Welding Metallurgy
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Lecture – 22
Heat Flow in Welding

Welcome to the lecture on heat flow in welding, so in the last lecture we discussed about the different heats sources, which is used in welding and typically, we also got the idea about the efficiency of those heat sources. Now, we will try to understand how there will be heat flow in the welding, so we will try to have the understanding about the fundamentals of the heat flow equations.

And then also in our coming lectures, we will try to have an understanding of the temperature distribution in the welding. So, in this lecture we will try to have the understanding about the basic equations which govern this, if you talk about the heat flow in the welding.

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Introduction

- Heat flow in welding is mainly due to heat input by welding source in Limited zone and further its flow into body of workpiece by conduction.
- Limited amount of heat loss occurs by convection as well as radiation.
- Welding can be considered typically a problem of heat conduction with moving heat source.

So, heat flow in welding basically is due to the heat input by welding source in limited zone and further it is flow into body of work piece by conduction, so certainly we know that we are you know using a heat source that heat source will generate the heat and that heat will be basically in a local manner, it will be doing its job by melting you know but then as the heat which is there that will be basically flowing into the body.

Because all around that zone where the metal is melted, you have the metallic material, so that takes the heat from it, so that goes by conduction and certainly when you have a liquid metal pool and which is exposed to the surrounding which is exposed to environment, so there will be some you know loss to the surroundings by radiation and there will also be some you know loss or heat flow by convection also.

So but then we neglect because that is in a very small you know amount as compared to their heat which is flowing into a body using the conduction mechanism. Now, what we see in case of welding process is that you have a source which is the heat source that is where the arc is striking the you know the metal. So, in that zone, so that basically heat there basically, large amount of heat is generated.

And also that is moving along certain direction so basically, you know if you try to see then you can I mean consider this welding process can be considered typically a problem of heat conduction you know with a moving heat source because the source which is there its basically moving in certain direction. Now, so certainly that you must have the concept about the heat flow, what are the governing equations for heat flow?

And then only you will be able to have the idea about what will be the temperature at certain point, so you will have some assumptions also.

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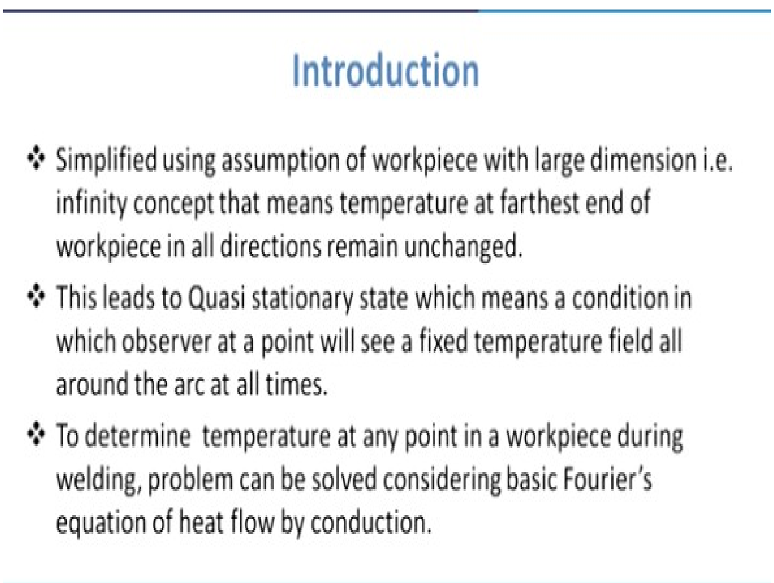

And there will be further simplification using the assumption of work piece with larger dimension, so if you take the work piece of larger dimension in that case, the you know infinity concept is you know coming into picture and that tells that the temperature at farthest end of the work piece in all directions remain unchanged. So, wherever we are typically thinking about the heat transfer at that particular point and if you take the point which is quite distant from there.

So there you know the temperature in all directions basically, remain unchanged so, you will have a you know temperature distribution pattern can be seen and that can be; so that will be another simplifying assumption, so that basically leads to the quasi stationary state, we call it as and this quasi state means that a condition in which the observer at a point will see a fixed temperature field all around the arc at all times.

So, if you have an observer and you know and he looks at the temperature field, so what he will see that you know around the arc, a field which is generated, so you will have a fixed temperature field, so if you have an arc all around you will have you know the constant you know, temperature field.

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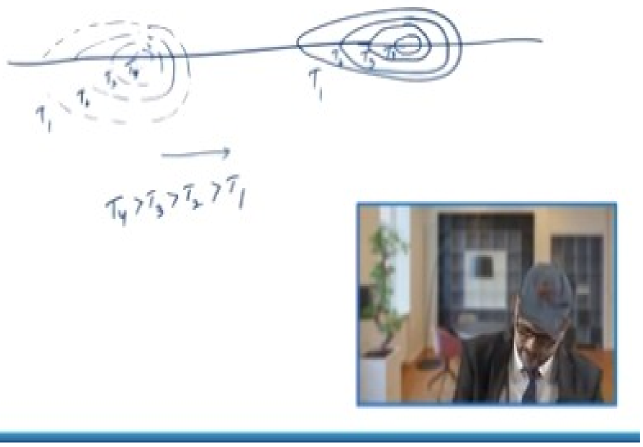

So, it will be something like you know you have so, you have a direction in which you are welding and what you see is that if you are having you are you are at this point at this moment, so what you see is that your temperature distribution will be moving like this, so this will be your this way your temperature distribution will move like so, you know this is temperature field in that can be considered to be in the case of quasi stationary, a state of welding.

And you will have you know the temperatures you know so, you will have suppose this is a temperature T_1, this is temperature T_2, this is temperature T_3, this is temperature T_4, so and then if you are you know moving in certain direction, so after some time at present time suppose this was in the past, so at present you will have again you are having you know this point, so this way your temperature field will be looking like.

So, again you will have the you know temperature lines like this is T_1, this is T_2, this is T_3 and this is T_4, so this is your direction of welding and in fact the T_1 is smallest and T_4 is the largest one, so T_4 is more than T_3 then T_2 and then T_1, so that way your; so that is talking about so this way your; you know the heat flow will be taking place. Now, what happens that in the case of you know, so we are talking about the; this so, what we see is a fixed temperature field all around the arc at all time.

So that is what we are seeing that you have if you look at it, observer will see all around the arc he will be seeing this fixed temperature line now, if we try to determine the temperature at any point in the work piece during welding so certainly, the problem can be solved considering basic Fourier's you know equation so, equation of heat conduction is there in Fourier's heat conduction equation.

And as we know that you will have, so what we see that in the case of Fourier's conduction equation if you try to revise so, it tells that the rate of heat transfer.

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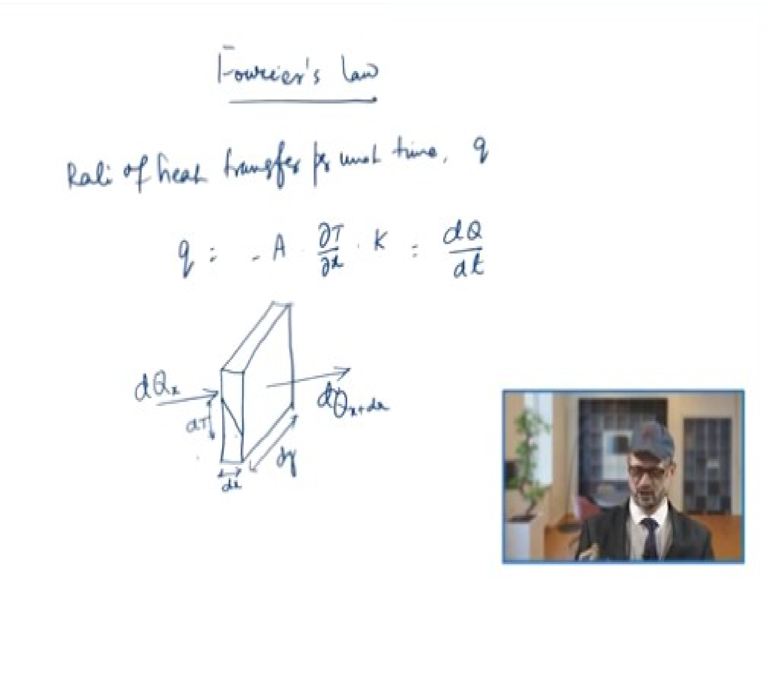

So, if you go to the Fourier's equation; Fourier's law of conduction so, it tells that the rate of heat transfer, so that is you know per unit time if you look at this unit that is if you call it as a q, so q will be the product of; so q basically will be product of area A, so it will be area A* $\frac{\partial T}{\partial x}$ so that is your temperature gradient at the section.

And then you have the thermal conductivity, so that is your you know the q defined as and you know sign negative because heat flow will be occurring in the direction of decreasing temperatures that is why we call it keep this I mean sign as negative and this is also known as the dQ/dt, so that is rate of heat flow so, we call it as the dQ/dt, so dQ/dt can be taken as this you know, so that is your Fourier's law of heat conduction and it can be derived by you know by if you go for the one-dimensional case, then you can have you know a geometry where we must have done earlier.

So, if you take a geometry of this type where there is unidirectional heat conduction, so you will have such a slab through which the heat is you know moving from this side to this side so, you will have this dimension as dx and then in this case, you will have the heat transverse, if you look at the temperature; this will be the high temperature and this will be at lower temperature.

So, this you know this difference that will be dT, so and then you will have other dimensions like this is taken as dy and you have, so this if you look at the value, if this is taken as dQ_x, so this distance will be x + dx and this is taken as $dQ_{(x + dx)}$, so that way you can have, so we can get the expression for that you can have the you know, some names to these phases and accordingly you can get the expression.

So, this is the condition of unidirectional heat flow and in this, there are many assumptions like heat flow in the y and z direction, so here you are having the assumption that you know you are still taking these you know in the heat flow in y and z direction is 0 and accordingly, so you solve this equation and try to have the temperature field.

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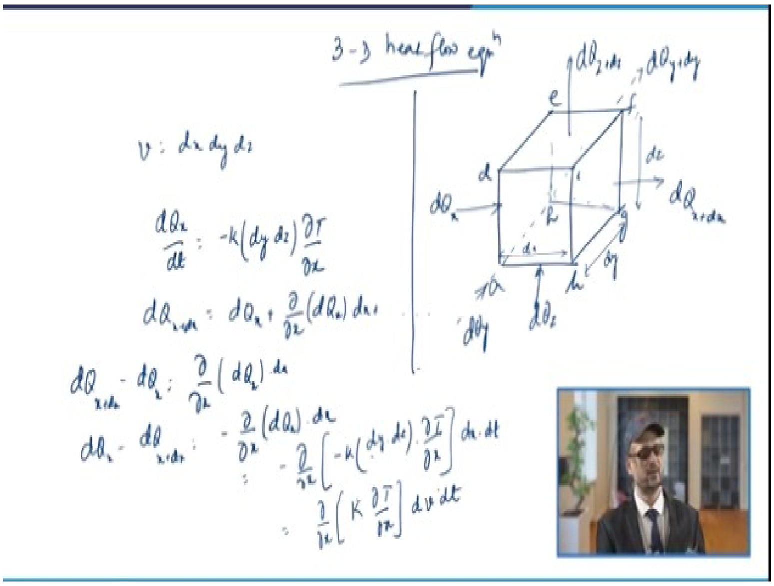

Now, if you take the 3 dimensional you know heat flow, so if you go to the 3 dimensional heat flow equation, so in that basically we are talking about the simple cube and if you take a cube of you know of this shape, so that is how the cubic shape can be taken and in this you can have this; this has a, b, c, d and similarly you have you know e, f, g, h, so you will have e, f, g, and h.

Now, in this case the heat transfer will be heat flow will be in all the you know 3 directions you know x, y and z, so you will have dQ_x you know here and this from this side, it will go dQ_{x+dx}, similarly you will have so, this is your y so, this is your x direction, this is your y direction and this is your z direction, so in this case you will have here a dQ_y and then from here it will you know move.

So that way it will go dQ_{y+dy} similarly, you will have the dQ_z and dQ_{z+dz}, so that way you will have the flow of heat from all the phases in the case of 3 dimensional geometries. Now, in this case as the geometry indicates you will have the volume of the you know geometry, so volume, v = dx.dy.dz, this will be the volume of this cubical element.

Now, dQ_x is the quantity of heat which is entering towards the x phase, so this is your direction x and this is the direction, this you know this dimension is dx, similarly this dimension will be dy and this dimension will be dz, so that is how you know, so we take it as dx dy and dz. Now, you can have the; you can use that Fourier's law of conduction and you can get the expression for dQ_x/dt, so $dQ_x/dt = -KA$.

So, A will be the area of this phase, so that will be you know this is your this side and this side, so this side is dy and this side is dz, so it will be dy.dz and then you will have the term that is temperature gradient, so that will be $\frac{\partial T}{\partial x}$, so that is your use of the partial differential term and if you take the dQ_{x+dx}, so if you find the dQ_{x+dx} term, now dQ_{x+dx} term can be written you know using the Taylor expansion theorem, you can write it as dQ_x, then you will have $\frac{\partial}{\partial x}(dQ_x)dx$.

Similarly, the other you know terms will be going out ahead of this, so if you take $dQ_{x+dx} - dQ_x$, so $dQ_{x+dx} - dQ_x$, $= \frac{\partial}{\partial x}(dQ_x).dx$, so that will be your you know this, so but if you get the value of $dQ_x - dQ_{x+dx}$, so it will be $\frac{-\partial u}{\partial x} \times dQ_x \times dx$, so that way you get this you know term $dQ_x - dQ_x + dx$, it will be $\frac{-\partial u}{\partial x} \times dQ_x \times dx$.

Now, if you substitute you know this value of dQx and if you, so in this value you have the if you find this dQx value, so if you substituting this value, your this value becomes $\frac{-\partial u}{\partial x}$ and then you take -K and then dy/; you know dy * dz and then you have $\frac{\partial T}{\partial x}$, so that is what you are taking the value of dQx and then you will have the *dx* * *dt*, so that is your this dt will go into this side.

So that way you are getting this term and what this way you are coming to what you see is that this dx, dy and dz will come from this side, so you will have you know so, you have minus sign here and minus sign here also, so that way it will be a positive sign so, you will have you know $\frac{\partial}{\partial x}$ of K *dx dy dz*, so you will have you know and *dx dy dz* can be taken out.

So, you will have; so that will be your dv, so it will be K and $\frac{\partial T}{\partial x}$and your dx dy dz because a constant values, so it can go out, so you will have the term dv you know and *dt*, so that way you are getting the value of the dQx - dQx + dx, similarly you can have the value for the dQy - dQy + dy , so that is you know heat is flowing from here to and the difference of rate of heat flow that we can find.

And similarly, dQy - dQy + dy that will be according to this you know this equation you can have ∂ by; you know ∂y of [K $\frac{\partial T}{\partial y}$] dv.dt, so that way you may have the dQ z – dQz + dz.

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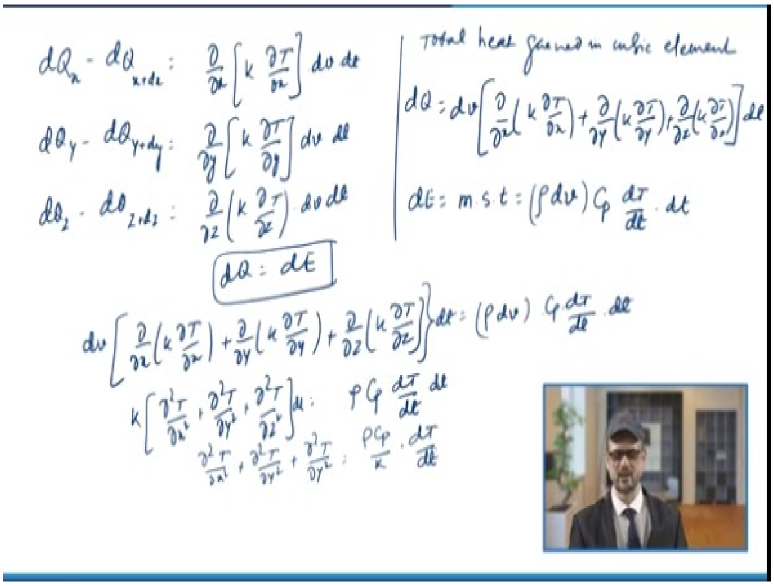

So, in a nutshell what we get is dQ_x – dQ_x + dx, it will be

$$d\,Q_x - d\,Q_{x+dx} = \frac{\partial}{\partial x}\left[k\frac{\partial T}{\partial x}\right] dQ.dt$$

$$d\,Q_y - d\,Q_{y+dy} = \frac{\partial}{\partial y}\left[k\frac{\partial T}{\partial y}\right] dQ.dt$$

$$d\,Q_z - d\,Q_{z+dz} = \frac{\partial}{\partial z}\left[k\frac{\partial T}{\partial z}\right] dQ.dt$$

So that way you are getting you know the 3 expressions; so if you talk about the total heat gained so, if you talk about total heat again in this cubic element; total heat gained in cubic element that will be the dQ_x – dQ_x + dx + dQ_y - dQ_y + dy + dQ_z – dQ_z + dz.

So, in cubic element if you talk about the total heat gained, it will be dQ and it will be the summation of all these and if you write you know this, it will be so you can take the $dQ = dv\left[\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)\right]dt$ coming into picture, so that way what you see this dQ term is coming as the dv into this.

Now, further if you talk about the heat gained in the system, it is nothing but it will be expressed in terms of increase in the internal energy of the system so, if you talk about the internal energy increase of the system, so that is your DE, it will be so, we know that will be mass into the specific heat into temperature difference so, it will be $m.s.t.$ and you know, so you have mass as volume into density.

So is it density is ρ and volume we know that volume is dv, so that is your dv, the volume of the that element, then you have specific heat that is and then you have temperature difference, so that is dt and dt, so that way you will have this expression and basically both can be equated so, if you try to equate now, in this equation you know that all the things you see this is the C_p, specific heat at constant pressure.

And you have other terms known to you, so what you do is; you have to equate these two values, so you will have dQ = dE, so if you do that you will be getting the expressions and that will come as so, you will have $dv\left[\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)\right]dt = (\rho.dv).C_p\frac{\partial T}{\partial t}.dt$ So that way you will have this term and if you know if you take the K as uniform in all the directions, if the K becomes constant, then this K will come out and this dV will be also be cancelling, the K will be coming out so, your equation will come as $K\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right]dt = ¿$now, this will be; dv term will be cancelling.

So, you will have the $\rho.C_p\frac{dT}{dt}$ and this dt term will also get cancelled, so that way you know this so, if you keep dt here, it will be also coming as dt and anyway that dt has to go now, if you talk about that condition in which with respect to time basically, there is no temperature difference so, from here you are getting the expression that is you can have these equations written like $\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}$.

That can be taken as $\frac{\rho . C_p}{K} . \frac{dT}{dt}$, now this is basically the equation of the 3 dimensional you know heat conduction in solids now, what happens that we try to define this term $\frac{\rho . C_p}{K}$K, so $\frac{K}{\rho . C_p}$ is taken as the term α.

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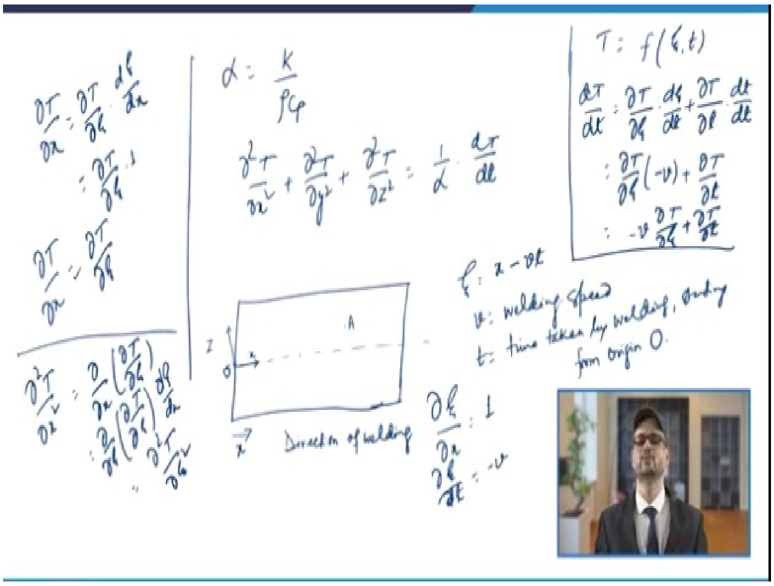

So, if you take αas $\frac{K}{\rho . C_p}$, so in that case your equation can be written as you know so, Fourier equation becomes $\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha} . \frac{dT}{dt}$, so this is known as the Fourier's, you know law; Fourier's equation of the 3 dimensional heat flow. Now, this equation is basically implied in the case of welding.

And then the temperature distribution basically is tried to be you know found out so, in the case of welding as we discussed that you have you know what we see in those cases that if you take you know a very large plate where we had seen that we are using this infinity concept so, now in this case suppose, you have you know you define so, what you see is that that this is your x direction and this is your z direction.

So, what you see and this is your x is your direction of welding now, in this case what happens that your source which is there, the heat source which is there, it is not fixed at any particular point basically, it is moving so and moving in the in this x direction, so and you know so there is the arc which is here at this point it will be moving to this point at you know, some point of time.

So, if you take you know, if you want to find you know the temperature distribution at any you know point that is if suppose, you are taking this point as A and if you want to find the temperature distribution at this point x, y, z, now in this case your tip of the electrode will be moving in the x direction, so other you know distances will remain fixed except the you know x coordinate system.

So, what we do normally in such cases we are basically changing this coordinate system you have to take the you have to shift the coordinate system, so along the x direction basically, you will have the another parameter which will be coming in place of x and what we do is in that coordinate system we are taking this because if suppose V is the velocity of you know welding.

In that case, we are basically taking ζ and ζ is taken as x – vt, so at any you know time taken by welding at any time, if you try to find so, you will have y and z coordinates remaining fixed but the x coordinate because this if you talk about the distance on this point A, so that will be changing, so because of that change we are changing this you know coordinate system with respect to you know, the tip of the electrode.

And zeta we are taking as x – v.t, where the v is the welding speed and t is the time taken by welding, so that is starting from origin O, so if you take you know this origin as O, so after time t, it has to travel; it will travel a distance of suppose v.t, so this distance which is x, it will be x – v.t, so that way your ζ will come into picture, so apart from so instead of the coordinate as x, y, z, it will be zeta y, z that will be taken.

Because there is movement in certain direction and that will lead to certain changes because if ζ is taken as x - vt, so your dζ/ , if you take $\frac{\partial \zeta}{\partial x}$, so $\frac{\partial \zeta}{\partial x}$ will be coming as 1 and if you take the $\frac{\partial \zeta}{\partial t}$, so that will be certainly as you see, it will be taken as - v, so T will be the function of basically zeta and time, so because the T is the function of ζ and time.

So, what we get further T is since it is a function of zeta and time, so we have to have the expression for ∂T by; so the differentiation of the dT with t, so if you take the differentiation of dT/dt, so it will be taking so, we will have the partial you know differentiation principle, so once you will differentiate in terms of zeta and then $\frac{dT}{dt} = \frac{dT}{d\zeta} . \frac{d\zeta}{dt} + \frac{dT}{dt} . \frac{dt}{dt}$ and as we know $\frac{dT}{d\zeta}$, so $\frac{dT}{d\zeta}$ will be as usual and $\frac{d\zeta}{dt}(-v) + \frac{dT}{dt} = -v\frac{dT}{d\zeta} + \frac{dT}{dt}$, so that way you are getting the expression for $\frac{dT}{dt}$, with respect to time.

Similarly, we can have the expression for $\frac{dT}{dx}$ also, so you will have the expression for

$$\frac{dT}{dx} = \frac{dT}{d\zeta} . \frac{d\zeta}{dx} = \frac{dT}{d\zeta} .1$$

$$\frac{dT}{dx} = \frac{dT}{d\zeta}$$

Similarly, if you; so further we can differentiate these you know expressions and try to have the expressions for the $\frac{\partial^2 T}{\partial \zeta^2}$, so if you further differentiate say you are differentiating with respect to you know, if you are differentiating further with respect to x, this equation so, it will be $\frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{dT}{d\zeta}\right)$.

So that will be basically $\frac{\partial}{\partial \zeta}$, so you can write as $\frac{\partial}{\partial \zeta}\left(\frac{dT}{d\zeta}\right)\frac{d\zeta}{dx}$, so you can write it as $\frac{\partial^2 T}{\partial \zeta^2}$. So, what you see that $\frac{\partial^2 T}{\partial x^2}$ will be $\frac{\partial^2 T}{\partial \zeta^2}$ and $\frac{dT}{dx}$ is $\frac{dT}{d\zeta}$ and $\frac{dT}{dt}$ will be -v of $\frac{dT}{d\zeta} + \frac{dT}{dt}$.

Now, these expressions will be used to find the temperature distribution in the case of welding that we will do in our next lecture, where we try to find in actual case, when you have a moving you know heat source, what will be the temperature distribution, so that we will see it in our coming lecture. Thank you very much.