

Welding Metallurgy
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Lecture - 15
Problem Solving on Strengthening Mechanism in Metals

Welcome to the lecture on problem solving on strengthening mechanism in metals. So we in the last week we have discussed about the different strengthening mechanisms and just we will solve few problems to have the concepts more clear to us because we need to know that how you know these numerical values are achieved suppose in terms of the yield strength or how the grain size changes, increases the value of stress value to move the dislocation or so. So for that we will have the different type of problems.

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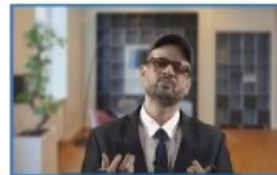
Q.

	σ_y	K	n
steel	520	1270	0.15
Copper	55	317	0.54

To calculate the difference between flow stress at 0.1 plastic strain & yield stress (for two metals).

for Steel: flow stress: $1270(0.1)^{0.15} = 900$ | Difference = $900 - 520 = 380$

for Cu: flow stress: $317(0.1)^{0.54} = 91.4$ | Difference = $91.4 - 55 = 36.4$



So first problem is related to you know finding you know difference between flow stress, so say we have a problem like for 2 materials the yield strength value is given and also the work hardening exponent is given and for a particular plastic strain you have to find for the you know separately for 2 materials you have to find the difference between the flow stress at typical plastic strain value and the yield stress.

Because you know plastic deformation starts after the yielding, so we have to calculate the change in or the difference between that flow stress at particular strain value and the yield stress value. So suppose you have 2 materials that is one is steel and another is copper and for

these 2 materials the values are given so suppose yield stress value is given in terms of you know so this is the $\Omega\text{N/m}^2$ something like.

So this is not required to be given and that is given as 520, so this is not here, this is for steel, this is σ_y and we can write here that this is 520 and for copper it is 55, this is annealed copper. Similarly, the K value which is used in the case of you know this work hardening you know equations, so that is given as 1270 and in this case it is given as 317 and similarly the other value which is given is the strain hardening exponent and this is 0.15 for the steel and 0.54 for the copper.

So this is the table which is supplied and this is the yield strength value, this is the strength coefficient value and this is the you know the work hardening exponent value and you have to find so to calculate the difference between flow stress values, flow stress at 0.1 plastic strain and yield stress for 2 metals. So suppose if you have you know to sort you have to revise that concept.

And we know that we know that that will be say flow stress is nothing but $K * \epsilon^n$. So if you take for steel say, for steel you know for that the flow stress value will be K. So k is $1270 * \epsilon^n$. So we will have $0.1^{0.15}$, so this is flow stress. So that will be, this will be coming as = 900 MPa or so.

And if you calculate for the copper, for annealed copper so again flow stress value at 0.1 plastic strain will be again $K * \epsilon^n$, so it will be $317 * 0.1$, but n value in the case of copper is 0.54, so we will have the power 0.54 and that becomes equal to you know 55, that is becoming 91 if you do the calculation it will be 91.4.

Now what we need to find is the flow stress and the yield stress values. So for steel the difference in the value, difference in the value will be its yield point is 520, so it will be $900 - 520$, so it will be about 380 MN/m^2 and similarly you will have for copper annealed copper the difference will be you know it will be $91.4 - 55$. So this will be about 36.4.

So this way you can calculate the values of the difference and you know that concept has to be used that is flow stress will be $K * \epsilon^n$. So strain hardening exponent that value has to be

given. ϵ you must know ϵ may be given in terms of you know you may have the initial length and final length and based on that you can calculate the epsilon and then that also can be used.

So depending upon the problem you have to solve such kind of problems. Now we will have another type of problem. Say if we get the problem based on the dislocation density. So next problem will be.


(Refer Slide Time: 07:29)

Q. For a cold worked Cu crystal, having dislocation density of $3 \times 10^{14} \text{ m}^{-2}$, Shear stress required to move a dislocation is 100 MN m^{-2} . The value reduces to 1 MN m^{-2} on annealing. To find stress required to move dislocation if Cu crystal cold worked to dislocation density of $2 \times 10^{12} \text{ m}^{-2}$.

$\tau_0 = ?$

$$\tau = \tau_0 + A\sqrt{\rho}$$

$$\begin{cases} 100 = \tau_0 + A\sqrt{3 \times 10^{14}} \\ \tau_a = 1 + A\sqrt{2 \times 10^{12}} \end{cases} \rightarrow A \rightarrow \frac{99}{\sqrt{3} \times 10^7}$$

$$\tau_a = 1 + \frac{99}{\sqrt{3} \times 10^7} \times \sqrt{2} \times 10^6 = 1 + 9.9 \times \frac{\sqrt{2}}{3} = 1 + 8.085 = 9.085 \text{ MN m}^{-2}$$


You know like you have a cold worked copper crystal so for a cold worked copper crystal, so having dislocation so it has the dislocation density some value dislocation density of 3×10^{14} per m^2 and for that the shear stress required to move a dislocation so that was calculated is found to be 100 MN/m^2 . So that is the data given.

Now the value reduces to 1 MN/m^2 on any link. So this is when you anneal the material in that case it reduce to 1 MN/m^2 it means that once we know that when we do the annealing, it becomes soft. So you can think of this value as the stress required when you know the dislocation is not there. So this is the constant value which is used in that equation.

So now you have to find, so to find stress required to move dislocation so in that case you know if copper crystal cold worked to dislocation density of 2×10^{12} per m^2 , so that we know so in such you know problems we have to simply use the equation so what we the equation is known to us that τ will be $\tau_0 + A\sqrt{\rho}$. So τ is for this equation when your tau is 100 MN/m^2 , τ_0 is given as 1.

So it will be A and rho is given as 3×10^{14} . Now you have to find the τ that is required, so τ required you know for the you know, so if suppose if τ_x is required suppose then in that case τ_x will be $1 + A\sqrt{10^{12}}$. So these 2 equations are there and these 2 equations need to be solved simultaneously to find the value of τ_x .

So you can have the from this equation you can have the value of A. $A = \frac{99}{\sqrt{3} \times 10^7}$. So you will

use in this equation. So if you use this equation τ_x will be $1 + \frac{99}{\sqrt{3} \times 10^7} \times \sqrt{2} \times 10^6$. So that way you can solve and then this will be you know this will be cutting to 10.

So it will be 99/10 so it will be $9.9 \times \sqrt{2}/\sqrt{3}$, so you can write it as $1 + 9.9 \times \sqrt{6}/3$. So that can be you know so root 6 will be somewhere close to 2.45 and divided by 3, so if you do this calculation it will be $1 + 8.085$ something like, so it will be 9.085 MN/m^2 . So this way you know you can solve the problems if you have a problem related to the you know dislocation density where the dislocation density is changed.

And you have to find the value of you know stress required when the dislocation density will be changed. Now the next problem is related to again this time the effect of grain size.

(Refer Slide Time: 12:58)

Q: Estimate the yield stress of polycrystalline Fe-Si alloy when grain size is ASTM 1, 4 & 8 respectively. Assume $G_i = 80 \text{ MN m}^{-2}$ & $k = 0.63 \text{ MN m}^{-3/2}$

$\sigma_y = G_i + k d^{-1/2}$

ASTM 1 $\rightarrow d = 0.254 \text{ mm} = 0.254 \times 10^{-3} \text{ m}$

ASTM 4 $\rightarrow d = \frac{0.254}{\sqrt{8}} = 0.089 \text{ mm}$


ASTM 8 $\rightarrow d = \frac{0.254}{\sqrt{2^3}} = \frac{0.254}{8\sqrt{2}} = 0.022 \text{ mm}$

ASTM n $\rightarrow d = \frac{L}{\sqrt{16 \times 2^{n-1}/64}}$

$\sigma_y = 80 + 0.63(0.254 \times 10^{-3})^{-1/2}$
 $= 120.4 \text{ MN m}^{-2}$

$80 + 0.63 \left(\frac{\text{mm}^2}{\text{mm}^2} \right) = 146.5 \text{ MN m}^{-2}$

213 MN m^{-2}



So say the question is that you have to estimate the yield stress of polycrystalline Fe silicon alloy when grain size is ASTM 1, 4 and 8 respectively. So for this again you have been given certain values assume σ_i as 80 MN/m² and K is 0.63 MNm^{-3/2}. So using these values you have to find the yield stress of polycrystalline iron silicon alloy, 3% silicon alloy is there.

And the grain size is ASTM 1, 4 and 8, so we must have the concept of ASTM numbers and accordingly we have to calculate you know once you know the grain size that is d then in that case we can calculate you know the values, so suppose what we have understood is normally when you go from ASTM 1 to ASTM 2, the grain size will decrease by a factor of $\sqrt{2}$. So that is the rule for the ASTM you know number we have already refined it.

So and as you know that ASTM 1 grain size is somewhere close to 0.254 you know millimetre size, so that is the diameter for the you know ASTM 1 grain size. So and also so in that case once you know the grain size then you will have the finding of my yield stress at those values $\sigma_i + K \times d$ once you know the $d^{-1/2}$. So that is how, so your you know equation becomes like σ_y will be $\sigma_i + K \times d^{-1/2}$.

So d is in terms of meter, so in this way you know we get the value of so ASTM 1 as you know we have understood that it is normally defined for at 100X you know and then based on that we calculate these values of you know number of grains are there in per square inch and based on that we find the value in mm. So it comes to the order of 0.254 mm. Now for that also there is a calculation like if you have ASTM n calculation it will be

$$d = \frac{1}{\sqrt{10^4 \times \frac{2^{n-1}}{645}}}$$

So this 645 comes for the inch conversion to millimetre and then further this 10^4 is because it is at 100X magnification. So that is why you have and we know that for n you must have 2^{n-1} . So this 2^{n-1} comes into picture multiplied by 10^4 and divided by 644. So that way if you go for you know d.

So if you go for n = 1, so it will be 1, so 10^4 by 645 it comes close to 15.5 or so and then it is multiplication and that will be about you know 3.93 or so and then once you get the reciprocal of that that will be = 0.254. So this way we can calculate the different value of the

grain size for different ASTM numbers. So if you calculate for ASTM 1 as we know the grain size will be 0.254 mm.

So that will be this one and it will be $0.254 * 10$ raised to the power - 3 meter. Similarly, for ASTM 4 as we know so every time you have the increased number of ASTM number it means you know this at this point you will have the 2 raised to the power now it is becomes 3, 4 - 1 that becomes 3. So, basically the grain size will reduce by $\sqrt{2^{n-1}}$.

So in this case the d will reduce and ASTM in this case it will be 0.254 mm divided by 2^3 so that is 8, so $\sqrt{8}$. So it will be you know that accordingly you will have so you can have it like $2\sqrt{2}$. So 0.127 and then you know below also $\sqrt{2}$ is there, so you can have $\sqrt{2}$ by in fact 4, so accordingly you can have the d values and in that case you get the value of 0.089 mm.

So this is ASTM 4, similarly if you have ASTM 8 again in ASTM 8 you will have d as

$$0.254/\sqrt{2^7} = \frac{0.254}{8*\sqrt{2}} = 0.022 \text{ mm.}$$

So 0.7 with that we will multiply it and once you know you will divide it so it will be something like 0.03 and then again further you multiply with 0.7. So that way it will be coming as 0.022 mm something like that.

So what you will do so you can simply put in the equation and you can have the calculation of the yield stress. So yield stress calculation can be you know very simple so for ASTM 1 case your yield stress value will be you know it will be sigma i given as 80 and + you will have K value given. So it will be 0.63 and then then you will have d value. So d value is calculated as $(0.254*10^{-3})^{-1/2}$.

So if you do that if you, you know do this calculation then you will get it as this as 40.4, so it becomes 120.4 MN/m². So this way you can calculate these values. So without calculating even the value of d also actually you can have this expression only and use it in the equation.

So similarly for ASTM you know 4, you can have the expression like

$$80+0.63 \text{ } \hat{c}$$

So this way if you calculate this value this becomes 66.5, so it becomes you know 146.5 or 147 MN/m². So this way you can calculate similarly if you calculate for this it will be coming as you know for ASTM 8 it will be 213 MN/m², you can calculate that.

So you have to simply put in the equation and accordingly you have to calculate the values. So this is how you know you calculate the values of the you know yield strength as the so you can see that when ASTM 1 grain size is taken then your yield stress value is coming to 120.4, when it is ASTM 4, that is the grain size is increased to $2\sqrt{2}$, I mean decreased in fact.

Number is increased to 3 numbers, but the grain size is decreased to $2\sqrt{2}$. So you know it will be 146.5, it is increased from there and further it is increased to 213 if you go to ASTM 8.

(Refer Slide Time: 23:28)

Q. To calculate mean grain dia corresponding to ASTM 0, 3 & -2.5.

$$\text{ASTM 0: } 0.254 \times \sqrt{2} = 0.359 \text{ mm}$$

$$\text{ASTM 3: } \frac{0.254}{\sqrt{2^3}} = 0.127 \text{ mm}$$

$$\text{ASTM (-2.5)} \rightarrow \frac{0.254}{\sqrt{2^{-3.5}}} = 0.254 \times \sqrt{2^{3.5}} = 0.85 \text{ mm}$$



So maybe if you are to find so now it will be clear that if you may have to find the grain sizes so if you have to calculate mean grain dia corresponding to ASTM 0, 3 and -2.5. If you have to calculate you know the different ASTM numbers and for that you have to calculate the you know grain size, grain dia we know that for ASTM 1 it is 0.254 mm. So for ASTM 0 will be grain size will be larger than ASTM 1.

So it will be you know, you have $0.254 \times \sqrt{2}$. So you know it will be something like 0.359 you know mm. If you come to ASTM 3, now as we know that ASTM 1 is 0.254, so for ASTM 3 you have to simply you know divide it with 2 because you have 2 numbers, $\sqrt{2^2}$ and

it is under root, so it will be 2. So it will be you know if you go to ASTM 3 it will be

$$\frac{0.254}{\sqrt{2^2}} = 0.127 \text{ mm.}$$

And if you take ASTM -2.5, now in this case the grain size will be further larger than ASTM 0. So for that you will have 0.254 and basically you have to have $\sqrt{2^{-2.5-1}}$, so basically -3.5.

So in fact you have to multiply this with you know $\sqrt{2^{-3.5}}$, and it will be becoming as somewhere close to 0.85 mm. So this way you can calculate the different values of the you know grain diameter for different ASTM numbers. Next question is that suppose you have a problem like.

(Refer Slide Time: 26:16)

Q. A grain size of ASTM 7 is refined to ASTM 14 (for mild steel). Estimate increase in yield strength. ($K = 0.71 \text{ MN m}^{-3/2}$)

$$\sigma_7 = \sigma_i + K d_7^{-1/2}$$

$$\sigma_{14} = \sigma_i + K d_{14}^{-1/2}$$

$$\sigma_{14} - \sigma_7 = K \left[d_{14}^{-1/2} - d_7^{-1/2} \right] = 0.71 \left[\left(\frac{0.254 \times 10^{-3}}{\sqrt{2^{14}}} \right)^{-1/2} - \left(\frac{0.254 \times 10^{-3}}{\sqrt{2^7}} \right)^{-1/2} \right]$$

$$= 0.71 \times \left[\frac{1}{\sqrt{2^{14}}} \right]^{-1/2} - \left[\frac{1}{\sqrt{2^7}} \right]^{-1/2} \right] \approx 300 \text{ MN m}^{-2}$$



If suppose a grain size of ASTM 7 is refined to ASTM 14 for mild steel, so you know estimate increase in yield strength. So for we know that the K value is given for steel as $0.71 \text{ MN m}^{-3/2}$. So as we know that you will have for 1 grain size you will have the yield stress value calculated like sigma y will be $\sigma_i + K d^{-0.5}$.

So you know for if you take the you know σ_7 at ASTM 7 number it will be $\sigma_i + K d_7^{-0.5}$ and similarly σ ASTM 14 it will be $\sigma_i + K d_{14}^{-0.5}$. Now d_7 and d_{14} has to be calculated. So you have to now the d_7 will be 0.254. So you know 0.254 is for ASTM 1 and if you go for d_7 , so it has to be divided by $\sqrt{2^6}$, that is 2^3 you have to divide.

So you can directly calculate you know $\sigma_{14} - \sigma_7$ it will be, so this will cut and you will have

$$\sigma_{14} - \sigma_7 = K [d_{14}^{-0.5} - d_7^{-0.5}] = 0.71 \left[\left(\frac{0.254 \times 10^{-3}}{\sqrt{2^{13}}} \right)^{-0.5} - \left(\frac{0.254 \times 10^{-3}}{\sqrt{2^6}} \right)^{-0.5} \right]$$

and then we will multiply that. So basically it will be coming, so if you see this value will be coming as close to 596 and you know and this will be coming close to 171 or you know you have to multiply so 177 so like that it may come. So you can even not write that, so basically if you calculate this value whatever comes here that will be coming close to 300 MN, you can check it. So this way you can calculate the change in the you know yield strength value if there is change in the grain size.

So similarly you may have another type of you know examples like you know for any mild steel specimen suppose we may have another you know problem like you have a mild steel specimen and it has you know.

(Refer Slide Time: 30:46)

Q. Mild Steel Specimen of 0.03 mm \rightarrow refined to 0.003 mm.

$$\begin{aligned} \sigma_1 &= \sigma_i + K (0.03 \times 10^{-3})^{-1/2} \\ \sigma_2 &= \sigma_i + K (0.003 \times 10^{-3})^{-1/2} \\ \sigma_2 - \sigma_1 &= 0.71 \left[(0.003 \times 10^{-3})^{-1/2} - (0.03 \times 10^{-3})^{-1/2} \right] \\ &\approx 275 \text{ MPa} \end{aligned}$$



So suppose you have mild steel specimen and you are basically you know you have the grain dia of 0.3 you know 0.03 mm and it is refined to 0.003, so it is refined to 0.003 mm. So what will be the increase in the yield strength. So again you have to use the similar formula like σ_1 or σ yields for this value σ yield for that value and that will be again equated to, so you have to simply give the values, so you will have

$$\sigma_1 = \sigma_i + K [(0.03 \times 10^{-3})^{-0.5}]$$

you will have d. So d in this case will be

$$(0.03 \times 10^{-3})^{-0.5}.$$

Similarly,

$$\sigma_2 = \sigma_i + K \left[(0.003 \times 10^{-3})^{-0.5} \right]$$

So simply you have to calculate

$$\sigma_2 - \sigma_1 = 0.71 \left[(0.003 \times 10^{-3})^{-0.5} - (0.03 \times 10^{-3})^{-0.5} \right]$$
$$\cong 275 \text{ MPa}$$

So that way you can practice more and more you know problems you can have the concept of the grain size you can have the concept of dislocation densities, you can have also the concept about the length you know which comes into while discussing the dislocation.

So based on that you can calculate these you know dislocation densities and you can solve you know problems and that will help you when quantitatively you have to deal with the problems where basically in the strengthening has to be achieved up to certain level you know by increasing the yield stress values or so. So what mechanism is used and how it is used that will be more clear to you in the near future. Thank you very much.