Thermo-Mechanical and Thermo-Chemical Processes
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Lecture-16
Constitutive Analysis

Hello friends, today we will start with a new topic or new module of five lectures that is on constitutive analysis. So, sometime we say constitutive analysis or you can hear may be term called constitutive equation or constitutive model. So, constitutive model means the basic idea on which you have developed the analysis or the equation. So, this analysis equation model will be used in this particular lecture.

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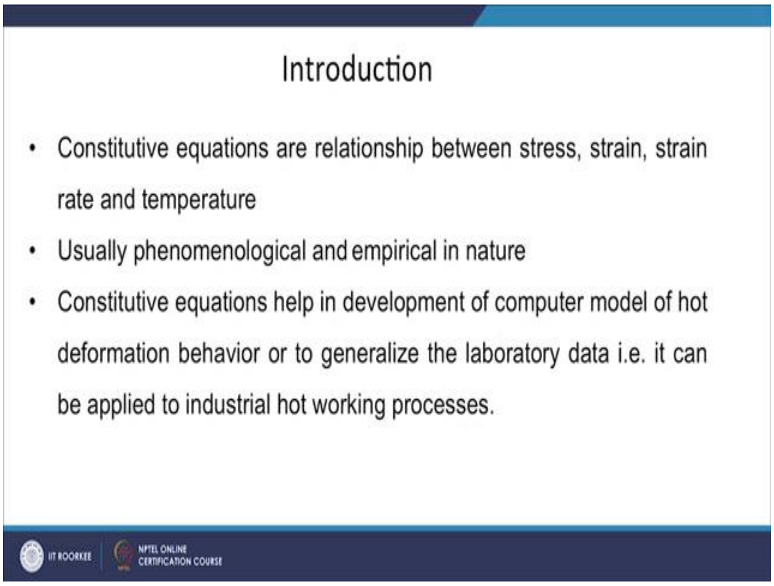

So, what do we mean by constitutive analysis or constitutive equations? So, constitutive equations are relationships basically between, stress, strain, strain rate and temperature. So, it can be between any of the two of these variables, between stress and strain or it can be between a stress and strain rate or it can be between a stress and temperature or you can combine all strain, strain rate, temperature and relate it with the stress.

So, the relationship between the stress and these parameters that is what we call as constitutive equation. So, wherever you hear about constitutive it will be dealing with these parameters. We will see it in more detail there are different type of models, the most popular one is the phenomenological model, we will see what do we mean by a phenomenological and these are empirical in nature because the actual process which is taking place is so complex.

So, you cannot kind of get an equation from the first principle usually in hot deformation especially because of the complexities of the processes which are taking place. So, you are having dislocation generation, you are having dislocation recovery and then you have dislocation glide, dislocation climb and vacancies are there, solutes are there and they are also diffusing. So, there are so many effects which are kind of combining together that making a simplified model or making a first principle model is very difficult process to account all these effects in one equation.

So, what is usually a practised is that you develop empirical relationships. So, constitutive equation when you develop for any material, it will help you to first understand the what kind of metallurgical processes are going on in the material during hot deformation. So, it will tell you about the mechanical behaviour under high temperature deformation. Because we are talking about high temperature deformation it depends on what type of deformation you are doing.

And another thing is it will be able to help you in developing FEM model. So, suppose you want to develop a model for your hot rolling or extrusion or forging process. So, to develop that model, you need this constitutive equation to tell the model that how the material is going to behave under these conditions.

So, you will apply the boundary condition and mechanics on the model by putting constraint. This is how the deformation is going on, but you also have to put in that how the material is going to behave under those imposed condition. And that will be told by the constitutive equation so, the constitutive equation has to go at the appropriate place for proper modelling. So, depending upon how you well you are developing the constitutive equation your modelling will be of that level, that it will be able to predict all the stresses, the strain and strain rate during the deformation accurately if you have an accurate constitutive model.

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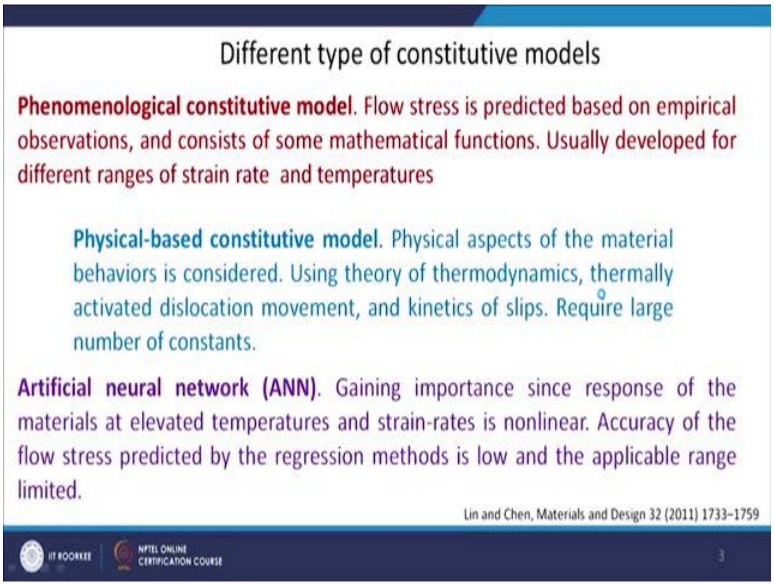

Now, there are different type of constitutive models as I was telling you, Phenomenological is very popular, in this flow stress is predicted based on empirical observations. As the name suggests, it is kind of you understand the phenomena and applying some logic you are using those ideas to develop an empirical relationship. So, which parameters to consider, which parameter to take in the numerator or denominator that will be decided by the phenomenology or phenomena which is taking place.

And that will give you a clue that what type of equation has to be developed. So, by doing the empirical the analysis, you have to find out that what is the relationship between different parameters, so, that when you put it you will be able to develop an empirical relationship between them and you will get ultimately a some mathematical function. And of course, it is usually developed for different temperature ranges and the strain rate ranges because the material behaviour changes a function of a strain rate or temperature.

At lower temperature you can see that only you have dislocation glide as the deformation mechanism, as you go to high temperature you also have dislocation climb and then the diffusion, self-diffusion and solute diffusion and so on. So, your material behaviour changes, so, you develop the constitutive equation for different strains rate range and temperature ranges. Then there is some physical based constitutive model which is based on the actual processes which takes place during the deformation.

So, the physical aspect of the material behaviour is considered for example, dislocation generation or dislocation recovery, how dislocations are directly taking part in the deformation process. So, basically that is what you try to develop through first principle in this case, using theory of thermodynamics, thoroughly activity dislocation movement and kinetics. But because as I told you it is not a very clear picture where you can easily get this equation from first principle.

So, it requires large number of constants to fit the equation or to get your required constitutive equation. Then now, nowadays a very important technique which is coming up is called artificial neural network, ANN models. The importance of ANN model is that you do not have to understand the actual metallurgical process. So, basically, you have some input data and you have some output data. So, input data will be in form of strain rate and temperature for example, and our put is in the form of a stress.

So, you have these parameters, these are input and you have some output. So, basically you train the ANN model using this data and then you try to predict that at any other condition what will be the stress. So, the ANN models are also gaining a lot of importance because when you develop this phenomenological or physical based constitutive model the problem is that all your response in terms of stress is nonlinearly dependent on the temperature and strain rate.

Whenever we develop empirical equation we try to make a graph between the response and the input parameter so that we get some linear curve because when you have linear curve or straight line, it is easier for me to do a regression analysis and get an empirical equation. If it is a non-linear function then it is a very difficult one to find out whether it is a exponential curve or parabolic curve or hyperbolic curve or so and so on.

But in a straight line, using best fit line I can find out whether how much this straight line is close to the actual data. So, you want to have some linear relationship whenever you develop the empirical relationships, but our stress and the response is actually nonlinear as a function of temperature and strain rate that is why developing these empirical relationships are difficult and you have so many models to do that.

So, ANN helps there, because it is nt worried about all these problems i.e. non-linearity, it fits lot of polynomial equation between these parameters, and do something in that black box, which we call ANN model and gives you give you the output. Only problem with ANN model is that you would not be able to understand the material behaviour from that and why that is importan, because when you are designing a material, if I don't know the material behaviour how I am going to design.

So, that is the only problem with the ANN model that I would not be able to get a clear picture about the material behaviour that how material is behaving or what parameter is controlling the response that will be very difficult for me to understand from ANN model. So, I won’t to get a very good first-hand experience of the material however, you have very good accuracy of flow stress predicted by these models.

Whereas with regression analysis; developing this empirical relationship you have lower accuracy. Now, to give you a feel of constitutive equation, one of the most familiar constitutive equations you must have come across is Hooks law. So, that is also constitutive equation just to take the fear out of this particular term. So, basically in that what we do is we related stress with the strain, very simple equation.

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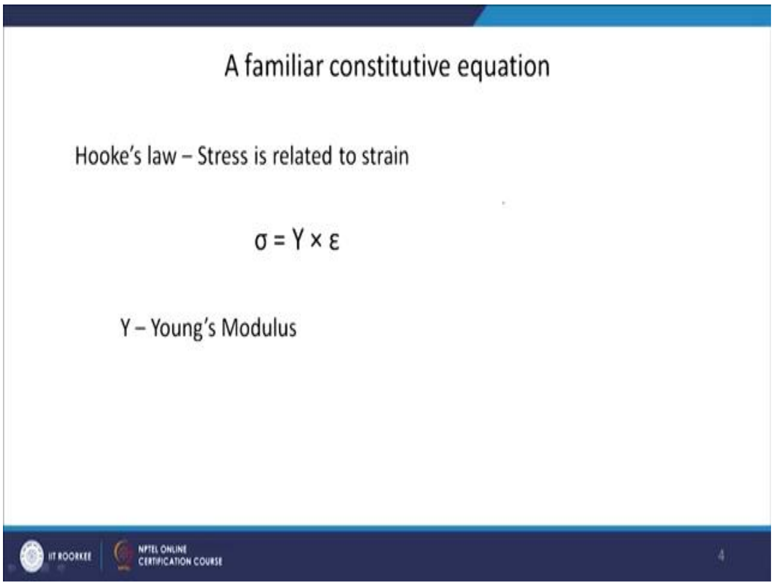

And basically, you can see that it is already linear, because when you have plot of stress and strain in the elastic regime, it is a straight line. Linear behaviour in the elastic regime and the slope of this particular line gives you the Young’s modulus. Very simple equation and it is already linear, because the curve starts from zero you do not have a constant here.

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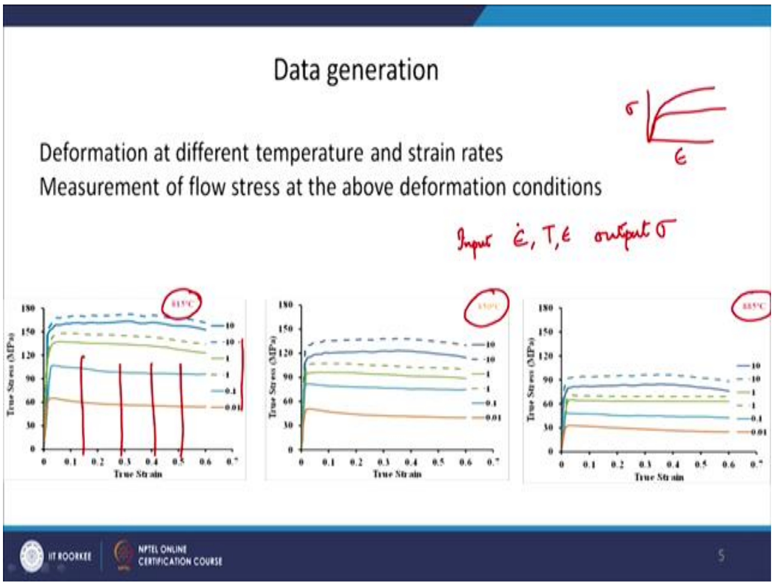

Now coming to hot deformation, first process which you do is to generate data. So, whether you do phenomenological modelling or if you do physical based modelling or you do ANN modelling first you have to find out that what is the response of the material to different parameters. So, the first thing which you do is generate data and if you generate good quality of data, your model will be that much accurate.

So, you should pay your utmost attention during this process i.e. data generation. You should not take any shortcut here, do good experiments. If any in any experiment suppose you are not sure about that the deformation was progress properly do another one because the amount of work which you put in here, that will kind of reflect in your constitutive equation.

So, first thing is always to put maximum attention and maximum care to develop or to get the data for your constitutive equation. So, in high temperature deformation, there are two very important parameters. These are strain rate and temperature. So, you have these two as the main input. And of course, output is your stress. Why we do not consider strain because if you see in high temperature deformation, if it is dynamic recrystallization also, if it is only having one single peak that you get a steady state condition.

That means the stress is not changing much with the strain. For example, you can see in these different curves at different strain levels more or less stress is constant. So, you have reached a steady state. If in your flow curve you also are seeing maybe a strain hardening part. So, it is something like this (refer to above figure) for example then you should also consider strain in this term. Because your flow stress is continuously changing as a function of strain, but if it is a steady state then I would be mainly interested in strain rate and temperature.

This you should understand that why is most of the time you do not have a strain in this constitutive equation when you are developing for hot deformation. Similarly, if you develop for low temperature, temperature below 0.4 of melting point you will see that strain rate is not there because the strain rate dependence of stress reduces as you go to lower temperatures. So, you will see this effect that constitutive equations are changing as a function of temperature.

So, what do you do in this data generation, as you can see, you are deforming at different temperatures and at different strain rates. So, you are generating data like that deformation at different strain rate and different temperature and then you will use this input to develop the constitutive equation.

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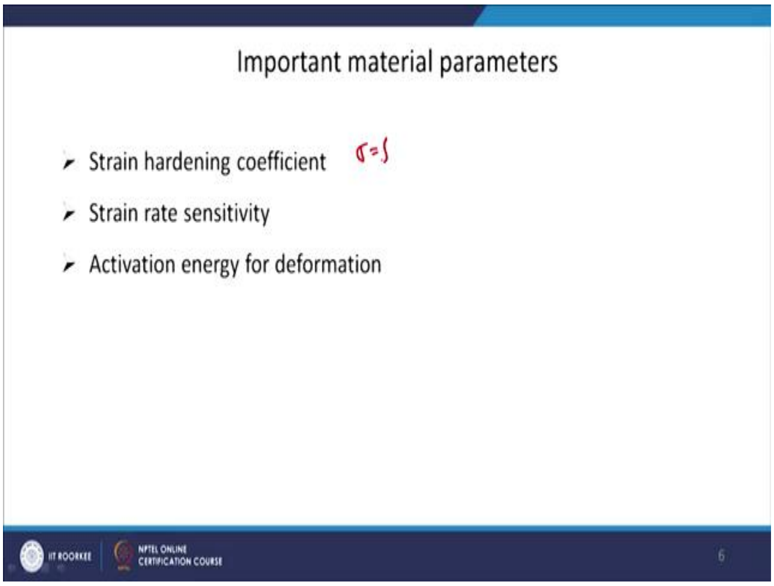

Now, what important material parameters you will be trying to find out here is basically, if your stress is function of strain then a strain hardening coefficient, if your stress is function of strain rate then your strain rate sensitivity, if your stress is function of temperature activation energy for deformation. So, these three are very important material parameter which you want to find out when you are developing the constitutive equation and these material parameters actually gives you some indication about how the material is behaving under the deformation conditions.

So, I will tell you one by one that how you can get these parameters from the data in a simplified way later on, right now, I am just giving you the sense of these parameters by doing an analysis.

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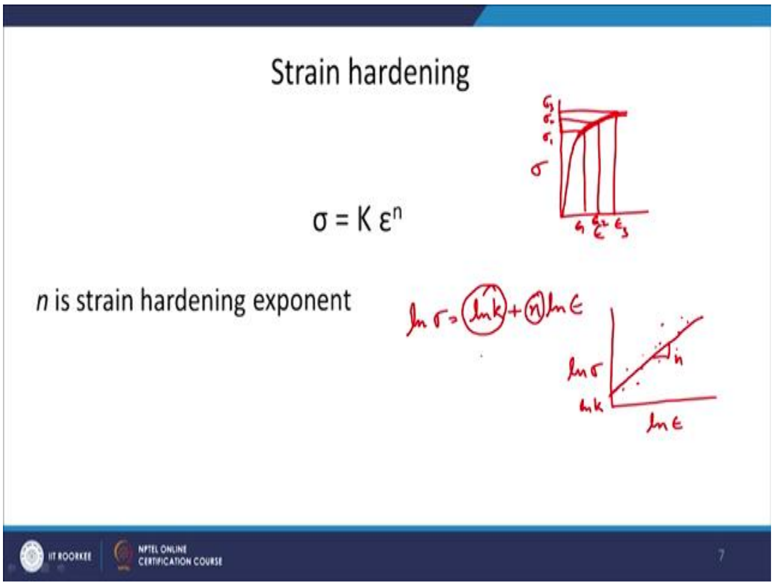

So, the strain hardening, I think we must have seen earlier also that stress is a function of strain. and there is an exponent on the strain term. So, it has some exponential type of behaviour. So, if you remember we just do it earlier also that this (refer to above figure) is my the linear part after yielding it will have something like this. So, basically, this n is the slope of this exponential curve, some kind of exponential curve is this.

And how to find out this exponent n, so, basically, you can find out at different strain levels what is the value of stress. So, you have ε1, ε2, ε3, ϭ1, ϭ2, ϭ3 and so on. So, now, if you see this particular equation if I take logarithmic on both the side the exponent will come down here now it will be something like this (refer to above figure). Now, it becomes a linear equation as I told you that whenever we want to develop empirical equation I want to linearize the particular equation.

So, whatever parameter the k parameter and n parameter I get from the equation, I can easily say with surety ok this is the best fit straight line I am fitting and this is the regression coefficient, how close it is to the to the value of 1 that gives me the confidence in the constitutive equation. So, for example, now, I will be plotting lnε vs lnϭ here and of course, you will get some points like this and then I will fit a straight best fit line to this (refer to above figure).

So, the slope will be equal to n and the intercept will be equal to k. So, of course, it will be log k here (refer to above figure) and you have to take anti log of that. So, this slope will be n and this will be your constant C. So, doing a simple analysis like this I will be able to get the strain hardening coefficient which is n here similarly, you can do for the strain rate sensitivity.

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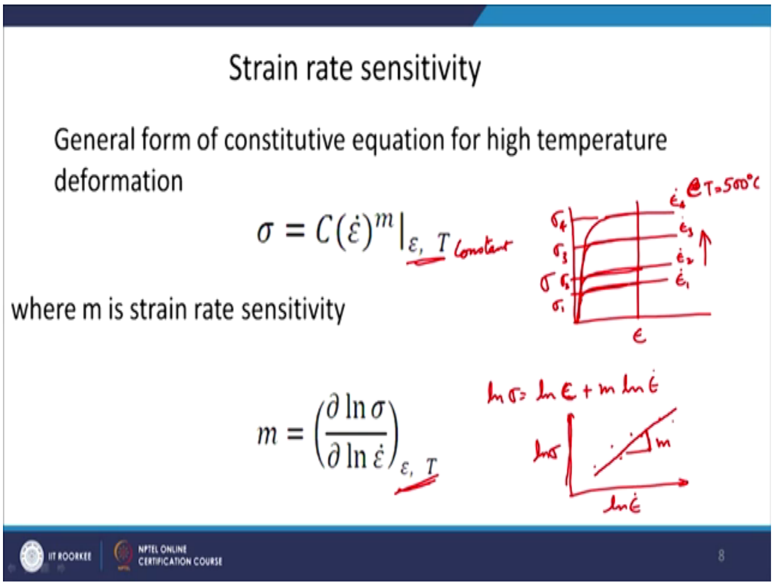

If you see a general form of constitutive equation for high temperature deformation you will see something like this.

$$\sigma = C(\dot{\varepsilon})^m.$$

Whenever you want to find out a particular constant you have to make other input parameters as constant then only you will be able to get this for example, if you want to calculate the strain sensitivity I will keep the strain and temperature as the constant.

So, again you can see that there is some kind of exponential dependence of stress on strain rate. For a fixed temperature, let us say just for argument's sake 500°C, at a temperature of 500°C I am doing test and finding out the; something like this (refer to above figure).

So, in different strain rates I am deforming the material at a fixed temperature. Now, I want to fix my strain also. So, let us say I want to find out the stress value at a particular strain. So, a strain is now constant. And now I will find out what is σ value for differently strain rates σ1, σ2, σ3, σ4.

So, now what I have got for different strain rate and you can see strain rate is increasing in this direction. So, as strain rate is increasing my stress is increasing. So, there is a dependence of stress on the strain rate. And again, I can do the same thing, if I take log on both the side so,

$\ln\sigma = \ln C + m \ln\dot{\varepsilon}$

So, again, I have linearized the equation. And now, taking all these values again, you will get some kind of again data points.

And a best fit line, straight line and the slope will give you the strain rate sensitivity m (refer to above figure). So, as you can see very clearly here, this is basically slope of the curve $\ln\sigma$ and $\ln\dot{\varepsilon}$.
So, so you are differentiating that particular one, so basically you are trying to find out the slope, where your strain and temperatures are constant.

So, this is the way I can calculate the strain rate sensitivity of the material during high temperature deformation.

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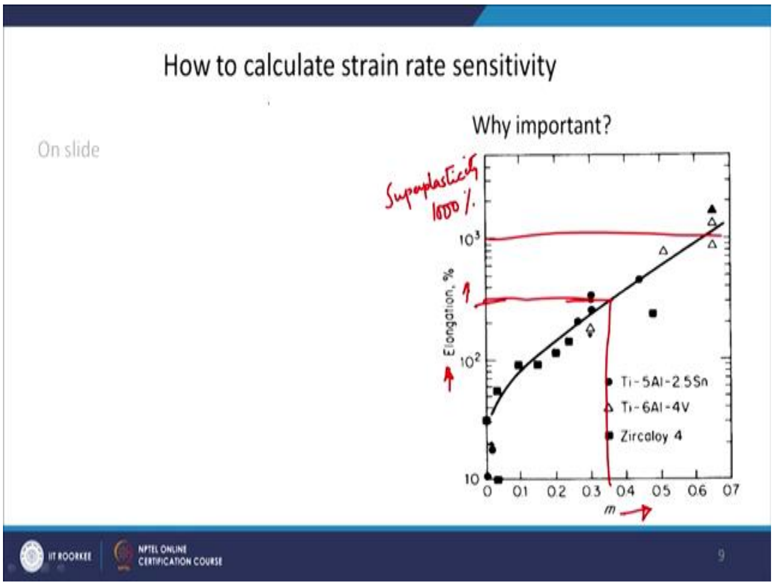

Now, why it is important? If you see this particular curve here that as my m is increasing my elongation is increasing. That means the ductility of the material is increasing as the strain sensitivity is increasing. So, it will be nice for me if I will be able to deform material where the strain rate sensitivity is high. That way I can impose more strain in the material because the ductility is good.

And you can see here for different material they have plotted it. So, as m is increasing from 0.2 to 0.6 you can see that the ductility is increasing and even it is reaching 1000% here, very high ductility and this is where if you remember I told you about super plasticity, this is where you will be able to say with confidence that super-plasticity is there in the material. Basically, anything above 300% I should be able to say it is superplastic.

And if you see around 300% my m is coming around 0.3 to 0.4. And this is what is the definition for super-plasticity that if any material has to behave in a superplastic condition, the strain rate sensitivity should be more than 0.33 or some people say point it should be 0.4. In general it can be above 0.3. So, you can see very clearly around 300 to 400% if I take as a cut off for superplastic deformation or elongation, you will be able to see m is in the range of on .3 to .4 and it should be above that, then you will be able to see super-plasticity in the material. So, this is how I can calculate the strain sensitivity and that is an important parameter during high temperature deformation.

I want to have higher strain rate sensitivity for higher elongation. Another very important parameter is activation energy for high temperature deformation.

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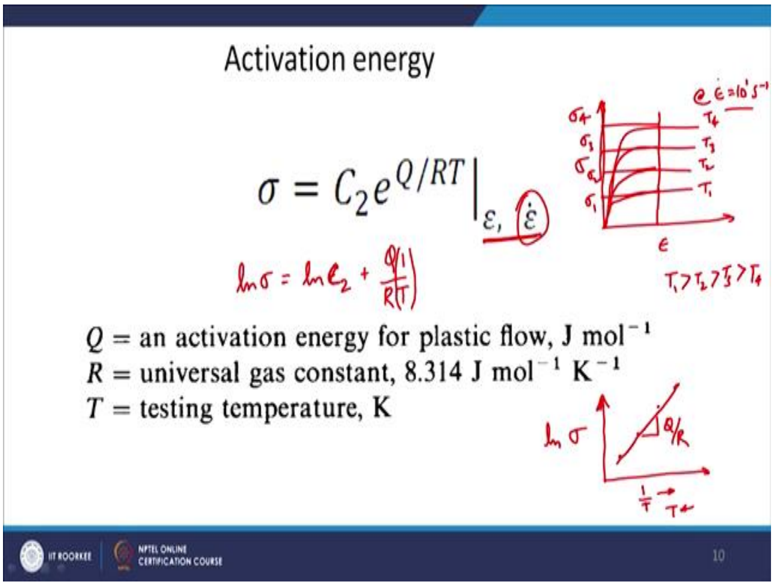

So, now, these are very simplified relationships because we are taking other parameters as constant, but when you develop the whole constitutive equation all this parameter will come together. So, if you see this particular equation here

$$\sigma = C_2 \exp(\frac{Q}{RT}).$$

And it is again you can see that we are keeping few things constant here, the strain and strain rate is constant.

So, for this the data plots which I am going to use will be like this (refer to above figure). So, let us say I am doing it at a strain rate of 10 and I am again kind of getting the stress strain curve for different temperature T_1, T_2, $T_3$and T_4. So, my strain rate is constant this is taken care now and I can do it for a particular strain here (refer to above figure).

So, strain is also constant and for different temperature I will get the stress value σ1, σ2, σ3 and σ4. So, this is how I will be able to calculate now the activation energy. So, if you see that the activation energy of deformation, this function as the temperature is increasing your stress is increasing. So, its temperature is increasing 1/T is decreasing. So, if I want to take $\frac{1}{T}$ here as increasing my temperature is increasing in this direction (refer to above figure).

So, your flow stress has to be low as temperature is increasing. So, it is T_1 is more than T_2 is more than T_3 is more than T_4. So, temperature is more in T_1 and less in T_4 but the stress is higher. So, there is an inverse relationship. So, when temperature is more stress will be low whereas temperature is reducing my stress is increasing. So, it will have some kind of data like this for stress as a function of $\frac{1}{T}$.

So, when temperature is reducing here $\frac{1}{T}$ is increasing here and that is how the stress is changing. This σ I have to take as log because to make this equation linear. What I will be doing, again taking logarithmic on both the side it will be something like this (refer to above figure)

$$\ln\sigma = \ln C_2 + \frac{Q}{RT}$$

So, now, if you see this particular equation, it is a linear equation lnσ is Y, this is your constant C and $\frac{1}{T}$ is your x-axis, so, $\frac{Q}{R}$ will becomes your slope.

So, if I plot a straight line here, now, my $\frac{Q}{R}$ becomes the slope of this equation. R is your universal gas constant and so, whatever value comes you multiply it by R then you will get the Q in joules per mole.

And testing temperature you have to take in Kelvin. Now if you see there are different strain ranges for different processes. So, at high temperature also you have large variation of strain rate where you have different type of processes which can takes place.

So, I am just kind of defining here different type of processes. Because these we will take later on when we will discuss constitutive equation in more detail.

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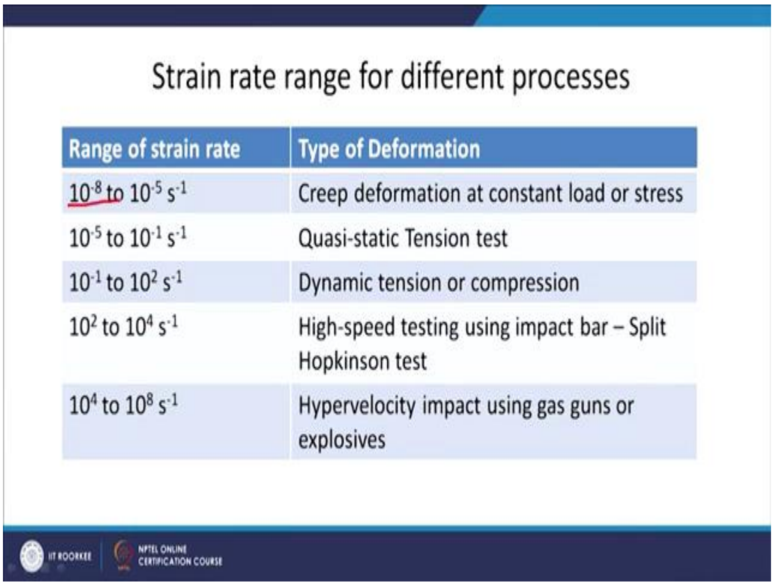
Strain rate range for different processes

Range of strain rate	Type of Deformation
10^{-8} to 10^{-5} s^{-1}	Creep deformation at constant load or stress
10^{-5} to 10^{-1} s^{-1}	Quasi-static Tension test
10^{-1} to 10^{2} s^{-1}	Dynamic tension or compression
10^{2} to 10^{4} s^{-1}	High-speed testing using impact bar – Split Hopkinson test
10^{4} to 10^{8} s^{-1}	Hypervelocity impact using gas guns or explosives

So, the first one in here is that you have at very low strain rate, 10^{-8} to 10^{-5}strain rate at that you get the deformation called creep and that is a very common failure mechanism in any material which is used in high temperature condition. So, at very low flow stresses also under the applied load structure on the structural material if it is at high temperature it keeps deforming as a function of time.

So, it deforms at constant load or constant stress. So creep deformation is a very slow process time dependent process, but material keep deforming under the very small applied stress which can be its own weight or maybe if it is a moving part under the forces of centrifugal forces or something like that keeps deforming and what will happen your dimensions will change after some time for your component and that will lead to the failure of the material.

So, this is one mechanism which takes place it very slow strain rate. Then the next strain rate ranges 10^{-5} to 10^{-1},where you actually do a Quasi-Static Tension test. So, normal tension test which you do in laboratory, these are usually in this strain rate range for example, super plasticity we will be doing in this particular strain rate range.

Or when you do your room temperature tensile tests also, you do at strain rate of maybe 10^{-2}. So, that is why these are called Quasi Static. It is not exactly a static, it is Quasi static. Very slow deformation you are doing. Then comes the another range 10^{-1} to 10^{2}.

Now you are going to high strain rate kind of regime, these are dynamic tension or compression and this is where you do basically hot working also.

So, all your industrial hot working processes thermal mechanical processing are done in this strain range. And this is where I would be doing super plasticity for example. Then you can go even ahead furthermore with 10^2 to 10^4 strain rate, these are high speed testing using impact bar one of the equipment can be what we call it a call as a Split Hopkinson Test bar.

So, basically you keep the sample and you hit it with a with a very heavy rod. So, the strain rates are very high. And the range for that the strain rate is 10^2 to 10^4. Very high strain rate deformations can be possible here. Then you can go even ahead with 10^4 to 10^8 strain rate. These are hypervelocity impact using gas guns or explosives. So, very high impact you can have to get through explosive techniques.

So, basically if you want to characterize an armour plate you should be going for this kind of strain rate that how the material is behaving under the impact condition. So, these are the different stain rate range in which different type of deformation mechanisms are operated or would be operating and as I was telling you in the beginning that empirical equations are usually made for particular strain rate range.

So, these are the ranges. So, I will be developing a different constitutive equation for creep deformation, another constitutive equation for a super plastic deformation, another type of constitutive equation for hot working kind of problem and even another constitutive equation for even higher strain rate range 10^2 to 10^4.

So, for different strain rate range depending upon these what kind of actual deformation mechanism, which is taking place in the material you will be developing different set of constitutive the equation for these strain rate range

So, this is how I can find out the strain rate range. So, thank you with this I have kind of covered the initial part of constitutive equation, what are the important material parameters, strain rate range and what are the different type of constitutive equation phenomenological, physical based and ANN type of modelling and what parameters see you are actually going to find out there, which kind of character is a particular deformation process. Thank you.

Key words - phenomenological physical based or through ANN type of modelling