced by ζ.

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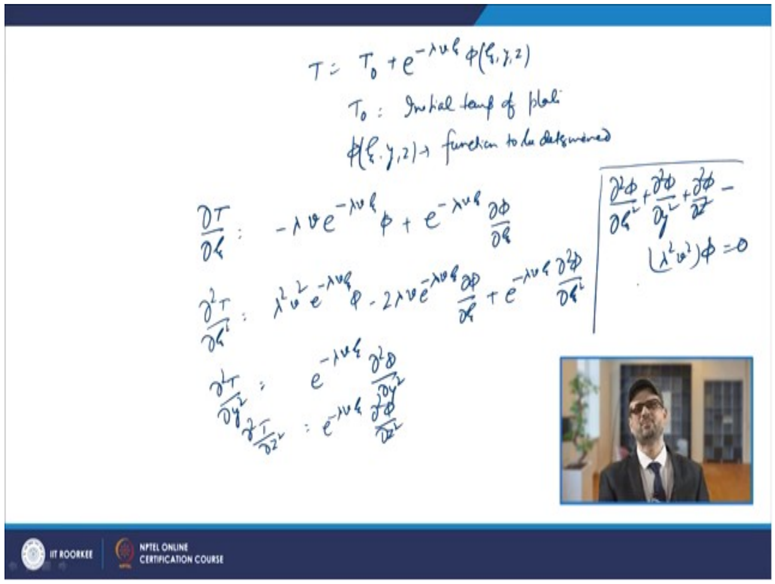

So you can have the T as $T_0 + e^{-\lambda v\zeta}.\varnothing(\zeta, y, z)$. So because of the velocity term you will have this λ will be x will be replaced by this ζ term. So and in this case T_0 is the initial you know temperature of the plate and this $.\varnothing(x, y, z)$ is the function which is needs to be determined $\varnothing$so $\varnothing$in fact that becomes ζ, y, z. So this is the function which is required to be determined.

Now what happens that so you need to find the expressions for the $\frac{\partial T}{\partial \zeta}$. So if you try to find the expression for $\frac{\partial T}{\partial \zeta}$. So

$$\frac{\partial T}{\partial \zeta} = -\lambda v e^{-\lambda v\zeta}\varnothing + e^{-\lambda v\zeta}\frac{d\varnothing}{d\zeta}.$$

$$\frac{\partial^2 T}{\partial \zeta^2} = \lambda^2 v^2 e^{-\lambda v\zeta}\varnothing - 2\lambda v e^{-\lambda v\zeta}\frac{d\varnothing}{d\zeta} + e^{-\lambda v\zeta}\frac{\partial^2 \varnothing}{\partial \zeta^2}$$

$$\frac{\partial^2 T}{\partial y^2} = e^{-\lambda v\zeta}\frac{\partial^2 \varnothing}{\partial y^2}$$

$$\frac{\partial^2 T}{\partial z^2} = e^{-\lambda v\zeta}\frac{\partial^2 \varnothing}{\partial z^2}$$

So that is you know this way once you have this function which is defined so accordingly you will have since T you are taking as this function.

So you will have to $\frac{\partial^2 T}{\partial \zeta^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} - (\lambda^2 v^2)\varnothing = 0.$

Once you have known these equations then you can further put them into the final equation and the final equation if you try to further simplify then that equation will come of the form

$$\frac{\partial^2 \varnothing}{\partial \zeta^2} + \frac{\partial^2 \varnothing}{\partial y^2} + \frac{\partial^2 \varnothing}{\partial z^2} - \left(\lambda^2 v^2\right)\varnothing = 0.$$

So basically after doing some simplification you can find this expression to be

$$\frac{\partial^2 \varnothing}{\partial \zeta^2} + \frac{\partial^2 \varnothing}{\partial y^2} + \frac{\partial^2 \varnothing}{\partial z^2} - \left(\lambda^2 v^2\right)\varnothing = 0$$

and this is the convenient form of equation for the heat flow in the case of quasi stationary case of you know state of welding and then from here we can find the temperature distribution inside in the welding process.

So this is that equation. Now we will talk about you know the equations which are you know used for finding the temperature distribution and for that there has been certain modification. So we assumed case of the point heat source however that is always not true in the case of welding. So if you do not take the you know finite you know this point heat source so you have a finite size weld pool.

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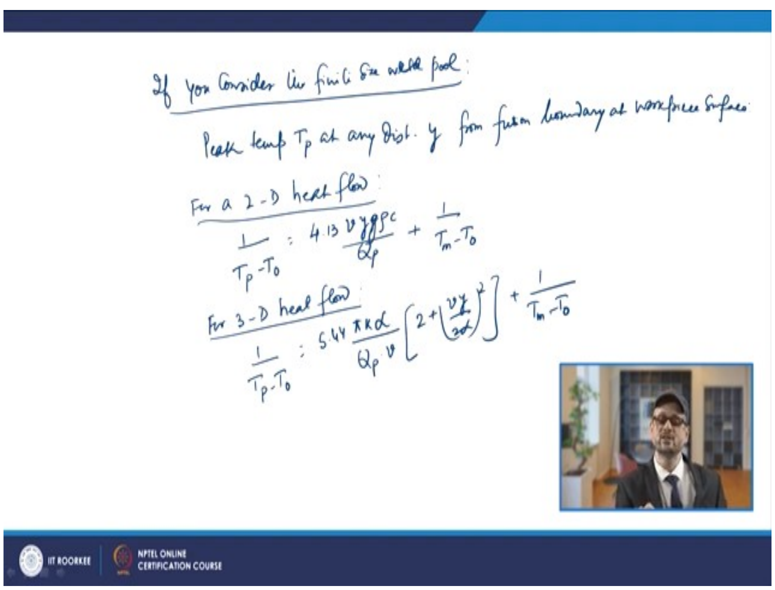

So you have basically if you consider the finite size weld pool. So in that case the derivation which has been given by the Rosenthal equation. So basically in that there has been some changes and following that equation basically Adams has given some modification. Adam and

Peak they have talked about the peak temperature you know T_p. So they found the peak temperature T_p at any distance you know y from fusion boundary at workpiece surface.

So basically you know so we have seen that what are the concept by which you know what are the basically mechanics or the theory by which you try to find the temperature distribution inside the welding and then we are you know to find the expression or we have to find basically the peak temperature T_p which will be added to the distance and y from the fusion boundary of the workpiece.

So Adams modification tells that for a two dimensional heat flow he has given this modification into the equation and they have given this equation as $\frac{1}{T_p - T_0}$. This will be so T_0 is basically the ambient temperature it will be $\frac{4.13\, vyg\rho c}{Q_p} + \frac{1}{T_m - T_0}$. So this is for the you know two dimensional case all the you know terms have the unusual meaning.

So that way you can put the values and you can have the value of T_p being calculated. Now in this case as you know that your T_m is the melting temperature of the material and if you talk about the three dimensional you know heat flow. So for three dimensional heat flow actually the equation reads as $\frac{1}{T_p - T_0}$ this is coming as $\frac{5.44\, \pi k\alpha}{Q_p v}\left[2 + \left(\frac{vy}{2\alpha}\right)^2\right] + \frac{1}{T_m - T_0}$.

So this is the expression for the peak temperature T_p at any distance y from the fusion boundary at the workpiece surface further. So this is your Adam and you know Adam has given this you know expression for the change in the Rosenthal equation. Then further you know you Wells modification is there.

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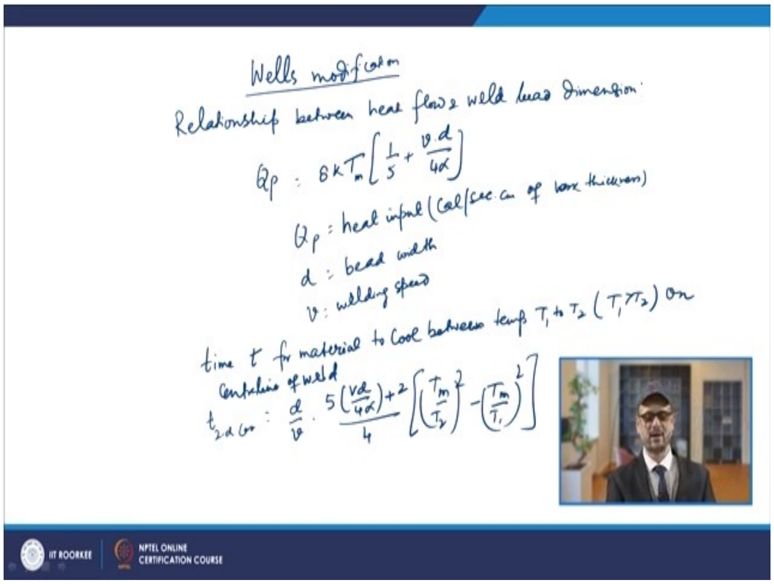

So the Wells modification tells that you know this is the relationship between the heat flow and the weld bead dimension and that is so this is the relationship between heat flow and weld bead dimension. Now in this case what Wells has suggested that you have Q_p that is your heat input that is in the case of in the units of calorie per minute $Q_p = 8 k T_m \left[\frac{1}{5} + \frac{vd}{4\alpha} \right]$.

So that is the expression for the Q_p here Q_p is the heat input in calories per second and centimetre of work thickness. Then as you know in this case d, d is the bead width v and other terms are having the usual meaning v is the welding speed and similarly α is the thermal diffusivity so that way we call this. Now from this you know equation it will be derived that so if you try to find the time t.

So from this equation from the time t for the material to cool between two temperatures. So for time t for the material to cool between you know temperatures T_1 and T_2. So if you are thinking of finding what will be the time for the specimen to cool from one temperature to other. So from temperature T_1 to T_2 where $T_1 > T_2$ on the centre line of weld. So this is for the two dimensional case.

And where you have assumed to be having the case of full penetration and in this case and you can have also the case for the three dimension also. So for two dimension case you know for 2D case this t is found as $\frac{d}{v}.\frac{5\left(\frac{vd}{4\alpha}\right)+2}{4}\left[\left(\frac{T_m}{T_2}\right)^2-\left(\frac{T_m}{T_1}\right)^2\right]$. So this way if you know the temperature T_1 to T_2 which is to be you know which for which you have to find the time t in which this temperature will be dropping from T_1 to T_2, $T_1 > T_2$.

In that case for two dimensional case you know the this value t can be calculated using this expression.

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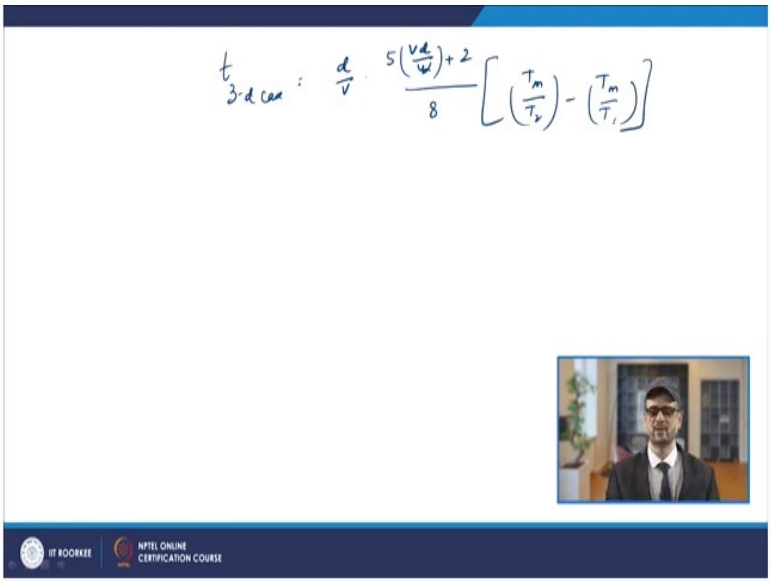

And if you go for the three dimensional case then it is calculated using the expression $\frac{d}{v}.\frac{5\left(\frac{vd}{4\alpha}\right)+2}{8}\left[\frac{T_m}{T_2}-\frac{T_m}{T_1}\right]$. So that way you calculate the expression for the two dimensional as well as the three dimensional cases. So this way I mean these expressions you must be knowing and you can read for you know how these you know expressions are to be used for solving certain cases.

For example, you may have the problems for on welding to be solved and you know suppose you are coming through such a situation where you have to find the Q_p. Suppose you know for using the Wells modification so you can accordingly you know you can accordingly use using the

Wells modification you can use for the so once you want you must know the values of this $k T_m$ and then you just put these values.

And you can have the values further substituted and get the value of Q_p further the time to change the temperature from T_1 to T_2 what will be the time required. So basically these you know values will be utilized when you are doing this welding process. In that process basically the time during which this any temperature drop is there you can also try to have the idea about the you know change of temperature with respect to time.

And that will be talking about the temperature gradient. Now accordingly you can predict the microstructural changes in that specimen. So that way you can have the microstructural simulation microstructural prediction in the welded you know specimen. Now once you have the different value of v. Now what happens that when you have the changing welding speed now what will be you know the so how this temperature change will be there.

And accordingly those predictions can be you know made like what will be the associated change in the microstructure under such circumstances. So use of these equations are basically important and at that time you need to have the proper understanding about the all the terms and how these equations have been derived. So basically all these derivations have been made by taking these plates of either finite size or infinite size that you can refer to.

And there has been you know the expressions being derived either for the temperature distribution in a large you know infinite plate of finite thickness. So you may have the cases like you may go for the temperature distribution.

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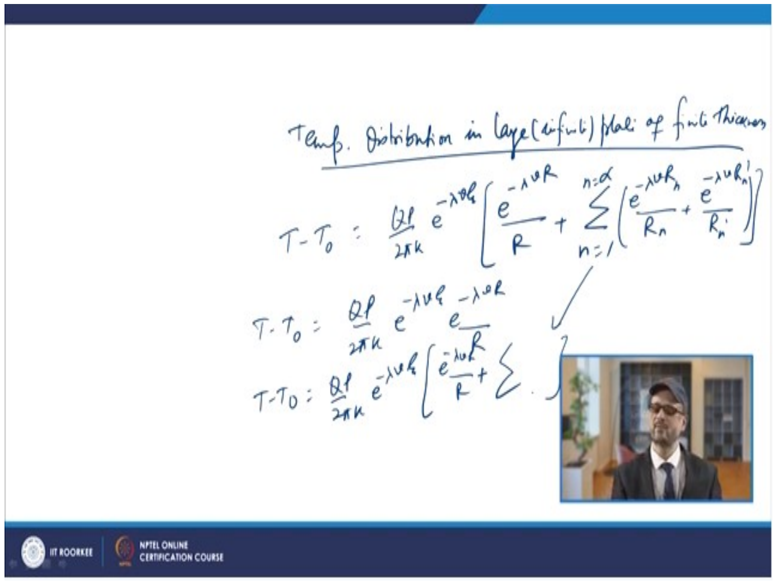

So you can go for the you know temperature distribution in large plates that is you take as infinite you know plate of finite thickness and then so the analysis goes with taking into account the because once you have a large plate so the plate thickness is taken as some value as G and then there will be you know again you will have the use of those equations λv and R and all these terms will be taken into account.

And in this case you get the final expressions as $T-T_0=\frac{QP}{2\pi k}\left[e^{-\lambda v\zeta}\frac{e^{-\lambda vR}}{R}+\sum_{n=1}^{n=\alpha}\left(\frac{e^{-\lambda v R_n}}{R_n}+\frac{e^{-\lambda v R_n^{'}}}{R_n^{'}}\right)\right]$.

So that way you know in this case you will have two terms coming up of T-T_0 is one is this and another is for this. So the two equations have been summed up so you will have this condition for the you know butt welding basically. So butt welding of large semi-infinite plate of finite thickness. And so that way if you look at the analyze these two equations you have this is the summation of two cores two equations that is $\frac{QP}{2\pi k}e^{-\lambda v\zeta}\frac{e^{-\lambda vR}}{R}$.

So this is for the you know first equation is this so and then you have infinite thickness and then you have the finite thickness if you look at it will be $\frac{QP}{2\pi k}e^{-\lambda v\zeta}\left[\frac{e^{-\lambda vR}}{R}+\sum\ldots\right]$. then you have all these terms coming up. So that will be for the finite thickness.

So that way this is the coming the two equations which are used for either the you know large semi-infinite plate you know of infinite thickness as well as for the finite thickness. So these are the two equations and from there you know the Adams and Wells modification has been further incorporated into and you get the expression for the peak temperature at any point at distance y from that fusion line. So accordingly you can practice more and more and have more understanding about the phenomena in the case of welding. Thank you very much.