Thermo-Mechanical and Thermo-Chemical Processes
Prof. Vivek Pancholi
Department of Metallurgy and Materials Engineering
Indian Institute of Technology-Roorkee

Lecture-18
Higher Strain Rates: Hot Working

Hello friends. So, continuing with our constitutive analysis discussion, in present lecture we will be covering the relatively higher strain rates where we do actually hot working. And actually, we have missed the part where we can cover super plasticity. But I am not doing that right now. So, we have covered very low strain rate where creep is important. And then next one in high temperature deformation, the strain rate which comes between let us say 10^{-1} or10^{-2}. There you can see super plasticity in material and then we can come to the hot working processes, the conventional one like rolling, forging and extrusion from 10^{-1} to at least 10^{2} and even strain rate more than 10^{2}.

So, we will see two models here right now, which is still going to come under phenomenological model, which covers relatively higher strain rate more than 10^{2} and the strain rate where you do the hot working. So, that analysis will be more.

So, slightly the sequence is a little bit different. We have done very slow strain rate now, we are doing relatively very high strain rate 10^{2} and more and then we will be discussing about the hot working which is the main focus of our discussion.

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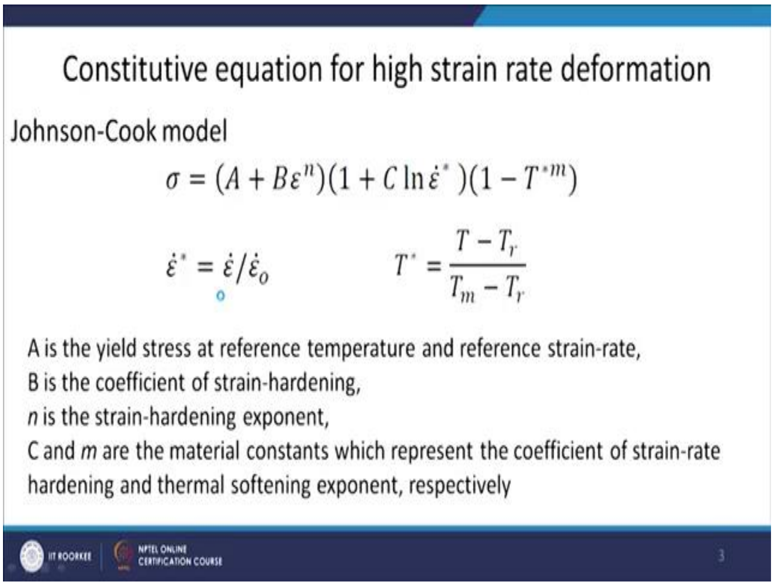

Constitutive equation for high strain rate deformation

Johnson-Cook model

$$\sigma = (A + B\varepsilon^{n})(1 + C\ln\dot{\varepsilon}^{*})(1 - T^{*m})$$

$$\dot{\varepsilon}^{*} = \dot{\varepsilon}/\dot{\varepsilon}_{0} \qquad T^{*} = \frac{T - T_{r}}{T_{m} - T_{r}}$$

A is the yield stress at reference temperature and reference strain-rate,
B is the coefficient of strain-hardening,
n is the strain-hardening exponent,
C and *m* are the material constants which represent the coefficient of strain-rate hardening and thermal softening exponent, respectively

So, again phenomenological based constitutive equations and so the constitutive equation for high strain rate deformation. So, as I was telling you that in this case my strain rate will be more than 10^2. One of the most important or used model is what we call as Johnson-Cook model , this model takes care of all the three parameters basically, because here again you can see that we are working on very high strain rates. So, there is not enough time for dislocation to recover. So, the strain hardening is a important consequence of that, that is strain hardening will be there in this type of deformation, very high strain rate deformation. And this Johnson-Cook model is the one of the most used models in that segment. And I am only discussing a couple of models in each of this category. Actually, if you go to any review paper or something like that book, then you will see there are large number of models with slight variation.

Or sometime the more famous models are modified for particular material or particular condition. So, this is not an exhaustive list you have very large number of models in each of these categories. I am just covering maybe 1 or 2 just to give you a flavour of the whole idea of constitutive equation development. So, if you see here (refer to above figure) the stress is dependent on the all the three parameters. So, you have a strain, strain hardening exponent and then you have a strain rate. And then you have a effect of temperature. Strain rate it is in the form of a strain rate some star, which is basically you are normalising it with the with some reference strain rate. Similarly, temperature is being normalised here. And it is used as T^*. So, the other parameters are A and B as A is the yield stress at reference temperature and reference strain rate and B is the coefficient of strain hardening and n is a strain hardening exponent, c and m are the material constant which represents the coefficient of the strain rate, strain rate hardening and thermal softening respectively. So, one of the important ideas which you should understand here is that when we do any high strain rate deformation whether in hot working or in this case even higher strain rate as more than 10^2, there is always a temperature rise in the material.

Actually, you understand that when we are working on the material, when we are deforming the material we are inputting energy into the material. So, some energy which you are inputting goes in the deformation process that means plastic deformation and some energy goes in form of heat. So, there will be heat generation whenever you do a deformation that you can also experience when you are for example, you take a metallic wire and you do repeated deformation and you will if you touch the material after some time it will be sufficiently hot means you can easily feel it.

So, the heating will always take space when you are working on the material. Some energy will go in of course, in the deformation process, but most of the energy goes in heating the material.

When you are doing at a slower strain rate, this heat you are giving have enough time to dissipate. So, there would not be any temperature rise in the material as such, but if you are doing a deformation at very high strain rate, then you are not giving enough time for this heat to dissipate and that is what we call as from your thermodynamics, you must be knowing that is what we call as adiabatic heating or adiabatic temperature rise because you are not allowing heat to dissipate. So, you have created an adiabatic condition. So, at very high strain rate when you are deforming because you are not allowing heat to dissipate temperature will rise above the temperature of deformation.

So, for example, you have set the temperature of deformation as 900°Cand you are doing a very high strain rate deformation, what will happen the temperature will lie rise to 950°C, I am just taking an example. Now, this 50°C rise you should understand that temperature will always come in the exponential term. So, stress is dependent exponentially with temperature. So, linear rise in temperature which will give an exponential drop in the stress.

So, when you develop the constitutive equation, you will take temperature as 900°C, because you are thinking that you have done at that your furnace was set for that, but actually the material is deforming at 950°C because of adiabatic heating. So, you have to compensate for this temperature. So, basically what you have to do is, you have to change the stress level to compensate for this temperature rise.

As I was telling you that there is exponential dependence between temperature and stress. So, basically you if you plot logσ vs $\frac{1}{T}$ then you will be able to know that what is the dependence of stress on the temperature. So, any rise in the temperature you will be able to compensate by changing the stress. So, basically the you have to bring down the temperature from whatever your measured is for 950.

So, you have to bring down that stress for 900 that means, in a stress strain curve if this (refer to above figure) is my stress which I have measured after the compensation the stress will be obviously higher than that because the temperature should have been lower than that. So, this type of temperature compensation in stress you have to give when you are doing a high strain rate deformation.

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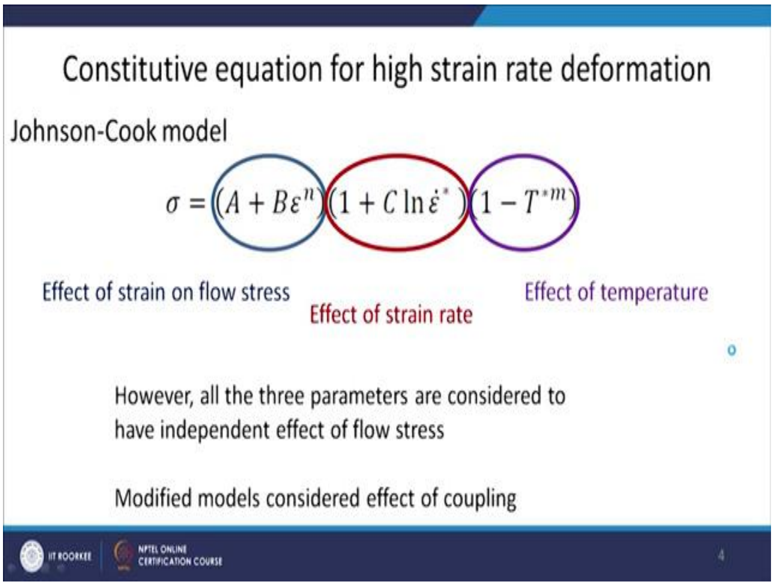

Constitutive equation for high strain rate deformation

Johnson-Cook model

$$\sigma = (A + B\varepsilon^{n})(1 + C\ln\dot{\varepsilon}^{*})(1 - T^{*m})$$

Effect of strain on flow stress

Effect of strain rate

Effect of temperature

However, all the three parameters are considered to have independent effect of flow stress

Modified models considered effect of coupling

So, basically Johnson-Cook model if you see (refer to above figure), it contains three terms. So, the first term brings out the effect of strain on flow stress, you have a strain term with this strain hardening coefficient, the second term brings out the effect of strain rate and the third term bring out the effect of temperature on the flow stress. So, all the important parameters are considered in the flow stress term.

However, in this Johnson-Cook model, the parameters are considered to have an independent effect that means, independently change in strain rate is not not going to influence the temperature and strain. Similarly, independently changing temperature is not going to affect the strain and strain rate and so on.

There is not going to be any effect of strain and strain rate, which is usually may not be the case because there is an interdependence of all this parameter and they sometime kind of have a combined effect on the stress. So, but Johnson-Cook model was a very simple model. And that is why it is very popular also because you can easily use this model to make a constitutive equation.

So, for a rough kind of work, you can easily use it very quickly. However, some modifications proposed by other researchers on this model to take care of this coupling effect.

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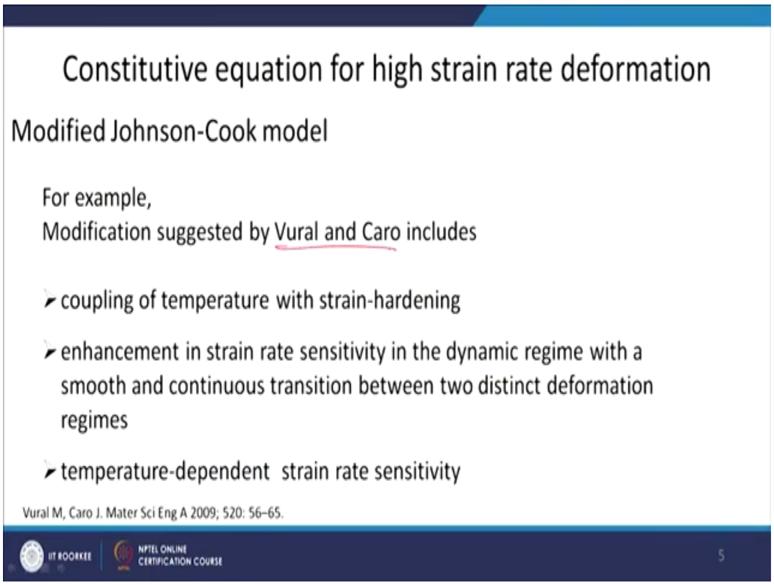

Constitutive equation for high strain rate deformation

Modified Johnson-Cook model

For example,
Modification suggested by Vural and Caro includes

- coupling of temperature with strain-hardening
- enhancement in strain rate sensitivity in the dynamic regime with a smooth and continuous transition between two distinct deformation regimes
- temperature-dependent strain rate sensitivity

Vural M, Caro J. Mater Sci Eng A 2009; 520: 56–65.

One of the models, I am just discussing it here, which was proposed by these two gentlemen Vural and Caro, they have modified this Johnson-Cook model, they have suggested some modification. So, they have coupled temperature with strain hardening. So, that they have proposed that it should be the temperature and strain hardening should be coupled because when you increase the temperature there is always going to be effect on the strain hardening exponent. So, these are not independent.

Enhancement in strain rate sensitivity in the dynamic regime with a smooth and continuous transition between two distinct deformation regimes. So, strain rate sensitivity is also going to be affected by the dynamic regime and there will be a temperature dependent strain rate sensitivity also. So, you should understand that these are all parameters are all coupled to each other. So, we cannot treat them independently and there are lot of modifications proposed by different researchers on Johnson-Cook model.

But, for a quick development, this is a very good model if you are working in a high strain rate regime i.e. more than 10^2 to use a Johnson-Cook model or any model, any deformation process which is working at that particular strain rate. So, this is for high strain rate deformation.

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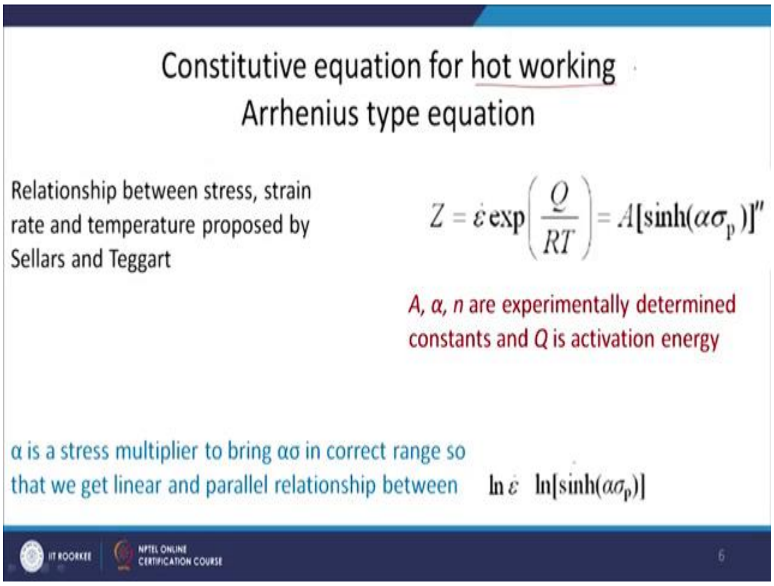

Constitutive equation for hot working
Arrhenius type equation

Relationship between stress, strain rate and temperature proposed by Sellars and Teggart

$$Z = \dot{\varepsilon}\exp\left(\frac{Q}{RT}\right) = A[\sinh(\alpha\sigma_p)]^n$$

A, *α*, *n* are experimentally determined constants and *Q* is activation energy

α is a stress multiplier to bring ασ in correct range so that we get linear and parallel relationship between $\ln\dot{\varepsilon}$ $\ln[\sinh(\alpha\sigma_p)]$

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Now, we are going to discuss the constitutive equation for hot working. So, now, again my strain rate range is let’s say from 10^{-1} to 10^{2} a strain rate. So, in this regime, we are looking at the constitutive equation and again you can relate it with what we were discussing during phenomenological constitutive equation that these phenomenological constitutive equations are developed for different temperature and strain rate regime.

So, you can also now see (refer to above figure) very clearly that for different strain rate windows we are developing different type of constitutive questions. One window we have developed for creep kind of deformation, one we have already developed now for very high strain rate more than 10^{2}. So, for each regime you have to develop different set of equations and in that regime also there are multiple models. So, you should not consider that there is only one model there are multiple models to relate the stress with these 3 parameters.

So, in this particular regime, we are going to discuss this Arrhenius type equation (refer to above figure) which relates the stress as a function of temperature with exponential term. So, basically again we are developing relationship between stress, strain rate and temperature and this particular equation was proposed by these two scientists Sellars and Teggart.

So, you have Z, again Zener-Hollomon parameter which brings out these two parameter together i.e. strain rate and temperature, our temperature will always come under exponential term and they relate this Z with the stress. We are taking here P, I am using it for saying peak stress. So, when you are deforming the material, if you remember if you have a single peak this is my peak stress σ_p. So, you can use some time steady state stress or peak stress.

So, here we are using peak stress to find out the constitutive equation. So, this stress is actually related with Zener Hollomon parameter here by this sine hyperbolic equation (refer to above figure). So, this is a very general equation to relate the stress with the Zener Hollomon parameter and of course, there are a couple of constants here α, A and stress exponent n. So, A, α, and n are experimentally determined constant.

And also, Q is also going to be determined experimentally only, it is activation energy. α is because you can see it is multiplied with the stress it is called stress multiplier and what it does is it brings $\alpha\sigma_p$ in correct range, so, that we get linear and parallel relationship between ln$\dot{\varepsilon}$ and ln[sinh($\alpha\sigma_p$).

So, basically α we used to bring the whatever plots we are going to get they should be all linear first and they should be all parallel to each other that means, they should have similar deformation behaviour. So, for that it is kind of adjustment parameter you can say which is we are multiplying with σ_p to bring these two curves linear and parallel.

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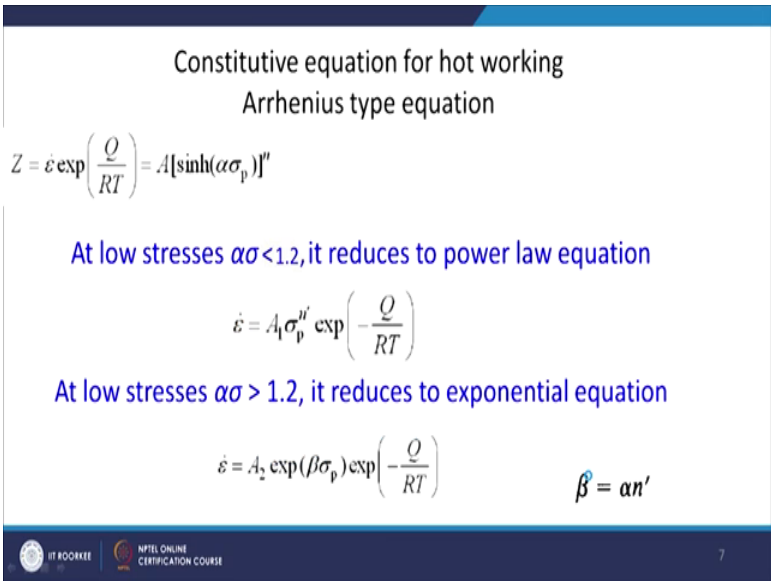

Constitutive equation for hot working
Arrhenius type equation

$$Z = \dot{\varepsilon}\exp\left(\frac{Q}{RT}\right) = A[\sinh(\alpha\sigma_p)]^n$$

At low stresses $\alpha\sigma < 1.2$, it reduces to power law equation

$$\dot{\varepsilon} = A_1\sigma_p^{n'}\exp\left(-\frac{Q}{RT}\right)$$

At low stresses $\alpha\sigma > 1.2$, it reduces to exponential equation

$$\dot{\varepsilon} = A_2\exp(\beta\sigma_p)\exp\left(-\frac{Q}{RT}\right)$$

$\beta = \alpha n'$

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Now, these Arrhenius type of equation this is general equation. So, I would call this hyperbolic equation as a general equation for a very large strain rate range. Of course, when you are talking about strain rate it is ultimately stress is also going to be related to that. So, at low stress this same equation can be reduced to a power law equation, already we have seen this type of equation a power law equation where $\dot{\varepsilon}$ and σ_p is related with some exponent n prime here and again temperature is in exponential term. So, this is one type of relationship where the $\dot{\varepsilon}$ and σ_p are related with some exponent here and these are called power law equation.

So, so at lower stresses you will have this power law equation. And incidentally this type of equation is also valid for super plasticity. So, you have a power law equation as you go to higher strain rate and basically higher stresses. So, where $\alpha\sigma > 1.2$, then this generalise equation can be reduced to an exponential equation which is given by equation like this (refer to above figure).

There is no exponent here now, but there is a multiplication of new very new constant which is called β. So, $\beta\sigma_p$ is there, but the σ_p instead of power now has come in the exponential term. So, these are some subtle changes in the way we are writing constitutive equation for different strain rate regime. So, sine hyperbolic is a generalised equation and these are equation for a specific domain.

So, for low strain rate, lower stress condition you will have power law equation, but there is a power law breakdown as you increase the strain rate. So, at higher stresses you have this exponential dependence of strain rate on the on your stress. So, this the multiplication here β is actually again the same α the stress multiplier is there and the n` exponent which is in the power law equation. So, you have to multiply these two to get β. So, these are the different variation of the constitutive equation in different strain rate regime (refer to above figure).

So, now we will use all these particular equations to calculate our all the parameters of our constitutive equation. So, you can see in this particular constitutive equation you have Q which you want to know, there is n which you want to know, there is α which you want to know. So, these parameters are required for development of the constitutive equation. So, calculation so, first we will start with the calculation of n` which is shown here (refer to below figure).

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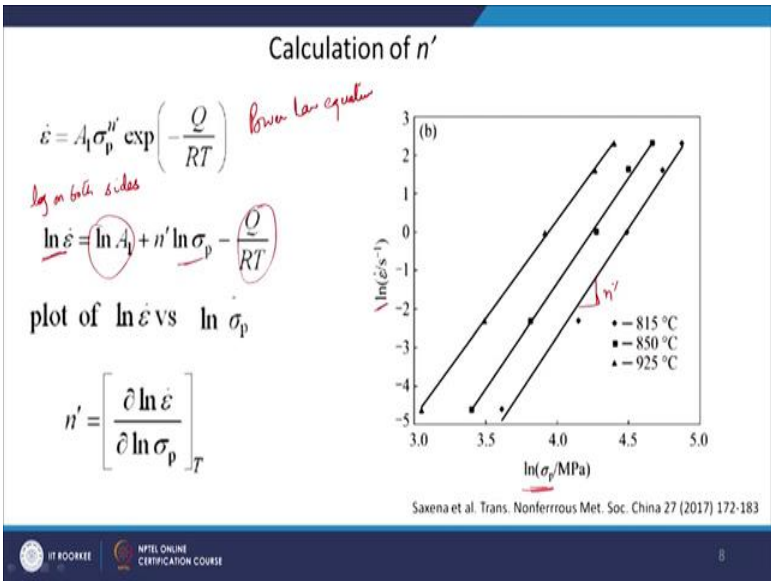

Calculation of n'

$$\dot{\varepsilon} = A_1\sigma_p^{n'}\exp\left(-\frac{Q}{RT}\right)$$ Power law equation

log on both sides

$$\ln\dot{\varepsilon} = \ln A_1 + n'\ln\sigma_p - \frac{Q}{RT}$$

plot of $\ln\dot{\varepsilon}$ vs $\ln\dot{\sigma}_p$

$$n' = \left[\frac{\partial\ln\dot{\varepsilon}}{\partial\ln\sigma_p}\right]_T$$

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So, calculation of n prime equation is this which is basically our power law equation for lower strain rate and lower stress condition. So, $\dot{\varepsilon}$ = A$_1\sigma_p^{\acute{n}}$ exp$\left(-\frac{Q}{RT}\right)$.

Again, we will do the same thing, if I take log on both sides then
ln$\dot{\varepsilon}$ = lnA$_1$ + $\acute{n}$lnσ_p- (Q/RT)
so, now if I plot a curve between ln$\dot{\varepsilon}$ and lnσ_p. So, that is what we have plotted here in this figure (refer to above figure) you have plotted ln$\dot{\varepsilon}$ and lnσ_p, then the slope of this, if I differentiate this equation the constant terms will go become zero and then n` will be equal to $\frac{\partial \ln\dot{\varepsilon}}{\partial \ln\sigma_p}$.

So, basically it is this slope of this curve. So, these three curves are at for these three different temperatures so, basically how we have got this these data, so 815°C you have deformed the material different strain rates, from there you have got this different data points, then again 850°C we have deformed for different strain rates, so again different data point and so on.

So, for each strain rate what was the stress that is measured. You take logarithmic of strain rate and logarithmic of stress.
That will give you these three straight lines more or less these are parallel that means there is no change in the deformation behaviour at these three temperatures. So, I can take the slope of this particular curves as n`. So, this is how I am going to get the n` for my constitutive equation.
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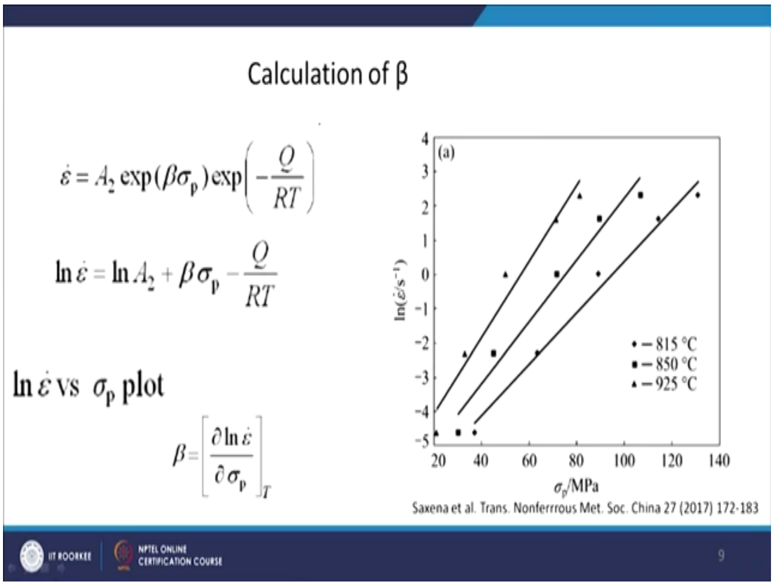

Calculation of β

$$\dot{\varepsilon} = A_2 \exp(\beta\sigma_p)\exp\left(-\frac{Q}{RT}\right)$$

$$\ln\dot{\varepsilon} = \ln A_2 + \beta\sigma_p - \frac{Q}{RT}$$

$\ln\dot{\varepsilon}$ vs σ_p plot

$$\beta = \left[\frac{\partial \ln\dot{\varepsilon}}{\partial \sigma_p}\right]_T$$

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Now, the next parameter I want to know is β. So, β is what we saw in the other one, the exponential equation which was valid for higher strain rate or higher stress condition Aϭ > 1.2. So, you have this β term here and β as I told you earlier also

$$\beta = \alpha ή.$$

So, n` already we have calculated now, we want to know β here.

So, again for from this (refer to above figure) equation, again take ln of this equation so, it will

$$\ln\dot{\varepsilon} = \ln A_2 + \beta\sigma_p - \frac{Q}{RT}$$

So, again we will take the constant temperature here. If I differentiate this equation then β will be $\frac{\partial \ln\dot{\varepsilon}}{\partial \sigma_p}$.

In ή it was $\partial \ln\sigma_p$.

So, now it is $\partial\sigma_p$, so, what we have plotted now σ_p on the x-axis $\ln\dot{\varepsilon}$ on y-axis again you can see.

You have got some straight lines; a straight-line fit is there again at three different temperature for different strain rates and from the slope of this you will be able to get β. Now, we will do the calculation of n.

So, already you can see that I have got the α parameter because I know β, I know n`. So, α will be simply equal to $\frac{\beta}{ή}$.

So, α is automatically calculated once I know the β and n` these two parameters.

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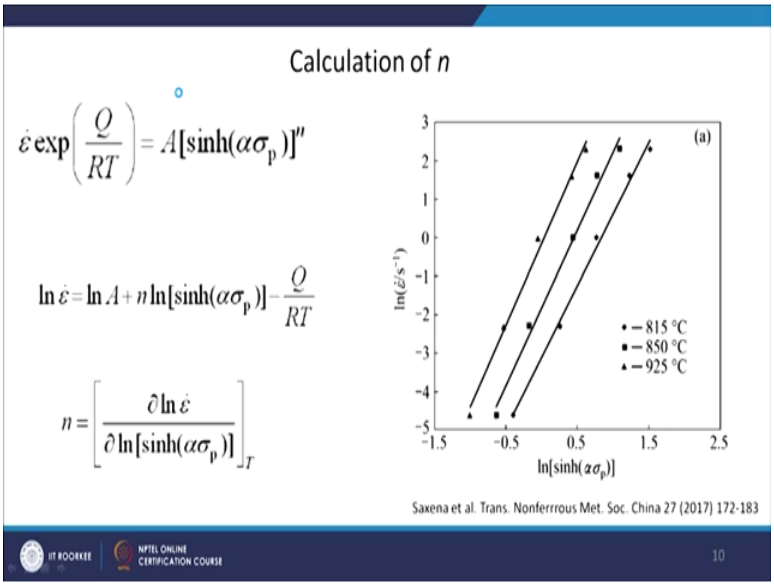

Calculation of *n*

$$\dot{\varepsilon}\exp\left(\frac{Q}{RT}\right) = A[\sinh(\alpha\sigma_p)]^n$$

$$\ln\dot{\varepsilon} = \ln A + n\ln[\sinh(\alpha\sigma_p)] - \frac{Q}{RT}$$

$$n = \left[\frac{\partial \ln\dot{\varepsilon}}{\partial \ln[\sinh(\alpha\sigma_p)]}\right]_T$$

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So, now I want to know what is the calculation for n. So, already we have seen this (refer to above figure) equation the sin hyperbolic equation

$$\dot{\varepsilon}exp\left(\frac{Q}{RT}\right) = A\left[sinh\left(\alpha\sigma_p\right)\right]^n$$

So, here it is n, in the power law equation it was n` so that is the difference. Again, we will take the ln on both the side so

$$\ln\dot{\varepsilon} = \ln A + \text{n}\ln\left[sinh\left(\alpha\sigma_p\right)\right] - \frac{Q}{RT}$$

So, now, I can differentiate this equation for constant tempreature and n will be $\frac{\partial\ln\dot{\varepsilon}}{\partial\ln\left[sinh\left(\alpha\sigma_p\right)\right]}$.

So, now, I am going to plot ln $\left[sinh\left(\alpha\sigma_p\right)\right]$ on the x-axis and ln$\dot{\varepsilon}$ on the y-axis.

And now you can see that we are again doing it at constant temperature for different strain rates. And these are also linear curves obviously and also parallel that is what was the purpose of using α to make these graphs as parallel and linear.

So, from the slope of these curves now, you will be able to find out what is the stress exponent n. So, this is how actually you will do calculations that is why I am doing the whole calculation in front of you. So, that once you have data you can do all this calculation yourself. All these things can be easily done on any spreadsheet whether you want to use excel, origin or any other software does not matter, you can use it very nicely. You just have to take logarithmic and taking sine hyperbolic multiplying by something and so on.

So, once you have the plot you can find out the slope very easily by fitting a trend line and finding the equation of that straight line. So, slope will give you the value of n. So, all these things you can do very easily using any software. Now, we want to know that what is the activation energy.

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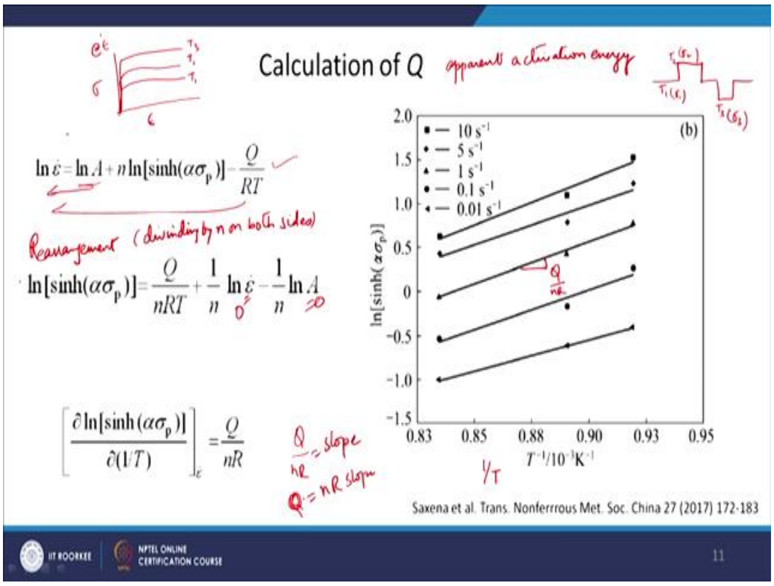

Calculation of Q

apparent activation energy

$$\ln \dot{\varepsilon} = \ln A + n \ln[\sinh(\alpha\sigma_p)] - \frac{Q}{RT}$$

Rearrangement (dividing by n on both sides)

$$\ln[\sinh(\alpha\sigma_p)] = \frac{Q}{nRT} + \frac{1}{n}\ln\dot{\varepsilon} - \frac{1}{n}\ln A$$

$$\left[\frac{\partial \ln[\sinh(\alpha\sigma_p)]}{\partial(1/T)}\right]_{\dot{\varepsilon}} = \frac{Q}{nR}$$

$$\frac{Q}{nR} = \text{slope}$$

$$Q = nR\ \text{slope}$$

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So, for calculation of activation energy our temperature will change and strain rate will remain same. Also please note down, I forgot to mention it earlier that these are all apparent activation energy. One activation energy you also get from the diffusion data, those are true activation energy. These are apparent activation energy the reason is because the microstructural is continuously changing during the deformation and these are not activation energy calculated from one specific mechanism. We are trying to predict a mechanism using the activation energy. So, the microstructure is not going to be a stable microstructure or for example, you are deforming at different temperatures here. So, now, if you are deforming at different temperatures earlier also if you remember we were seeing in dynamic recrystallization that as you change the temperature the flow curve changes from a single peak where it shows discontinuous dynamic recrystallization to multiple oscillating peak kind of stress strain curve.

Also if you remember we were seeing some flow stress curve were at low temperature it is showing a strain hardening then at some intermediate temperature it shows dynamic recovery then even at higher temperature it shows dynamic recrystallization, steady state sometime softening and so on. So, the idea is that the microstructure is continuously changing it is dynamic.

So, when we are calculating the activation energy from the data of different temperatures the activation energy which you are getting is going to be an apparent activation energy. If you want to calculate more accurate activation energy what you should do is when you are deforming at a strain rate let us say you are deforming it 0.1 strain rate. So, what do you do during the deformation itself you change the temperature.

And because then the microstructure would not have been different so much in during a very short duration. So, basically you are deforming let us say at one temperature you change the temperature, deform it, bring it to again the same temperature maybe go below and so on. So, by doing that, you are actually trying to have the same microstructure and in the same microstructure you are changing the temperature.

And for suppose this is T_1, this is T_2, this is T_3 you are you will get corresponding stresses. So, then you will be able to say with more confidence that this is my activation energy of deformation. The way we are calculating is we are not doing it like this. We are doing test at different temperature this T_1, T_2, T_3. So, T_1 is more than T_2 is more than T_3. And suppose we have done this all the at a constant strain rate.

So, this is what we are doing now. So, again taking the previous equation where we took the logarithmic. So, this particular equation again we are using it here (refer to above figure). So,

$$\ln\dot{\varepsilon} = \ln A + n\ln\left[sinh\left(\alpha\sigma_p\right)\right] - \frac{Q}{RT}.$$

And again, you are doing some rearrangement here. So, this will become

$$\ln\left[sinh\left(\alpha\sigma_p\right)\right] = \frac{Q}{nRT} + \frac{1}{n}\ln\dot{\varepsilon} - \frac{1}{n}\ln A$$

Slight rearrangement of the earlier equation and dividing by n on both sides you will get this equation and again we will do a differentiation.

Now, this is at a constant strain rate. After differentiation we will get,

$$\frac{\partial\ln\left[sinh\left(\alpha\sigma_p\right)\right]}{\partial\frac{1}{T}} = \frac{Q}{nR}$$

So, for this what we will do, we will do a plot between $\frac{1}{T}$ is shown here (refer to above figure) as T inverse or T^{-1} and $\ln\left[sinh\left(\alpha\sigma_p\right)\right]$.

So, if I do that, I will get again these curves more or less all the curves are linear and parallel to each other (refer to above figure). So, I would say that there is not much change in the deformation mechanism at different temperatures. So, now from the slope of this curve, this slope is equal to Q/nR. So, to get the activation energy I will multiply nR with the slope.

So, I multiply the slope with n×R, n already we have calculated earlier, R is your universal gas constant then I would be able to get the activation energy for deformation. Usually activation energy for deformation which you get through these kinds of processes comes very high compared to any known diffusion processes or solute diffusion processes because the dislocation movement also require certain amount of activation energy.

So, all those things get combined and it is sometimes very difficult to assign any physical meaning to this kind of activation energy calculation sometimes it is difficult. So, with this we have completed the calculations of different parameters which we are going to use for development of the constitutive equation. So, the example we have taken for constitutive equation is the Arrhenius type of constitutive equation.

So, whatever parameters are going to be required in that, that we have seen that how we are going to do the calculation, so once you know that, you can do it for some other models also, which you think is suitable for your material. And that will be able to give you a constitutive equation which you can use for doing maybe if you're doing any modelling work, FEM modelling or any other modelling, for your deformation process. With that I thank you.

Key words- Constitutive equation, High strain rate, Arrhenius equation.