Thermo-Mechanical and Thermo-Chemical Processes

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Lecture-17
Constitutive Analysis: Low Strain Rate

Hello friends so we have started with the constitutive analysis of high temperature deformation. In that today I will first start with the low strain rate or the constitutive equations which were developed for low strain rate or as we have just seen in the previous lecture where the creep kind of deformation takes place. Why I am taking this one is because this for high temperature deformation the development of constitutive equation actually started with the creep kind of deformation first.

The reason for that is for the high temperature or the hot working which we do in the industry initially the equipments were not there to do that kind of testing in the machine, the servo-hydraulic machines or the other machines were not to capable to give that kind of strain rate to the sample. So, only the data which was being collected is from the actual industrial processes where you cannot get the stress as a function of strain rate and temperature and so on.

So best you can could do at that time with the industrial process hot working is that you deform the material at different temperature, applying different strain rate and look at the microstructure but you could not develop the constitutive analysis for hot deformation at that strain rate range which is from 10^{-1} to 10^{2}.

But the creep kind of deformation was much easier because it is under constant load or constant stress condition. So, you keep a sample in the furnace and you apply some certain amount of load or stress on that and as a function of time you can look at how the deformation is progressing or how strain is accumulating in the material. So, initial work on the constitutive analysis especially for hot deformation, for high-temperature condition took place in the creep kind of deformation. That is why I am taking that one as the first one to understand the constitutive equation in the creep kind of condition.

And then we will come to the high strain rate conditions. So, I am taking the phenomenological based constitutive equations here. So, all this creep kind of deformation was done using this phenomenological approach.

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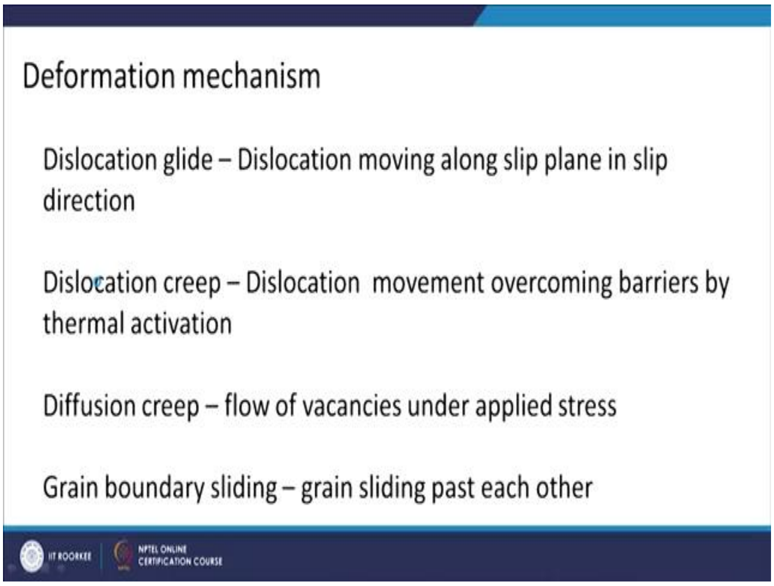

Deformation mechanism

Dislocation glide – Dislocation moving along slip plane in slip direction

Dislocation creep – Dislocation movement overcoming barriers by thermal activation

Diffusion creep – flow of vacancies under applied stress

Grain boundary sliding – grain sliding past each other

These are the typical deformation mechanism which takes place during the high-temperature deformation (refer to above figure). So, usually what we get at room temperature or low temperature is through dislocation glide, that the dislocation generation will takes place then the dislocation will glide on the slip plane in the slip direction. So, dislocation glide mechanism, this is one of the mechanisms always will be there.

Then there can be dislocation creep. In this now the dislocation movement is also added by the thermal activation. So, in this what happens is that your additional slip systems become sometime active and if there are any barriers to dislocation movement that can be overcome with the thermal activation for example dislocation climb process. So, there are different type of thermally activated process which can help the dislocation to move.

So, dislocation Glide is also helped through the thermal activation. Because you are at higher temperature, diffusion-based deformation also can take place. This is what we call as diffusion creep. So, flow of vacancies takes place under the applied stress. So, this is another very important mechanism at high temperature. Another very important mechanism you will see is called grain boundary sliding. See in dislocation mechanism what happens is the deformation is taking place within the grain.

So if you are deforming a grain by dislocation mechanism or diffusion mechanism it will get elongated in the direction of the principal strain. Whereas in grain boundary sliding the grain you can consider as a rigid body. So, there are two rigid bodies and the grains are sliding passed each other, so there is a sliding between the grain. So, deformation is not within the grain, deformation is at the grain boundary and grains are sliding passed each other. A very interesting mechanism at high temperature because diffusion will be very high grain boundaries are weaker compared to grain interiors. So, you can have this kind of grain boundary sliding. In super plasticity it is one of the prominent mechanisms of deformation. So, you can see this in super plasticity. Usually at lower stresses you will see diffusion creep whereas dislocation creep you will see at higher stresses.

So, here also there is kind of a demarcation where you will see dislocation creep and where you will see the diffusion creep (refer to above figure). The diffusion creep means it happens at very low stresses. So, the creep will be through diffusion mechanism. There is no dislocation movement at those low stresses.

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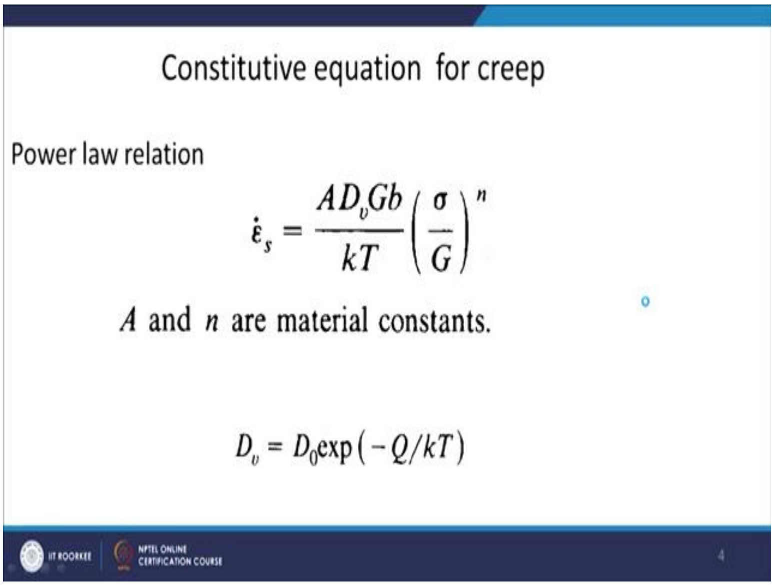

Constitutive equation for creep

Power law relation

$$\dot{\varepsilon}_s = \frac{A D_v G b}{kT}\left(\frac{\sigma}{G}\right)^n$$

A and n are material constants.

$$D_v = D_0 \exp(-Q/kT)$$

So, if you want to see a constitutive equation for creep this is a power law kind of relationship (refer to above figure). So, now you can see that when we talk about creep we do not relate stress with strain rate, temperature and so on. We relate a strain rate with the stress because in this our control variable is the stress, the load which we are applying on the material and looking at the strain as a function of time.

So, when we are looking at the strain as a function of time of course we are looking at the strain rate. So, under an applied constant load what is the strain rate of deformation. So, we are looking at the strain rate that is my dependent variable and this σ is my independent variable and these are called power law relationship because you have an exponential power here over whatever is your input variable, σ here in this case.

So, you can see that there are some material constants here for example A and n are material constants D is your diffusion coefficient which is basically D$_0$exp$\left(\frac{-Q}{KT}\right)$, Q is your activation energy for diffusion, G is your shear modulus, b is of course your burgers vector, K is your Boltzmann constant and T is of course temperature in Kelvin.

And the stress is actually as you can see is normalized with shear modulus here. So, whenever you see constitutive equation this you will see a lot that we are trying to normalize the parameter with some material constant here in this case we are normalizing it with the shear modulus. So, this is what we call as a power law relationship where strain rate is dependent on stress with power n.

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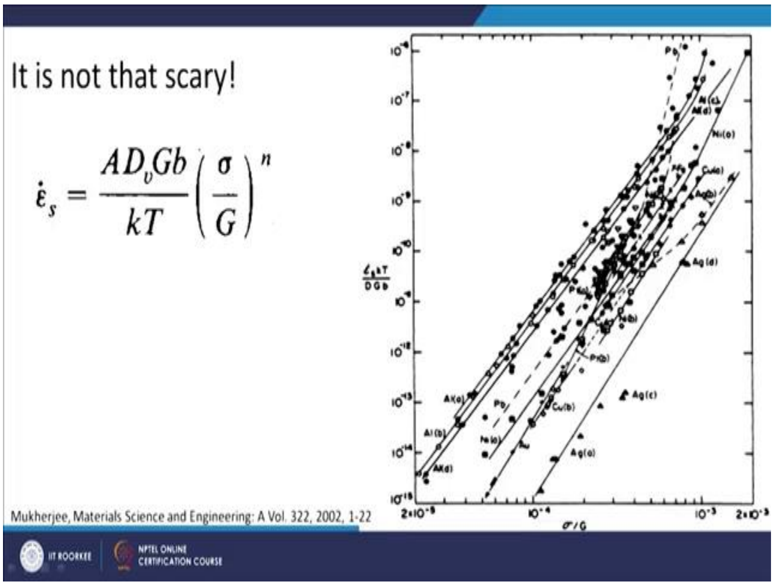

It is not that scary!

$$\dot{\varepsilon}_s = \frac{AD_vGb}{kT}\left(\frac{\sigma}{G}\right)^n$$

Mukherjee, Materials Science and Engineering: A Vol. 322, 2002, 1-22

Now how people have got this particular equation. So, this is where phenomenological comes in picture. So, this is your equation (refer to above figure). How I have got this particular equation? So, the idea is that you plot parameter. So, if you see on the y-axis the parameters are like this it is not visible clearly. So, I am just writing it here (refer to above figure). So, you are normalizing the stress with shear modulus and that you are plotting on x-axis of course it is also on in logarithmic scale because you have to take log on both side to make it linear. So, log if you take on both the side the n will come as a slope.

So instead of taking log actually if you see they have plotted on a log scale. That is why you do not see log on the parameter here and how they have found out this particular equation that should be interesting to see.

You are trying to find out that by what parameters I have to normalize the strain rate, what parameter to use the normalize the stress so that I can get a linear relationship. And then it is very easy for me to fit a straight line and get the equation. So, that is what is the regression in empirical analysis using phenomena that what type of parameters will be required to make it a linear relationship.

So you can see that how they are dividing by different parameters here and to get a linear relationship and by normalizing (refer to above figure). Another important thing which can takes place is that data from different material can be put on a single graph. So, to kind of demonstrate that this is a universal phenomenon. It is a general equation, not an equation for a particular type of material.

It can be applied to any material. Only difference will be that as material changes your n i.e. material constant will change, rest everything will remain same. So, it basically it will be linear mostly. And you can see here lot of different data is plotted so you have silver, nickel, lead, copper here.

So, lot of data is plotted on the single band of curve and through that by the slope I will be able to get the parameter n and from the I think the intercept I will be able to get the parameter A. So, this is how I would be able to get the constitutive equation for a creep kind of deformation.

Now what are the constitutive equation for diffusion creep. That was for dislocation creep there will be now some new parameter will be introduced.

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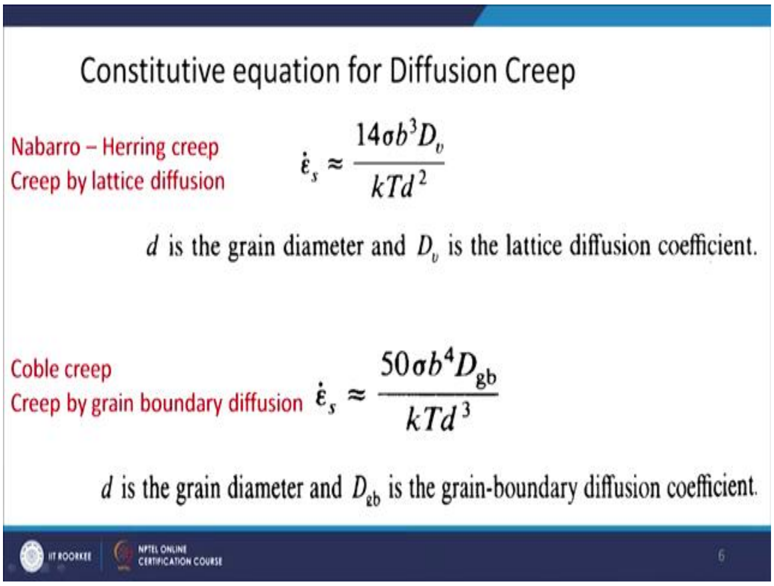

Constitutive equation for Diffusion Creep

Nabarro – Herring creep
Creep by lattice diffusion

$$\dot{\varepsilon}_s \approx \frac{14\sigma b^3 D_v}{kTd^2}$$

d is the grain diameter and D_v is the lattice diffusion coefficient.

Coble creep
Creep by grain boundary diffusion

$$\dot{\varepsilon}_s \approx \frac{50\sigma b^4 D_{gb}}{kTd^3}$$

d is the grain diameter and D_{gb} is the grain-boundary diffusion coefficient.

So, first one in this is called Nabarro-Herring creep which is creep by lattice diffusion. So, I will just give you a quick understanding of this suppose this is my grain (refer to above figure) these are the grain boundaries. Nabarro-Herring diffusion creep is at low stresses as I told you the dislocation will not be a mechanism here.

So, the creep will take place through diffusion mechanisms. So, suppose I am applying stress in this direction, this is a tensile stress (refer to above figure). So, what will happen is when I am applying tensile stress then you are creating more gap in this direction by stretching it. So, when you are creating more gap you are creating a driving force for atoms to diffuse from this direction toward this direction.

So, as you can understand that if I am applying the tensile stress in this direction there will be induced compressive stresses from the perpendicular direction. So basically, you are pushing the atoms to diffuse from this. So, the atoms are diffusing from this direction towards this direction. Similarly, it will be going like this in this direction (refer to above figure). And for atoms to diffuse here, there will be a reverse diffusion of vacancies.

So, vacancies are moving from tensile side to the compressive side and atoms are moving from compressive side to tensile side. And slowly you will see that the grain is getting elongated because you are removing material from this side and adding on this side. so, there will be continuous elongation of the grain because of this deformation.

So, now you can see some new parameter which is coming here which is you can see here D where D is your grain size grain diameter and D_v is of course your latest diffusion coefficient (refer to above figure). So, it because the diffusion is through lattice. Of course, it will be dependent on the lattice diffusion but there is also an effect of the grain size here which was not there in the previous equation. So, now you can see again that how we are trying to linearize the equation to get this empirical relationship. Because now we see that there will be an effect of grain size here. So, grain diameter is coming in the picture.

Another type of same in similar fashion through diffusion process is what we call as Coble creep. In this the creep takes place by grain boundary diffusion. So, in the earlier case it was through lattice and now if I see in this case now the diffusion is taking place through the grain boundaries. This grain boundary diffusion is going to take place.

So, the another type of creep which takes place in the through diffusion process is called Coble creep and this the creep takes place by grain boundary diffusion and you can see that if I plot a grain like this the diffusion will be taking place through the grain boundary here (refer to above figure). And because of that there will be an effect on the grain size. So, in Nabarro-Herring creep the grain size had a exponent of 2 whereas in this the grain size has an exponent of 3.

So, the Coble creep is more influenced by the grain size whereas in NH creep there is a less dependence on the grain size. So, as exponent is increasing you can understand that it will have more effect on the strain rate. In fact if you want to compare in the same material where the NH creep takes place and where the Coble creep takes place you should do an experiment where you are refining the grain size.

So you will see that for smaller grain size Coble creep is the important diffusion process because you have more grain boundary for diffusion to takes place whereas for the coarse grain for the same material NH creep will be dominant creep process where the diffusion will takes place through the lattice. So, there you can see very subtle changes which are taking place in the in the constitutive equation based on the what type of deformation mechanism which is operating in the material.

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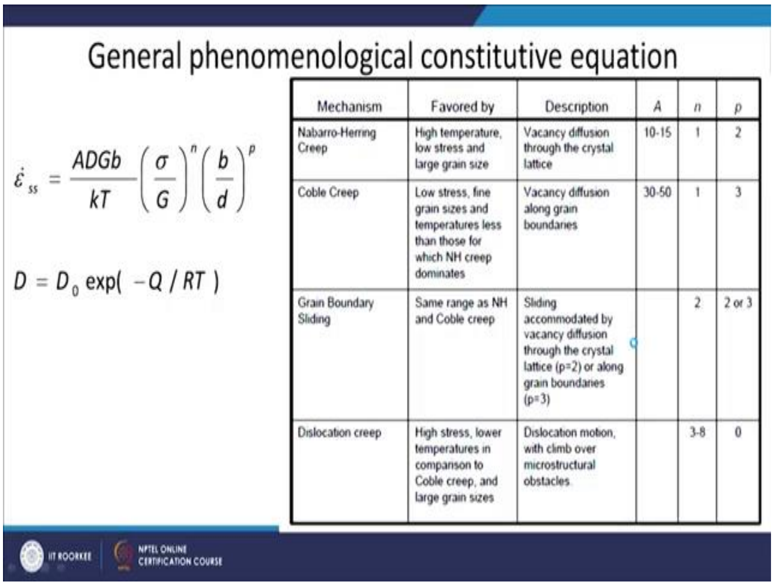

General phenomenological constitutive equation

$$\dot{\varepsilon}_{ss} = \frac{ADGb}{kT}\left(\frac{\sigma}{G}\right)^{n}\left(\frac{b}{d}\right)^{p}$$

$$D = D_0 \exp(-Q/RT)$$

Mechanism	Favored by	Description	A	n	p
Nabarro-Herring Creep	High temperature, low stress and large grain size	Vacancy diffusion through the crystal lattice	10-15	1	2
Coble Creep	Low stress, fine grain sizes and temperatures less than those for which NH creep dominates	Vacancy diffusion along grain boundaries	30-50	1	3
Grain Boundary Sliding	Same range as NH and Coble creep	Sliding accommodated by vacancy diffusion through the crystal lattice (p=2) or along grain boundaries (p=3)		2	2 or 3
Dislocation creep	High stress, lower temperatures in comparison to Coble creep, and large grain sizes	Dislocation motion, with climb over microstructural obstacles		3-8	0

So, in fact I can kind of summarize this in a one in a nutshell here (refer to above figure). So, instead of giving you different creep equation, I am giving you a generalized creep equation. So, you can see for NH creep it is favoured by high temperature, low stress and large grain size. At very low stresses and large grain size you will have NH creep we can say diffusion through crystal lattice, A will be in the range of 10 to 15, the exponent n is 1 that means it has already a linear relationship there and grain size exponent is 2 as we have just seen.

For Coble creep, again it is at low stress but in fine grain size and temperature is less than those for which NH creep dominates. So, it happens at lower temperature. Why it happens at lower temperature because grain boundary diffusion will be faster because grain boundaries are already having open structure so, my diffusion of atoms through grain boundary can takes place at lower temperature also.

Whereas diffusion through lattice require vacancies and vacancies you can have a higher concentration only at high temperature. So, Coble creep can dominate for fine-grain size as well as for lower temperature, vacancy diffusion through grain boundary, A is in 30 to 50 range, stress exponent is again 1 here whereas the grain size exponent is 3. Grain boundaries sliding can be in the same range as NH or Coble creep. Grain boundary sliding is accommodated by vacancy diffusion through crystal. So, again P is equal to 2 or 3 depends upon whether the diffusion is taking place through crystal lattice or along grain boundary but n in this case is 2. Dislocation creep will be taking place at higher stresses where now dislocation can start gliding under the applied stress, lower temperature in comparison to Coble creep and of course at larger grain size dislocation motion with climb over microstructural obstacles.

So, climb can takes place through high temperature activation or thermal activation, n in this range 3 to 8 a very large range so in each of these range also there will be subtle changes in the deformation processes, there is no grain size dependent. So, that is why you remember when I was discussing dislocation creep there was no term for grain size there.

Because it does not have any dependence on the grain size, on stress it has a very strong dependence and at the same time it has a big range. So, depending upon what type of deformation mechanism is taking place your n will change.

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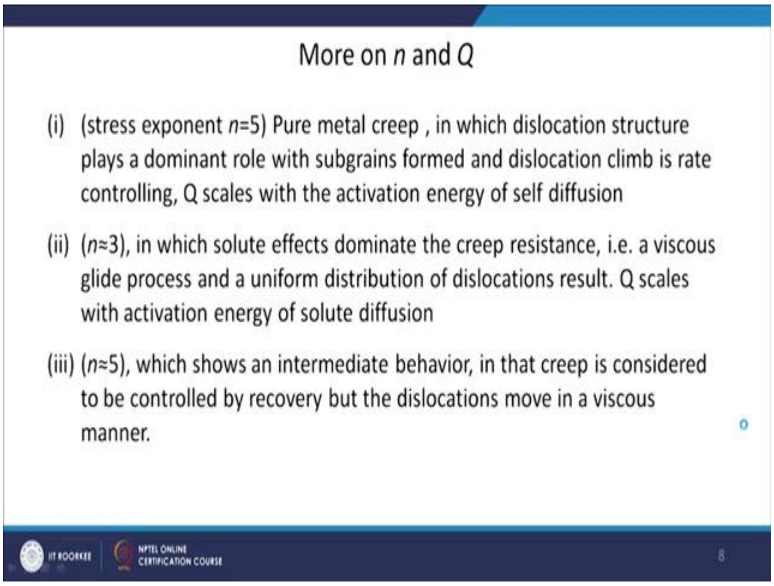

More on *n* and *Q*

(i) (stress exponent n=5) Pure metal creep , in which dislocation structure plays a dominant role with subgrains formed and dislocation climb is rate controlling, Q scales with the activation energy of self diffusion

(ii) (n≈3), in which solute effects dominate the creep resistance, i.e. a viscous glide process and a uniform distribution of dislocations result. Q scales with activation energy of solute diffusion

(iii) (n≈5), which shows an intermediate behavior, in that creep is considered to be controlled by recovery but the dislocations move in a viscous manner.

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More on n and Q (refer to above figure). Just to clarify this more because this way you will keep calculating whenever you are developing constitutive equation. So, stress is exponent of around 5 this is usually you will see in pure metal creep in which dislocation structure plays dominant role with sub grains formed and dislocation climb is rate controlling. So, in dislocation creep which takes place at relatively higher stresses usual your processes will takes place which also takes place during hot deformation that dislocation will generate, they will kind of realign, rearrange through climbing process into sub grains forming low angle grain boundaries. And because they are getting recovered through this rearrangement of dislocation through dislocation climb, the dislocation climb is the rate controlling process.

So, whenever we develop constitutive equation, we are also trying to understand which is the rate controlling process here that means which determines the kinetics of the process. And if you see Q here it will be very close to the self-diffusion of that particular material whatever is the matrix material. If it is aluminium alloy then Q will be close to the activation energy for self-diffusion of aluminium which is around 140 kilo Joule per mole.

Self-diffusion means aluminium atom will self-diffuse and what you need for self-diffusion is vacancies and atomic migration. So, the activation energy will be dependent on the creation of vacancies plus activation barrier for migration. So, when you calculate Q from as we have seen earlier that how I can calculate Q from the data if it is coming close to the activation energy for self-diffusion then we can say that it is a climb controlled process.

Why we say are saying that because you need vacancies for climbing also. So, when vacancy is diffusing of course atoms are also diffusing in the opposite direction, so these vacancies will be required for climbing process. So, vacancy diffusion will be related to the self-diffusion of the material. So, if climb is controlling then Q should be equal to the activation energy should be equal to the activation energy of self-diffusion.

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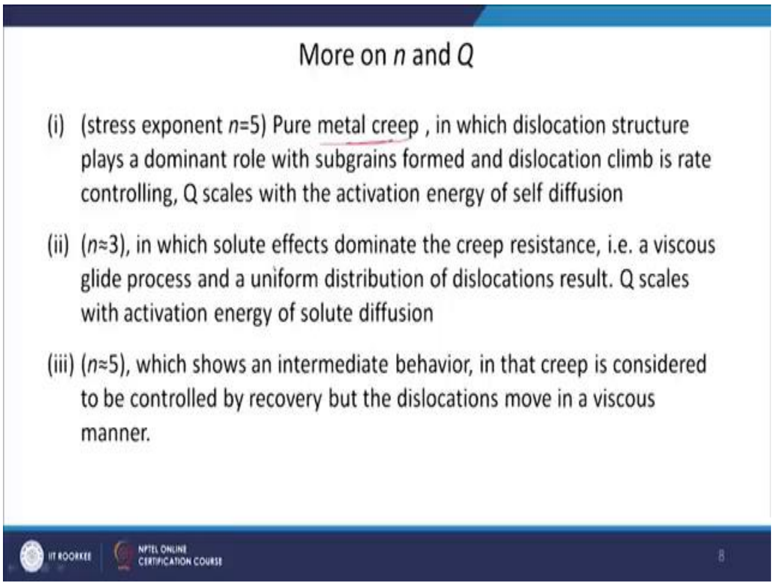

More on n and Q

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If n = 3 which is the case the solute effect dominates. You know that when we are developing alloys there will be some solute atom inside the material. And what this solute atom do is they go and collect in the core of the dislocation. I do not know whether you know that let say simple example of edge dislocation which I can draw like this is my extra half plane (refer to above figure).

So, because you have an extra half plane, you have compressive stresses here and tensile stresses below the slip plane. So, if you have solute atom in the material what they try to do is at high temperature solute will also be able to diffuse. So, what they like to do is if the solute atom is of bigger size than the matrix then those solute atoms will go and sit where the tensile stresses are there or tensile strain are there.

Because the bonds are stretch their bigger atom can go and easily sit applying the compressive stresses now and this tensile compressive will balance out. Whereas if the solute atom is of a smaller size it will like to sit where the compressive stresses are there because already atoms are compressed, a smaller atom will help them to reduce the strain energy. So, a smaller atom will go and sit in the upper side (refer to above figure).

So, a smaller solute atom will sit here a bigger solute atom will sit in the tensile side what it does when this solute go and sit in the dislocation core. Now, if suppose you are applying stress it wants to move it has to carry all this solute atom also with the dislocation. Sometime if you are applying higher stresses dislocation velocity is sufficiently high then sometime it can also leave this solute atom and will be free to go.

But sometime if the stresses are low it has to carry all this solute atom also with it so the stresses to move or dislocation will be now different because your it has to carry this extra load. So, in those cases you have n ~ 3.
when the solute effect dominates the creep resistance. A viscous glide process and a uniform distribution of dislocation result Q scales with the activation energy of solute diffusion.

Because how fast solute will be able to diffuse that decide how the dislocation is going to move and that decides the creep rate. If n = 5 which shows an intermediate behaviour in that creep is considered to be controlled by a recovery but the dislocation moves in a viscous manner. So, for different n and different Q, I can kind of take a guess that what will be the dominant deformation mechanics which is taking place in the material.

So, with this we have covered of course it is not related to the hot deformation which is our primary concern here. But the constitutive equation development for high temperature deformation is started with the creep when people were developing it for creep and in the creep I think they were developing from 1950s and 60s if I am not wrong, whereas for high-temperature or hot working it started after 80s or so because only then you started having that kind of equipment to apply a very high strain rate at high temperatures whereas in creep this kind of work was going on for a long time. So, the initial phase of development of constitutive equation during high temperature deformation was taking place in creep. So, by taking that we are able to kind of understand that how to develop a constitutive equation. So now we will go for the constitutive equation of at a higher strain rate, thank you.

Key words - Constitutive equation, Low strain rate, Creep. Dislocation.