Thermo-Mechanical and Thermo-Chemical Processes

Prof. Vivek Pancholi
Department of Metallurgy and Materials Engineering
Indian Institute of Technology-Roorkee

Lecture-04
Stress and Strain

Hello friends, we will start with a new lecture, lecture number 4 which will be defining stress and strain and which we will which you use in metallurgical parlance and the definition to start a stress or strain.

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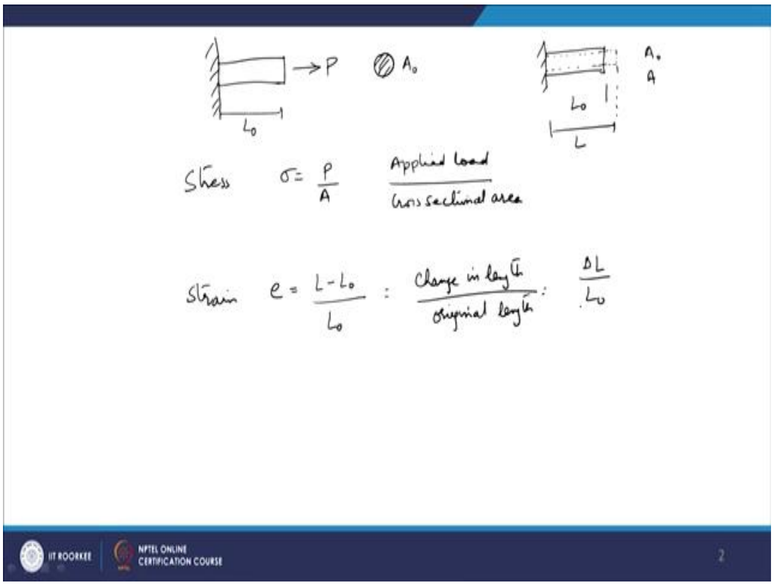

Let us first I will start with a simple geometry here (above figure). So, suppose you have a bar here which I have fixed at one end at this point and now I am applying load or force here in a uniaxial direction and let us say that the length of this initial bar is L_0 and the cross sectional area of this bar if I want to take a cross sectional area then let us say it is A_0. So, if I have these parameters, I can define a generalized stress or an average stress acting on this bar.

So, that will be defined as

$$\sigma = \frac{P}{A_0}$$

so, that is basically

$$\frac{the\ applied\ load}{cross\ sectional\ area}$$

So, this is what we usually know about stress so, nothing new here. What will be the new is later on you will see that I am also refining the definition of this particular stress. So, if I want to do that, let us say I do not want to put this A_0 here right now, I will bring it later on. So, simply it is;

$$\frac{\text{cross section } load}{cross\ sectional\ area}$$

And if I want to give a more exact definition of a stress, I would say you go to a book of mechanics there they define stress at a point and then they also define the stress in the stressed tensor terms and then you can use this tensor to do stress transformation and so on, but it is a different way of treating the stress we generally treat stress as an average stress. So, this is an average stress. Similarly, when I am putting a load on a; on any material on a block for example; here cylindrical block then it will also deformed.

In general term we call this a deform, in engineering terms I will be calling it as a strain that I am putting a strain on the material. So, what will happen after the loading suppose this is my initial bar maybe it will get stretch under the applied load. Now, I can see that my initial length which was L_0 has now become some another length L. So, it is from here, and initial cross-sectional area A_0 has become some new cross-sectional area A.

So, if I want to define a strain now, which I will put like this, so, this is my stress and this is my strain then it will be simply the change in length. So, basically

$$\frac{L - L_0}{Initial\ length}$$

So, this is I would be calling as

$$\frac{\text{change in length}}{\text{original length}}$$

or I will be calling L – L_0 can also be written as $\frac{\Delta L}{L_0}$.

Now, some refinement will come here that I will be calling these stress and strain in two different ways.

One I will be calling it as engineering stress and strain and another I will be calling as true stress and true strain. I will explain that why you need these two definitions. So, let us start with the engineering stress first.

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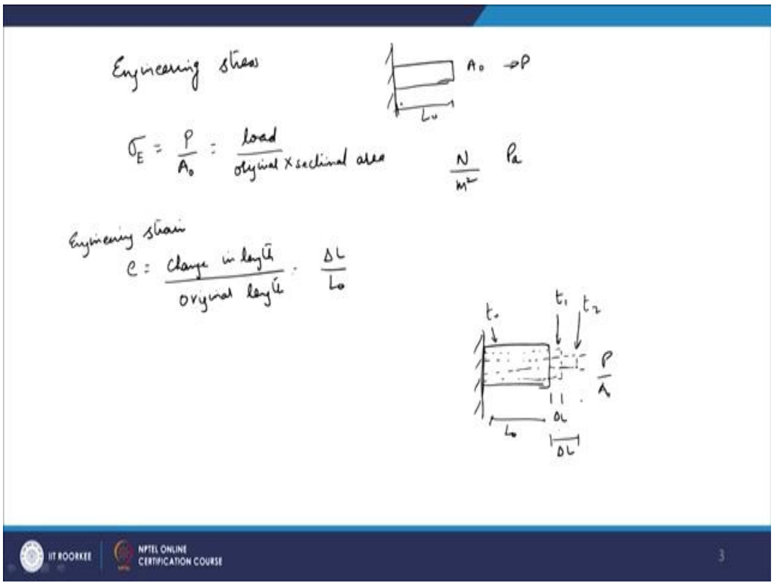

So, engineering stress; again, my same figure so, your cross sectional area is A_0, length is L_0 and we are applying force here P ok. So, if it is engineering stress that means I am dividing the load by the original cross-sectional area. So, if it is engineering is stressed so, let us now I will put subscript here E. So,

$$\frac{P}{A_0}$$

that is the original cross sectional area. So, that is my

$$\frac{\text{load or force}}{\text{original cross sectional area}}$$

and the units will be of course, load will be in Newton and area will be in meter square.

So, it will be the units or simply I can also call it as Pascal. So, these will be the unit. Similarly, I can define the engineering strain. So, this is my engineering strain now, as

$$\frac{\text{change in length}}{\text{original length}}$$

So, that is again

$$\frac{\Delta L}{L_0}$$

so, it is what we generally have as definition. Now the one aspect you might start thinking about is that when you are deforming a material. So, let us say take a bigger diagram here.

So, I am deforming the material at one instance suppose this is what will be the length and the cross-sectional area. So, t_0 this is the condition, this is the condition at let's say t_1 , after another time; duration let's say now this is the condition so, my cross sectional area has changed and my length has changed. So, if you see in terms of the instantaneous values of stress and strain that are continuously changing. So, if I say that the stress on the material at instant 2 is same

$$\frac{\text{Load p}}{\text{original cross sectional area } A_0}$$

Then I am not truthful as you can say because my cross sectional area has not remained A_0 now, in fact it has become this small area A. So, the stress on the material has changed during the deformation process. Similarly, if I talk in terms of strain initially I said this was my

$$\frac{\Delta L}{L_0}$$

But after this for the next instance which is t_2 coming here, whether I would take the new ΔL here, and divide it by or let us say I will be taking let us say ΔL from here and dividing by L_0.

So, I am continuously dividing it by the original length for every change in the length. Ok. So again, I am not being truthful here because if I take instance 2 here that means for that the original length should have been the

$$L_0 + \Delta L$$

And then you should have calculated the next increment or next strain. So, strains as well as the stress are continuously changing during the deformation. So, to being more truthful to the situation we will define a new stress and strain term.

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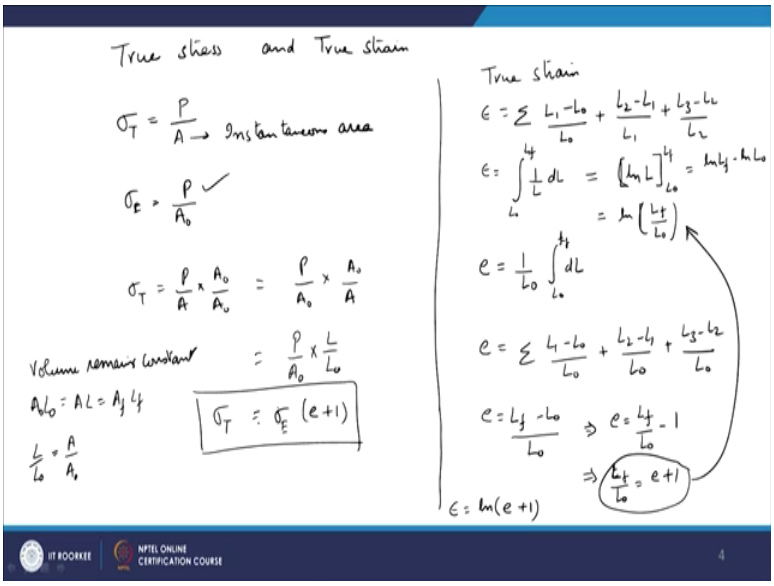

Which now you will see that we normally don't do define in these terms is what we will call as true stress and true strain, that means we are being truthful to the situation now. So, true stress means, now, I will define this as σ_T here ok. So, my load is whatever I am measuring in my load cell and the cross-sectional area instead of A_0 now it will become A that is the instantaneous area.

So, whatever is the area at that moment that will be the stress on the material. So, this A is now instantaneous area. And load is whatever is measuring you are measuring in the load cell. So, now, if I want to define this σ_T term in more detail, lets say or if I want to have now an expression between the σ_T and σ_E. So, let us do some jugglery here. So σ_E was

$$\frac{P}{A_0}$$

So, I can do some changes here. So, lets say this is σ_T

$$\frac{P}{A}$$

And I am dividing and multiplying by A_0 here. So, this will become

$$\frac{\mathrm{P}}{A_0} \times \frac{A_0}{A}$$

Now $\frac{A_0}{A}$. I have not defined it till now. But before coming to that, lets also define the true strain here, true strain my true strain is ε. So, engineering strain I have written as e so, I will be writing true strain as ε and that is the continuous change in the length. So, basically if I want to write in summation form it will be like this.

$$\frac{\text{New length from the original length}}{L_0} + \frac{L_2 - L_1}{L_1} + \frac{L_3 - L_2}{L_2} \text{ and so on.}$$

So, every time I am taking a new length or instead of writing in submission form let us say I write it in a integral form that is

$$\int_{L_0}^{L_f} \frac{1}{L} dL$$

This is what will be the definition and if I want to write the same definition for engineering strain.

Engineering strain should be something like this it will be

$$\frac{1}{L_0} \int_{L_0}^{L_f} dL$$

So, you can see the basic difference between engineering strain and true strain is that here we are keeping because L is continuously changing we are keeping it in the integral whereas, in this case a L_0 is a constant which has come out of the integral and only you are integrating the change in the length. So, it is again basically the summation and summation form if I want to see this it should be like this

$$\frac{L_1 - L_0}{L_0} + \frac{L_2 - L_1}{L_0} + \frac{L_3 - L_2}{L_0}$$

So, every time I am dividing by the same original length that is why it is L_0 will come out and rest will become the whatever is the increment that will be summed up. So, this is the difference between the engineering strain and the true strain. Now, similarily to finding out the relationship between the true stress and true strain I have not completed that part because I wanted to do this first.

I can now relate the engineering strain and true strain. So, if I do a complete integration of this, I can write it like this

$$\int_{L_0}^{L_f} \frac{1}{L} dL$$

will become with the limit and L_0 and L_f. Now, this can be written as

$[lnL_f - lnL_0] = ln\left(\frac{L_f}{L_0}\right)$

So, this is my definition. Now, and for engineering strain it is simply a

$$\frac{L_f - L_0}{L_0}$$

Now, if I do again some rearrangement here.

I can write it like this

$$e = \frac{L_f}{L_0} - 1$$

and that will make

$$\frac{L_f}{L_0} = e + 1$$

So, you can see that we have defined a term here and now this will go there. So, this particular expression is can be inserted here. So, now ε will become $\ln(e + 1)$..

So, now, this is my relationship between ε and that means true strain and engineering strain. So, this is the relationship between the two.

Now, I will come back to our problem here that we have got that true stress

$$\frac{\mathrm{P}}{A_0} \times \frac{A_0}{A}$$

So, I want to now relate this with the stress here and that is lets say now, I will also call it is ok. So, when we are doing any plastic deformation one thing you should always remember that my volume remains constant; volume is not going to change when you are doing a plastic deformation. So, this constancy volume will always be taken or considered during the plastic deformation.

So, that means that if I want to find out volume that will be cross sectional area for cylindrical sample of course into length that should be equal to whatever is the area or length during the deformation.

So, every point and then should be equal to finally, whatever final values you have got all these should be equal. So, if I want to write $\frac{L}{L_0}$ there will be $= \frac{A}{A_0}$.

So, that ratio will be there. So, we have $\frac{A}{A_0}$ here that can be replaced by then $\frac{L}{L_0}$.

So, this will become $\frac{L}{L_0}$,

$$\frac{L}{L_0} = e + 1$$

So, we will have σ; and $\frac{P}{A_0}$ is our engineering stress this one.

So, this becomes σ_e and $\frac{L}{L_0}$ becomes e +1. So, it becomes e+1. So, σ_T or true stress = engineering the stress × (engineering strain + 1)

So, this is the two these are the two relationships between true stress and true strain sorry true stress and engineering stress and true strain and engineering strain very important relationships.

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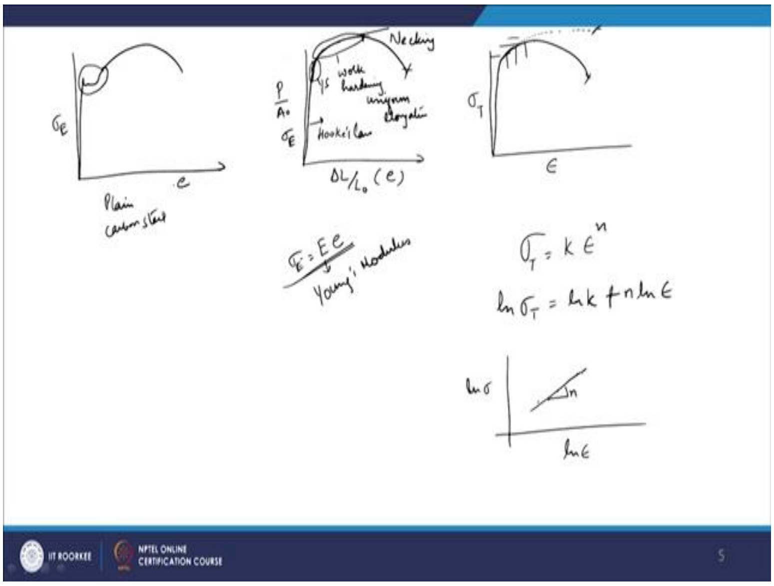

So, what you will do actually when you do any tensile test any uniaxial tensile in a tensile testing machine. So, the initial graph, if you want to plot it will be extension, and here it will be force. So only you are measuring force, actually, your load cell does not measure the stress. And one more thing you should be very careful to understand or you should understand is that when you are doing a tensile test, you are not applying load on the sample you are only stretching it you are applying a strain on the material.

So, when you are holding the sample, what you are doing is your cross head is moving, when the cross head is moving, what do you are measuring as force is which is we are measuring on the load cell of the machine is the response of the material to that strain that is why a strain is on the x axis and stress it also on the y axis. So, what you are doing is you are stretching the material and noting down the response of the materials. So response of the material is in terms of the basic data which you get is in terms of force.

And you are extending the material so that is on the x axis. So, extension versus force data you will get from this data you will calculate. So, extension means basically ΔL so, if I divide by L_0 then it will be the engineering strain and if I divide force P by A_0 then I will get the engineering stress. So, this data will be next plotted by doing this so, this you can do easily in any software which handles data for example, Excel Microsoft Excel you can easily do this kind of calculation.

Then from here by knowing the relationship between the engineering stress and engineering strain I can find out what will be the true strain and true stress very simple. So, you have this calculation we have already have relationship now between the true stress and engineering stress and engineering a strain similarly between the true strain and engineering strain. So, we have relationship you can easily find out the data.

Now, the interesting thing is that how these graphs will look when you are plotting it for example, I am not going into force attention it will be same as what you will get in engineering stress strain curve ok. So, the curve will look something like this, sorry; this is a normal curve which you will see one more type of curve which you will see I will just show you in the; because this is where lot of mistakes usually people do.

So, one type of curve is like this what I am plotting another one is this one and then you have something like this and then it goes like this. So, this is what you see in a plane carbon steel ok.

So, again, this is my engineering strain and engineering stress. So, only difference here is this part and this is what we call as yield point phenomena. So, this happens only in few materials. So, whenever somebody asked you to plot a stress strain curve it is not necessary that you will always plot it like this.

You can plot this is also a general stress strain curve. So, difference is that this in some material you will observe this kind of yield point phenomena not in all materials. So, I will be using only this stress strain curve most of the time. I am not going into all the details of the stress strain curve here. You might be knowing that but just to refresh your memory this is your elastic part where the material follows Hooks law.

And Hooks law means my σ_E will be proportional to strain and E is my modulus of elasticity or Young modulus. It is Young modulus. Then somewhere it will yield. That also you can find out by taking an offset of 0.2% strain and make a line parallel to the this linear part of the curve. So, you will get somewhere as yield point or yield strength. Then you see that actually the there is a increase in the stress as you are deforming.

As you are putting strain there is an increase in stress this part is called up to this point where you again it starts dropping this part is called work hardening or the strain hardening that you are putting a strain and it is getting hardened. And this happens because of the increasing dislocation density in the material. So, dislocation is start interacting with other barriers for example, grain boundary, precipitate, among themselves also all this interaction actually increase the strength of the material ok.

So, the stress for deformation keeps increasing as you are putting strain in the material. So, this is what we call as work hardening and this is where you get what we call as uniform elongation. And then at some point the stress start dropping that is where the actually the necking it starts. And then the drop in the stress start to happen. Now, if I want to plot the same curve in a true stress true strain curve or plot then initial part will look similar to the one here up to the more or less yield point ok.

And then lets say first I plot the engineering stress strain. And then I start plotting the true stress true strain curve. So, you can see that engineering stress strain curve is coming down after the necking whereas true stress strain d curve is not coming down. Why? Because we are taking the instantaneous area of the sample. So, stress has to increase the value of a stress has to increase because in the previous case we were taking only the original cross-sectional area though the cross-sectional area is continuously decreasing.

So, now because we are calculating stress by taking instantaneous area my true stress value will be more. Whereas, the engineering strain or basically true strain values will be lower than the engineering strain. Why? Because an engineering strain we are dividing the change in length by the original length which is a smaller length and as you deform the length will increase. And in case of true strain we are dividing by the instantaneous length which is more.

So, something is more in the denominator means the value has to be less. So, this is how the normal relationships will be there. And this is where actually the sample fractures, so, in true stress this is where the sample will fracture. So, these are the different stress strain curves for the two cases here. Now, if I want to have a more understanding of the work hardening for the elastic part, we have this relationship where the work hardening is there or where the uniform elongation is there. We can see that it is a nonlinear type of curve here this is a linear equation, but this curve is a nonlinear one.

Relationship can be now established between this one and more most acceptable relationship between the true stress and true strain in the deformation plastic deformation part is basically relationship like this

$$\sigma_T = k\varepsilon^n$$

So, you can see that it is a nonlinear equation now; it is some kind of exponential curve, because it looks like an exponential curve.

$$\ln \sigma_T = \ln k + n \ln\varepsilon$$

So, that kind of equation is there. And if I take logarithmic on both the side, now becomes a linear equation. And if it becomes a linear equation and if I plot ln σ versus ln ε, I will get different levels you have measured all these values and plotted these. So, you will get some points something like this. So, if I fit a linear line straight line on this, then the slope will be equal to the n. So, I can find out that what are the material parameters during the strain hardening of the material.

Now, I would like to tell you the advantages of measuring or performing or your calculation using true stress or true strain.

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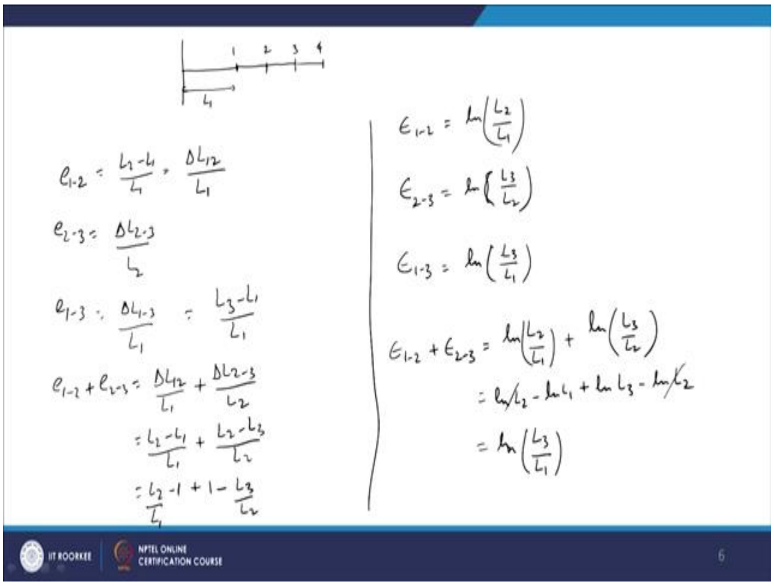

Let us give an example here. Again, I am starting with plotting on a linear scale. So, this is let's say, my original length of the material. And let us call it as L_1. Now, I am giving increment in different stages here. So, let's call them as L_2, L_3 and so on. So, 1 2 3 4 So, different length will be called as L_1, L_2, L_3 and L_4 and so on. So, if I am doing calculations using the engineering strain.

And this kind of a situation can come very easily during any operation for example, when you are doing deformation in industry. You start with a big ingot. Then you do first pass of rolling second pass of rolling may be heating in between third pass of rolling. And now you want to know that, what is the maximum deformation you have given total deformation not maximum total deformation you have given to the material during all these stages.

So, some deformation or some strain you have imparted at first rolling then it goes the material goes to the second rolling mill you have imparted some strain at that point and then you have imparted some strain in the third rolling stage. So, now, you want to know that from ingot to the maybe the final finishing what is the total strain you have imposed in the material. So, this is what is the similar case here I started with a sample which is having length L_1 ok and now, I am deforming the material in different stages.

Now I will see the advantage of doing the strain calculation using the engineering strain idea and true strain idea and want to say that what is the advantage of using one over another. So, let's start with first; so, I will call it e 1-2 that means the strain imparted in the material from going from length 1 to the length 2 that will be equal to basically $\frac{L_2-L_1}{L_1}$ that is I will call $\frac{\Delta L_{12}}{L_1}$.

Similarly, I can say what is $e_{2\text{-}3}$ is $\frac{L_3-L_2}{L_2}$, so, I am directly writing in terms of ΔL now the length has become 2 so L_2 then e_3 so, like that, and now, I would like to compare if I have gone from directly from 1 to 3. So, the idea is that if I have done something in the stages, so, length is same, I have gone from L_1 to L_3 in one case, I have gone directly from L_1 to L_3 in another case, I am going from L_1 L_2 to first and then L_2 to L_3.

So, the paths are different. So, we want to see whether it is a path dependent or not. So, if I directly go from 1 to 3, then I will be calling is $\frac{\Delta L_{13}}{L_1}$ is $\frac{L_3-L_1}{L_1}$

Now I will be taking L_1. So, now I want to see whether these two are equal. So, let us see $e_{1\text{-}2}$ + $e_{2\text{-}3}$ look similar to that. So this is $\frac{\Delta L_{12}}{L_1} + \frac{\Delta L_{23}}{L_2}$.

If you also want to simplify this in terms of $L_2 - L_1$ and so on let's say we do that also.

$$e_{1\text{-}2} + e_{2\text{-}3} = \frac{L_2-L_1}{L_1} + \frac{L_3-L_2}{L_2}$$

So, if I divide it let us see if it still looks same

$$e_{1\text{-}2} + e_{2\text{-}3} = \frac{L_2}{L_1} - 1 - 1 + \frac{L_3}{L_2}$$

whereas this is $$e_{1\text{-}3} = \frac{L_3-L_1}{L_1}.$$

So, it it is very clear to me that this expression does not look similar to this expression. So, when you are doing engineering strain calculation for this kind of incremental strain, then it is a path dependent kind of calculation. It will not say that in these two cases where you are going from 1 to 3 directly or you are going from one in stages 1 to 2 and then 2 to 3, both calculations are not same.

Let's do the same calculation using the true strain idea. So, I will be writing again that is

$$\varepsilon_{1-2} = ln\left(\frac{L_2}{L_1}\right)$$

Similarly, $\varepsilon_{2-3} = ln\left(\frac{L_3}{L_2}\right)$.

And if I want to go directly from 1 to 3, then it will be

$\varepsilon_{1-3} = ln\left(\frac{L_3}{L_1}\right)$.

Now, let's see whether these two expressions look similar. So, if I now add

$$\varepsilon_{1-2} + \varepsilon_{2-3} = ln\left(\frac{L_2}{L_1}\right) + ln\left(\frac{L_3}{L_2}\right).$$

and it becomes a $ln(L_2)$- $ln(L_1)$+ $ln(L_3)$.

I'm just expanding this using logarithmic relationships.

So, you can see that this and this will cancel out, this is negative. So, when I am writing in ratio terms it will go in the denominator. So, it becomes

$$\varepsilon_{1-2} + \varepsilon_{2-3} = ln\left(\frac{L_3}{L_1}\right)$$

So, now you can see that this edition looks similar to this. Now, this is a very important concept for a metallurgical engineer because he does all this deformation in stages. So, if he does calculation using engineering strain he will get every time he will get a different strain values.

Or if he misses any particular stage then the values will be continuously will be different from each other. Whereas, if he does the calculation using true strain, if he find out the strain in each stage and do the calculation or if he finds out the strain from only the initial stage and the final stage both the strain will this will be same. So, for a metallurgical engineer he will always use true strain concept to do calculation.

Whereas the mechanical engineer can use engineering strain or engineering stress concepts because they are only dealing with the elastic part of the deformation, they are not concerned with the plastic deformation that is only in the view of a metallurgical engineer because they want to do deformation. So, I hope that with this lecture is the ideas of stress and strain engineering stress and engineering strain, true stress and true strain will be clear to you and the relationship between each one of them and why we want to do the calculation using true strain concepts. Thank you.