Thermo-Mechanical and Thermo-Chemical Processes
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Lecture-12
Using Stereographic Projection

Hello friends, so today we will start with how to use stereographic projection to plot the standard stereographic projection of different projection planes. So, before coming to that there are some important points, we should remember i.e. for any plane angle between a plane and it is normal will be always 90°. Obviously, that is why we called it as a normal to that plane.

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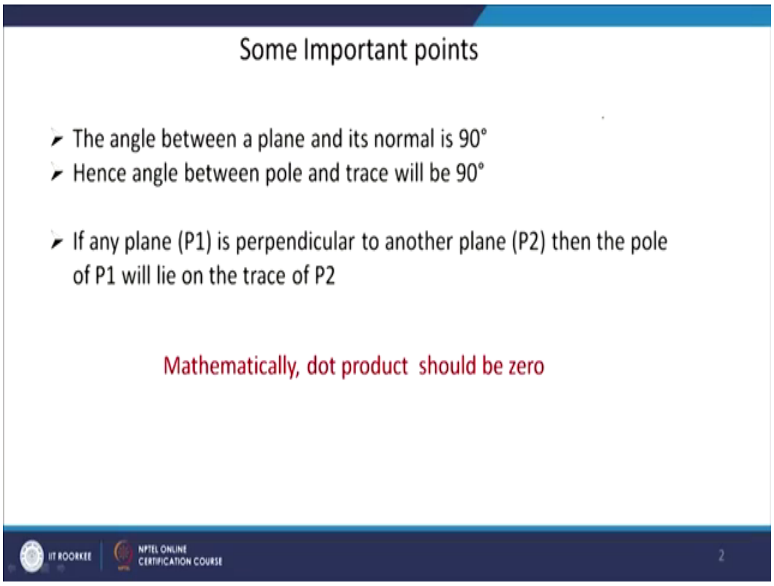

Since, we have already agreed that stereographic projection preserves the angular relationship that means the angle between a pole which we have already projected and a trace which trace of a plane which also we have projected on the equatorial plane, the angle between the pole and the trace of a plane will also be 90°..

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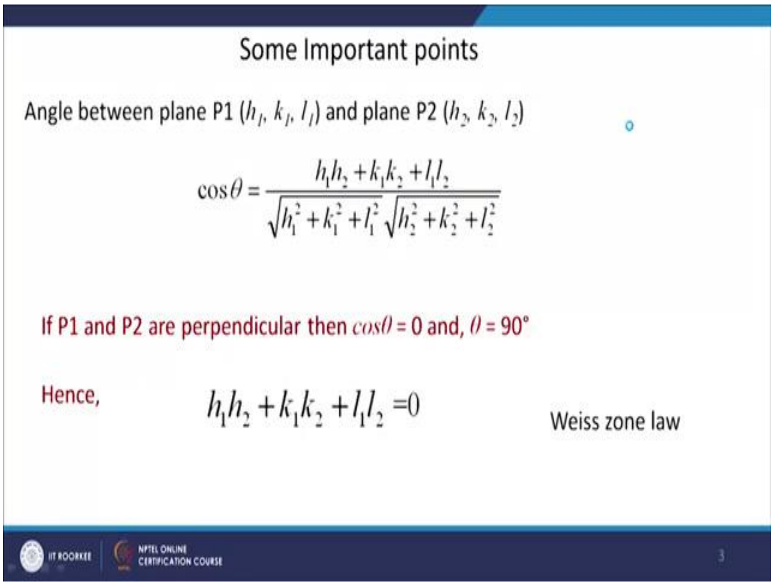

So, mathematically how we can understand this is that the dot product of P1 and P2 should be zero. So, if P1 has the indexing as h_1 k_1 l_1 and P2 as indexing h_2 k_2 l_2 to if they are having a 90° angular relationship then the dot product should be zero. So, that is what is shown here (refer to above figure) also for any two planes, I can always find out that what will be the angle between the two planes. So, if P1 is h_1 k_1 l_1 and P2 is h_2 k_2 l_2 then I can find out the angle between these two planes by a relationship like this which is true for cubic crystals.

Obviously if P1 and P2 are perpendicular then Cos θ = 0.

That is what we know already from trigonometry and theta should be 90°. If that is true if cos θ if I make it zero here (refer to above figure) what will happen this whole thing will be zero and

$$h_1.h_2 + k_1.k_2 + l_1.l_2 = 0$$

that is what I have written it here and that is what we call this Weiss zone law also. So, that h_2 k_2 l_2 is; is in the zone of h_1 k_1 l_1 and their dot product should come as zero.

And that also clear clarify that the pole of h_2 k_2 l_2 plane will be lying on the trace of h_1 k_1 l_1. Now how to use the Wulff net; which we have seen earlier to plot this standard stereographic projection.

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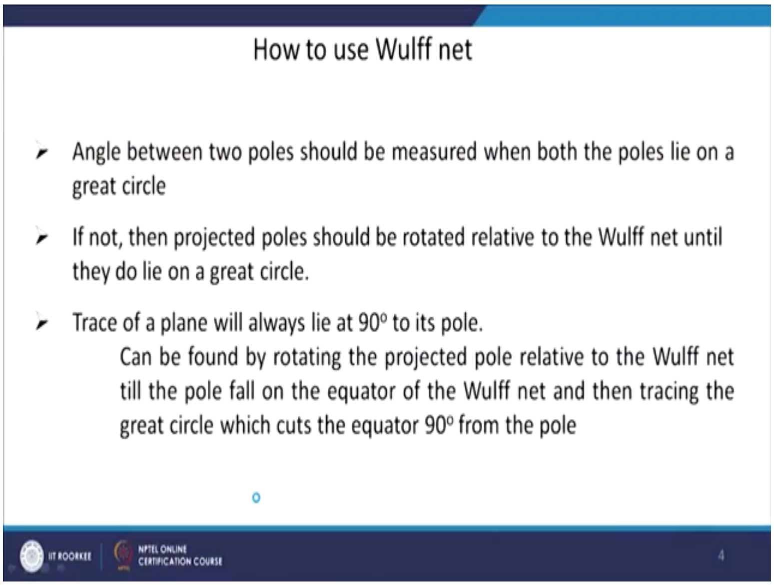

So, whenever you want to measure the angle, we know the two poles that should be measured when both the poles lie on a great circle. Why we want to measure only on great circle because only the great circles have their centre and the centre of the sphere coinciding and that is why when you measure any angle on the great circle it will be at true angle because both the centres are coinciding.

So, if we want to measure angle between any two poles then we have to measure it on the great circle only. If not, suppose if there are two poles such that they are not falling on the great circle I can always rotate the Wulff net or my projection relative to each other. So that is allowed. There is no harm in that I can always do a relative motion between them. And by doing that what I am trying to do is I am bringing both these poles on one of the great circles.

And the third point is that trace of a plane will always lie at 90° to its pole. So already we have discussed that the pole and trace will have a 90° relationship. So, if I want to find projection of trace of a particular plane if I have already a pole, I can easily find out that what will be the traces because I know that it will have a 90° relationship. So, again to do that we will be rotating the projected pole relative to the Wulff net.

And how much rotation will be there till the pole fall on the equator of the Wulff net. So, I have to bring the pole on the equator of our stereographic projection or of the Wulff net and then trace the great circle which cuts the equator 90 °C from the pole.

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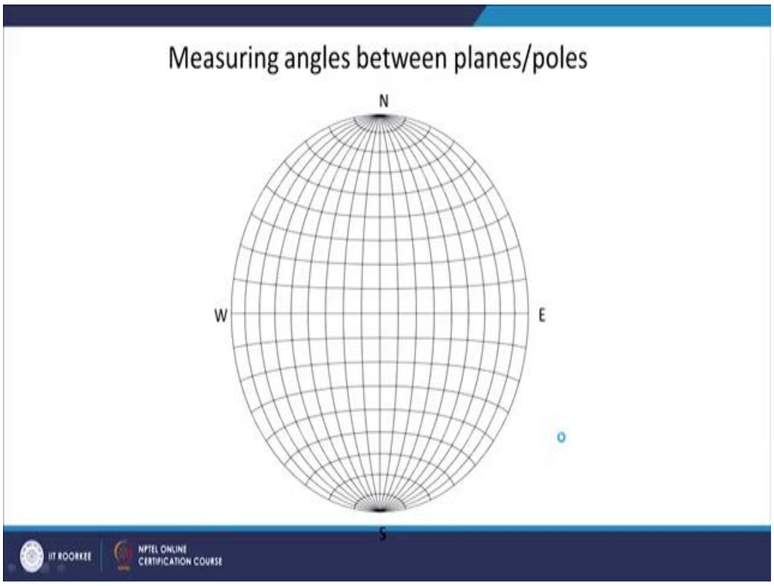

So first measuring the angle between the planes or poles, you have a Wulff net here (refer to above figure). And suppose there are two poles like A and B. Now fortunately A and B both are all lying on the great circle. The great circle here right now is the equator. So, I can directly measure the angle on the great circle. The graduation already we know it is at 10° interval.

So you have 10 20 30 40 50 60, so the angle between A and B is 60° (refer to above figure). So that is what is there. There is another two poles E and F. Again, fortunately they are falling on one great circle here (refer to above figure). So according to rule, which is connecting north and south that will be the great circle and for latitude only the latitude connecting west and east will be great circle, rest of them are small circle. So, I cannot measure the angle on the small circle only on that great circle.

So, in this case now, if you see the graduations here, it is 10 20. So, the angle between E and F poles is 20°.

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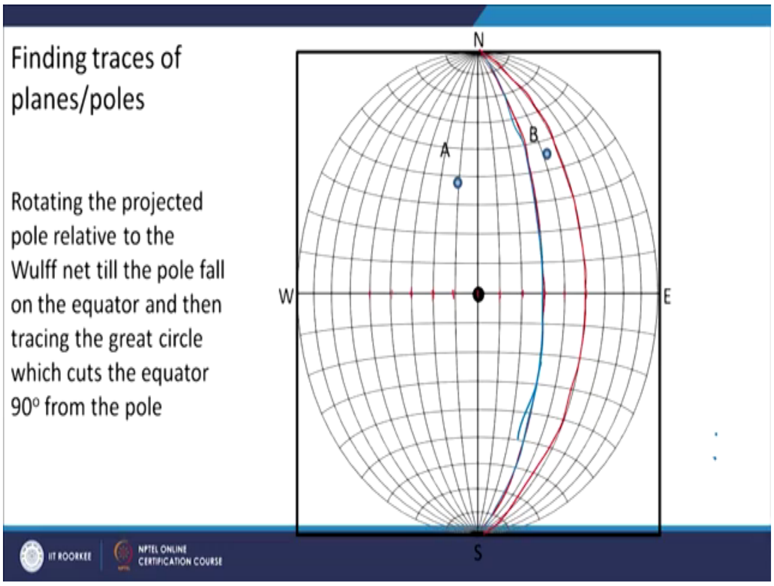

Now the problem starts, in this case now the there are two poles A and B but they are not falling on the great circle (refer to above figure). So, in this case I cannot directly measure the angle. So to measure the angle what I will do, I will rotate Wulff net and the rotation is done such that my both the poles are on particular Great circle.

So, both the poles should be on one single great circle it cannot be on two different circles. So, for example, I have brought these two A and B on one great circle. So, you can see this is particular great circle I have chosen while rotating, I saw that these two poles are falling on this particular great circle. So now I can directly measure the angle between the two so it will be 10 20 30 40 so 40° is the angular the angle between the two poles which are A and B.

And you can then bring it back to the original position. There is no harm because once we have measured the angle, we know what is the angle between the two poles A and B. Now, how to find that trace of the planes? Again, the same poles I am taking A and B. Now, if you remember that rotating the projected pole related to Wulff net, till the pole fall on the equator and then tracing the great circle which cuts the equator 90° from the pole.

So, I am going to do that now. I will rotate this one such that the A comes on the equator. So, is now A is on the equator (refer to above figure). This dark one is our equator. So, this is my equatorial line on which I have brought the A and now I will count the 90° from here.

So, it will be 10 20 30 40 50 60 70 80 and 90 and the 90, I will plot the trace. Whichever this great circle is coming that is my trace that I am tracing (refer to above figure). And next, I will do the same thing with pole B. So, these are our two trace of A and B.

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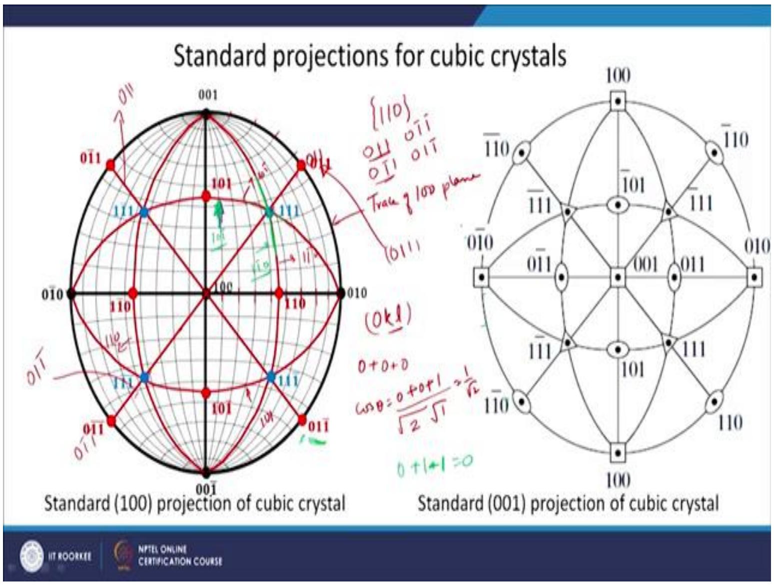

So now we want to plot using this idea that what will be the standard projection for cubic crystals. So, if you look at the standard 100 projection of cubic crystal if I am plotting that means I am taking the unit cell like this and taking this as the 100 plane (refer to slideshow). So, normal is going and hitting the top pole and it is projected. So, it is exactly at the centre. So, once that is at the centre, the other ones will be 001 and 010. So, that will be just 90° to this 001 and 010 (refer to above figure).

And obviously, if there is a 90° relationship between 100 and 001, there is also a 90° relationship between 100 and $00\bar{1}$ (which is just the opposite plane of 001). So, that will be plotted diametrically opposite to 001. So, any negative plane will be diametrically opposite on the Wulff net or on the stereographic projection. So, similarly, for 010 this will be the negative $0\bar{1}0$.

Now, I want to find out the other type of planes which are basically 110 types. So, 110 planes have 45° relationship with 100 planes or 90° relationship. You can understand that this particular periphery of the graphic projection is the trace of 100 plane. So, when I am taking the normal, it is hitting the top pole then if I extend the planes, it will be the equatorial plane which is cutting the sphere at the equatorial plane itself.

So, that will be the trace of 100 plane. So, and we have already seen that the angle between the pole and its trace should be 90° which you can see here also. So, it is 10 20 30 40 50 60 70 80 90, this is the trace of 100 plane. So, if I want to find any poles on the trace of this 100, we have already discussed that if any pole is on the trace of a particular plane, that should have the 90° relationship with this pole.

I take the dot product of between 100 and 001, which you can easily is 0.

So, cos θ is zero or dot product is zero that means it has a 90° relationship you can see it here also. And in general I can tell you if it is a 100 pole at the centre and this is the trace of 100 any plane which is which is a having indexing like 0 k l that means the first index should be 0 and remaining two can be any number that should fall on the trace of it (refer to above figure), because if I take the dot product of this 1×0 is 0 + 0×k will be 0 + 0×l will be 0.

The dot product will be zero.

So, if I want to find any 110 type of plane on this (refer to above figure), I know that any plane which has a dot product with 100 as 0 that will come on this particular trace. So, for example, it can be 011 types, it can be $0\bar{1}1$ type. And if 011 is there, $0\bar{1}1$ can this will also be there and if 011 is there diametrically opposite the negative of this will also be there.

So, we know that between 100 and 011 there is 90° relationship. We can also find out that what will be the relationship with 001 of this 011. So, we know that

$\cos\theta = (1*0 + 0*1 + 0*1)/\sqrt{1}*\sqrt{2}$

So, this is $\frac{1}{\sqrt{2}}$ which is 45°.

So, this will be somewhere here at 45° (refer to above figure). And since this is a great circle I can easily measure the angle directly on this. So, this is 10 20 30 40 45. So, the 011 should be coming somewhere here (refer to above figure). You can also check this by doing a vector addition because this is almost at the centre of these two because 001 and 010 have 90° relationship.

So, if I do a vector addition here. So, 0 + 0 will be 0, 1 + 0 will be 1, 0 + 1 will be 1.

So, vector addition proves that this should be 011. So, either you can find out from angle or you can find out from vector addition and there is another method to find out that what will be the pole at a particular location. So, if the 011 is here just diametrically opposite here if you can see here it will be $0\bar{1}\bar{1}$. Then $0\bar{1}1$ will be as I told you that there can be you can do a simple vector addition 0 + 0 will be 0, $\bar{1}$ + 0 will be $\bar{1}$, 0 + 1 will be 1.

So, this particular one should come somewhere here at 45°. Like this I can plot the 110 planes. Now, some 110 plane will be at 45 °C. So, that will come between the 100 and 001 types. So, for example, between 001 and 100 again doing vector addition you will get a 101 plane and again at 45° somewhere here (refer to above figure).

So, this 011, $01\bar{1}$, $0\bar{1}\bar{1}$, $0\bar{1}1$, just diametrically opposite negative of that (refer to above figure). Then in between 100 and 010 you will have 110 at 45°, between 100 and $0\bar{1}0$ you have $1\bar{1}0$ at 45°. Similarly, 101 and $10\bar{1}$. Now, what these (refer to above figure) lines are, these are the trace of these particular poles. How I can find that, suppose I if I rotate this $01\bar{1}$ such that I bring it here everything else also will rotate it. So, once I bring it here what will be the trace of this $01\bar{1}$ it will be at 90°. So, 10 20 30 40 50 60 70 80 90 and whatever great circle is going which is this is straight line that will be the trace of $01\bar{1}$. Similarly, I can find out the trace for 011 which will be like this and once I have plotted, I can bring it back to its original position. So, then you will have these two traces (refer to above figure).

So, this particular trace is for 011 plane and this particular trace is for $01\bar{1}$ and also true for $0\bar{1}1$, negative also. So, these two are the traces of these two-particular set of planes. Then I can find another trace which is for 110 planes here (refer to above figure). As you can see, it is already at the equator. So, I can directly measure the 90° here. So, from here to here it is 45 °C from here to here it is another 45° so 90° and whichever great circle you can have a smaller graduation also here at 2° interval (refer to above figure).

So, whichever great circle is passing from here that can be traced like this. You can also see whether if this particular pole is falling on the trace of this one that means there should be a 90° relationship between these two poles, which you can easily find out taking the dot product. So, 1× 1 will be 1, plus 1×-1 will be -1 and 0×0 will be 0. So, 1-1 will again be 0.
That means they have a 90° relationship between them. So, this is the traces for $1\bar{1}0$ and this is the trace for 110 (refer to above figure).

Similarly, you can find out the trace for $10\bar{1}$ and 101 by rotating it and bringing 101 first on the equator and tracing whichever the circle is and then bring it back to its original position.
So, this is the two traces for these two planes. So, this is the trace for $10\bar{1}$ and this is the trace for 101. Now, how to find 111 planes here. One way is to find out that what is the angle between 100 and 111 which you can easily find out by the cos θ relationship. The other way is that you can also find out from Vector addition, it is somewhere in between the 100 and 011.

So, you can see that it is a simply a vector addition of 1+0 is 1, 0+1 is 1, and 0+1 is 1.
This is 111. There is another way to find out that whether this is a 111 plane or not by knowing that whether it is falling on which trace. So, as you can see that 111 plane is falling on 3 traces here. We can take any two here. So, for example, one trace is this particular one let me change the colour a little bit so, that it can be easily seen. So, one traces this one for $10\bar{1}$, on which this 111 plane is coming.

And there is another trace this one on which 111 plane is coming and this trace is for $1\bar{1}0$. So, now, how I can use this information? Since I have told you if it is coming on the trace of a particular pole or plane, then it should have a 90° relationship with that pole. So, 111 should have a 90° relationship with $1\bar{1}0$ as well as $10\bar{1}$.

I can write two simultaneous equations here. So, using the dot product rule, suppose I do not know the 111 here right now. So, I am taking it as hkl. So, it will be
$1\times h + 0\times k - 1\times l = 0$
Similarly, I can write for $1\bar{1}0$
$1\times h - 1\times k + 0\times l = 0$
So, the first equation becomes $h - l = 0$.
and second equation becomes $h - k = 0$.
So, that means $h = l$ and $h = k$.
that means, $h = k = l$.
that means all the three indexes are same. So, there should be some of some type of 111 222 333 type of plane. Since we can also do this vector addition and find out that this actually is 111 plane. And also you can ensure that weather this is a 111 plane because this is also falling on these particular trace.

So, the pole 111 should have a 90° relationship with the $01\bar{1}$ also. So, you can check it

$0\times1 + 1\times1 - 1\times1$

So, this is coming as 0. So, whatever indexing we have done is correct. So, like that I can do indexing of other planes also. So, now, you can see that all these planes are indexed now, and now we are comparing it with a standard stereographic projection which you may see in any book. This is how it will look (refer to above figure).

Only slight differences there that we have plotted here as 100 projection whereas, in this case it is a 001 projection on the that changes so, slightly the poles will change, but rest of the things will remain same the angular relationship will be will be consistent in both the projections. Now, similarly, I can draw the standard stereographic projection also.

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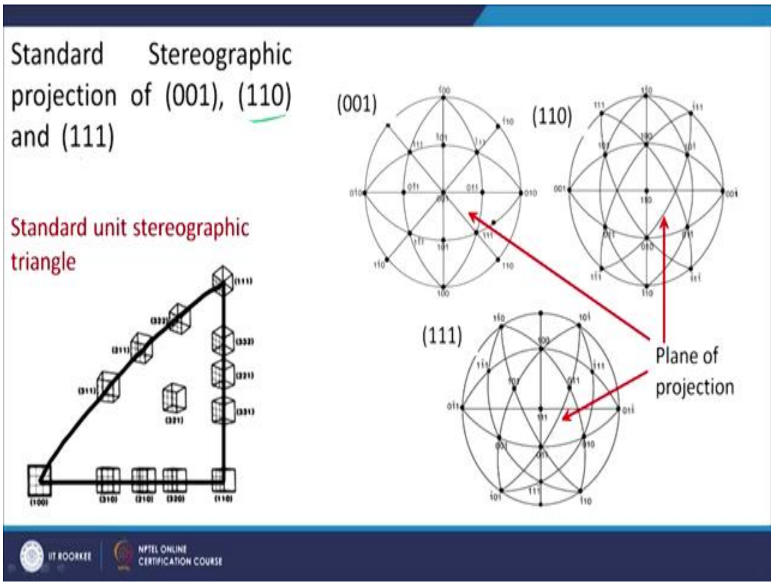

For example, for 110 planes or 111 plane. In that case, my unit cell will be like this. So, this is the 110 plane in that case. So, its normal is going and hitting the top pole so, it is coming exactly at the centre and then the rest of the relationship you can easily find out. So, if the position is like this or maybe let us makes it like this (refer to slideshow).

Then you can see that the 100 planes are going and hitting the sphere at the equatorial plane itself. This 100 is going towards in another direction, which is kind of making 45° with this one. So, you can see that only one set of planes are directly going and hitting this sphere whereas other 100 plane are coming at 45° with the 110. So, like that you can plot what we have done already you can do it for 110 or if you are interested in 111 you can draw a standard is stereographic projection of 111.

And you can also show the same relationship in a standard unit stereographic triangle which has basically a triangle like this (refer to above figure). So, you have 001, 101, 111, 100, 110 and 111. So, these are interchangeable nothing to worry about here and the other orientations you can show in between them. So, between any 110 and 111 type, you can again do a vector addition so 1+1 is 2, 1+1 is 2 and 0+1 is 1 so 221.

So, either it can be 221 type or 331 type or 332 type and so on depending upon what is the location of the of the pole. Whether it is closer to 111 or away from it or closer to 110. Similarly, between 100 and 110 if you do a vector addition it will be 1+1 is 2, 1+0 is 1 and 0+0 is 0.
So, exactly at the centre it will be 210 or it can be multiple of that 310 or 320 and so on. So, this is a standard stereographic triangle.

And because of the symmetry of cubic crystal whatever is there in this one triangle that will be repeated on other triangles. So, you can depict the whole thing in one standard triangle instead of showing it in the complete stereographic projection space. With this, I hope the you might have got some understanding of using stereographic projection. Now we will use these ideas in plotting the pole figures. Thank you.
Keywords – Stereographic projection, Wulff net.